

How Small is Small?

Non-linearities in Heterogeneous Agent Models

Javier Bianchi¹ Greg Kaplan²

¹Federal Reserve Bank of Minneapolis

²University of Chicago, e61 Institute & NBER

OVERVIEW

- Heterogeneous-agent models are a workhorse framework for policy analysis
 - ▶ Match the level and distribution of MPCs in the data

OVERVIEW

- Heterogeneous-agent models are a workhorse framework for policy analysis
 - ▶ Match the level and distribution of MPCs in the data
- MPC heterogeneity generates departures from Ricardian equivalence
 - ▶ These departures drive the aggregate effects of fiscal transfers

OVERVIEW

- Heterogeneous-agent models are a workhorse framework for policy analysis
 - ▶ Match the level and distribution of MPCs in the data
- MPC heterogeneity generates departures from Ricardian equivalence
 - ▶ These departures drive the aggregate effects of fiscal transfers
- Most common approach in the literature: first-order local methods
 - ▶ Sequence-space Jacobian (SSJ), Auclert, Bardóczy, Rognlie, and Straub (ECMA 2021)

OVERVIEW

- Heterogeneous-agent models are a workhorse framework for policy analysis
 - ▶ Match the level and distribution of MPCs in the data
 - MPC heterogeneity generates departures from Ricardian equivalence
 - ▶ These departures drive the aggregate effects of fiscal transfers
 - Most common approach in the literature: first-order local methods
 - ▶ Sequence-space Jacobian (SSJ), Auclert, Bardóczy, Rognlie, and Straub (ECMA 2021)
- ↪ Well understood: accurate for *small enough* shocks

OVERVIEW

- Heterogeneous-agent models are a workhorse framework for policy analysis
 - ▶ Match the level and distribution of MPCs in the data
 - MPC heterogeneity generates departures from Ricardian equivalence
 - ▶ These departures drive the aggregate effects of fiscal transfers
 - Most common approach in the literature: first-order local methods
 - ▶ Sequence-space Jacobian (SSJ), Auclert, Bardóczy, Rognlie, and Straub (ECMA 2021)
- ↪ Well understood: accurate for *small enough* shocks

- BUT: How small is small?

OVERVIEW

- Heterogeneous-agent models are a workhorse framework for policy analysis
 - ▶ Match the level and distribution of MPCs in the data
 - MPC heterogeneity generates departures from Ricardian equivalence
 - ▶ These departures drive the aggregate effects of fiscal transfers
 - Most common approach in the literature: first-order local methods
 - ▶ Sequence-space Jacobian (SSJ), Auclert, Bardóczy, Rognlie, and Straub (ECMA 2021)
- ↪ Well understood: accurate for *small enough* shocks

- BUT: How small is small?
- Our answer: for fiscal transfers, “small” must be *very small*

OVERVIEW (CTD)

- In calibrated HA models, MPCs are **highly non-linear** in wealth
 - ▶ MPCs fall sharply away from borrowing constraints

OVERVIEW (CTD)

- In calibrated HA models, MPCs are **highly non-linear** in wealth
 - ▶ MPCs fall sharply away from borrowing constraints
- Transfers shift households out of high-MPC regions
 - ⇒ Aggregate consumption response is **concave in the size of transfers**

OVERVIEW (CTD)

- In calibrated HA models, MPCs are **highly non-linear** in wealth
 - ▶ MPCs fall sharply away from borrowing constraints
- Transfers shift households out of high-MPC regions
 - ⇒ Aggregate consumption response is **concave in the size of transfers**
- Linear methods: **substantially overstate** response to transfers for small shocks

OVERVIEW (CTD)

- In calibrated HA models, MPCs are **highly non-linear** in wealth
 - ▶ MPCs fall sharply away from borrowing constraints
- Transfers shift households out of high-MPC regions
 - ⇒ Aggregate consumption response is **concave in the size of transfers**
- Linear methods: **substantially overstate** response to transfers for small shocks
 - ▶ Can be accurate for certain shocks. But those imply **HA $\approx$ RA**

OVERVIEW (CTD)

- In calibrated HA models, MPCs are **highly non-linear** in wealth
 - ▶ MPCs fall sharply away from borrowing constraints
- Transfers shift households out of high-MPC regions
 - ⇒ Aggregate consumption response is **concave in the size of transfers**
- Linear methods: **substantially overstate** response to transfers for small shocks
 - ▶ Can be accurate for certain shocks. But those imply **HA $\approx$ RA**
 - ↪ Unreliable when responses are driven by failures of Ricardian equivalence

OVERVIEW (CTD)

- In calibrated HA models, MPCs are **highly non-linear** in wealth
 - ▶ MPCs fall sharply away from borrowing constraints
- Transfers shift households out of high-MPC regions
 - ⇒ Aggregate consumption response is **concave in the size of transfers**
- Linear methods: **substantially overstate** response to transfers for small shocks
 - ▶ Can be accurate for certain shocks. But those imply **HA $\approx$ RA**
 - ↪ Unreliable when responses are driven by failures of Ricardian equivalence
- Second-order improves for narrow shocks, but deteriorate quickly

ROADMAP

1. Partial equilibrium environment
2. Fiscal stimulus in general equilibrium
3. Other shocks and results

HOUSEHOLD CONSUMPTION-SAVING PROBLEM

- Continuum of households with idiosyncratic income shocks

$$\max_{\{c_{i,t}, a_{i,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t})$$

$$\text{s.t.} \quad c_{i,t} + a_{i,t} = (1+r)a_{i,t-1} + (1-\tau)y_{i,t} + T_t \quad \text{and} \quad a_{i,t} \geq 0$$

HOUSEHOLD CONSUMPTION-SAVING PROBLEM

- Continuum of households with idiosyncratic income shocks

$$\max_{\{c_{i,t}, a_{i,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t})$$

$$\text{s.t.} \quad c_{i,t} + a_{i,t} = (1+r)a_{i,t-1} + (1-\tau)y_{i,t} + T_t \quad \text{and} \quad a_{i,t} \geq 0$$

Intertemporal Euler equation:

$$u'(c_{i,t}) \geq \beta_i(1+r)\mathbb{E}_t u'(c_{i,t+1}) \quad \text{with equality if } a_{i,t} > 0$$

HOUSEHOLD CONSUMPTION-SAVING PROBLEM

- Continuum of households with idiosyncratic income shocks

$$\max_{\{c_{i,t}, a_{i,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t})$$

$$\text{s.t. } c_{i,t} + a_{i,t} = (1+r)a_{i,t-1} + (1-\tau)y_{i,t} + T_t \quad \text{and} \quad a_{i,t} \geq 0$$

Intertemporal Euler equation:

$$u'(c_{i,t}) \geq \beta_i(1+r)\mathbb{E}_t u'(c_{i,t+1}) \quad \text{with equality if } a_{i,t} > 0$$

Optimization + aggregation $\Rightarrow$ aggregate consumption function:

$$\mathcal{C}_t(\{T_s\})$$

FISCAL TRANSFER POLICY

Starting from steady state, a one-time unanticipated lump-sum transfer at $t = 0$ [MIT-shock]

$$T_0 = T + \Delta, \quad T_t = T \text{ for } t > 0$$

FISCAL TRANSFER POLICY

Starting from steady state, a one-time unanticipated lump-sum transfer at $t = 0$ [MIT-shock]

$$T_0 = T + \Delta, \quad T_t = T \text{ for } t > 0$$

Object of interest. Aggregate consumption response:

$$C_t(\Delta) - C_t(0)$$

MODEL COMPUTATION: NON-LINEAR VS. LINEAR TRANSITIONS

Steady-state $\mathcal{C}_0$: endogenous-grid point method for decision rules + aggregation

MODEL COMPUTATION: NON-LINEAR VS. LINEAR TRANSITIONS

Non-linear

- Use household policy to solve exactly consumption response

$$\mathcal{C}_0(\Delta) = \int_Y \int_{-\infty}^{\infty} c(\tilde{x} + \Delta, y) \, dG(\tilde{x} \mid y) \, dF(y)$$

cash-on-hand before transfer

Linear

- Taylor expansion of aggregate consumption around steady state

$$\mathcal{C}_0(\Delta) \approx \mathcal{C}_0(0) + \mathcal{C}'_0(0) \cdot \Delta$$

Slopes computed by finite differences or SSJ toolbox. Then, extrapolate.

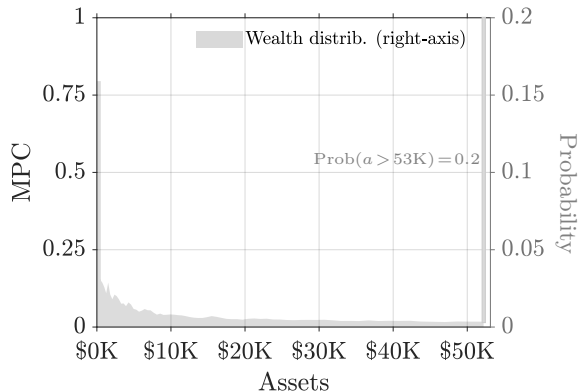
CALIBRATION

- A period = a quarter. $u(c) = \log(c)$
- Earnings process, $y_{i,t}$, calibrated to match second moments of annual earnings (Kaplan and Violante 2022)
- Interest rate 2% annual
- Tax rate $(\tau) = 27.4\%$
- Steady-state transfers = 15% of GDP ($\approx \$25\text{K}/\text{yr}$)
- Heterogeneous discount factors:

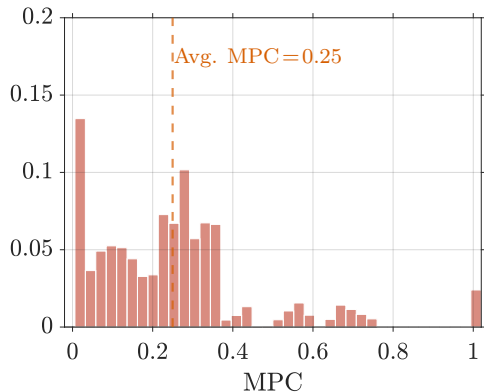
$$\left. \begin{array}{l} \text{Mean wealth / annual GDP} = 100\% (\approx \$166\text{K}) \\ \text{Avg. quarterly MPC (out of \$1,000)} = 25\% \end{array} \right\} \Rightarrow \bar{\beta} = 0.972, \sigma_{\beta} = 0.010$$

STEADY-STATE WEALTH DISTRIBUTION AND MPCs

(a) Wealth distribution

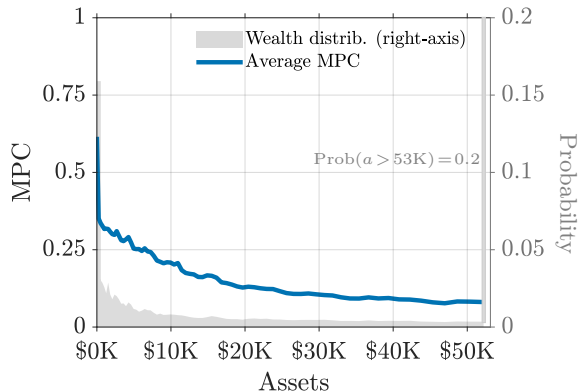


(b) MPC distribution

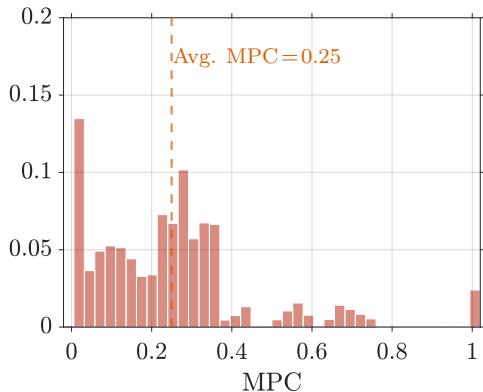


STEADY-STATE WEALTH DISTRIBUTION AND MPCs

(a) Wealth distribution and MPCs

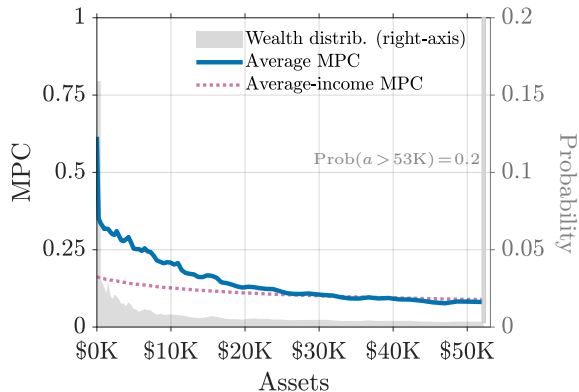


(b) MPC distribution

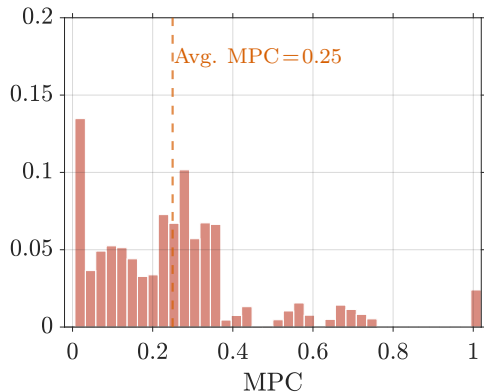


STEADY-STATE WEALTH DISTRIBUTION AND MPCs

(a) Wealth distribution and MPCs

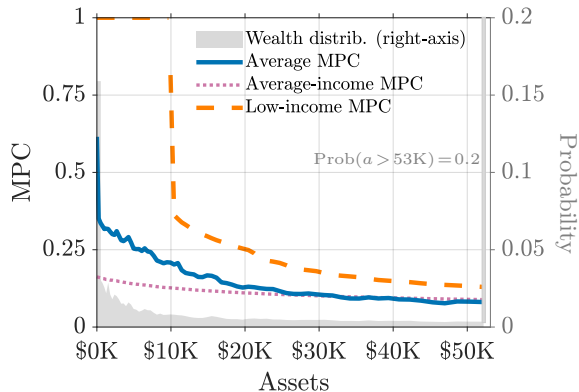


(b) MPC distribution

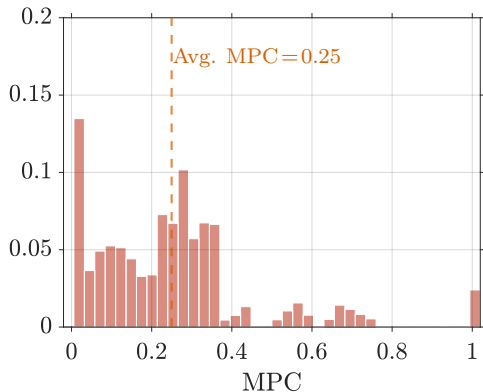


STEADY-STATE WEALTH DISTRIBUTION AND MPCs

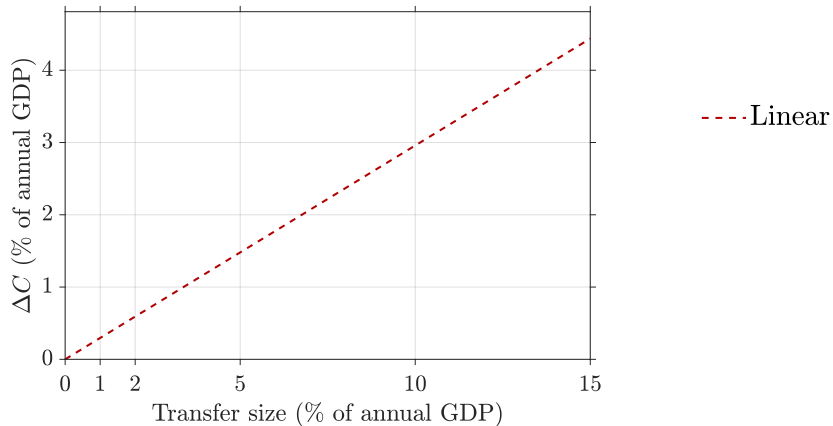
(a) Wealth distribution and MPCs



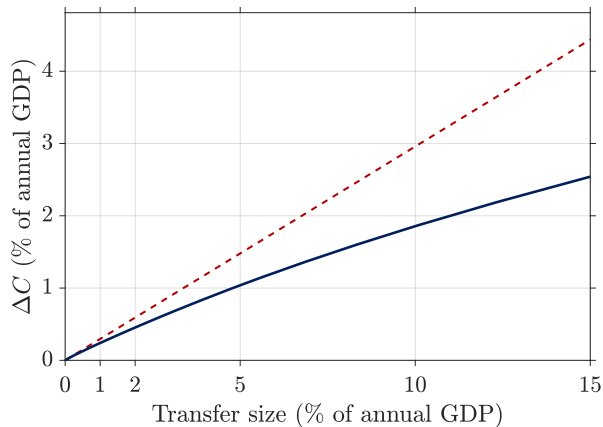
(b) MPC distribution



AGGREGATE CONSUMPTION RESPONSE



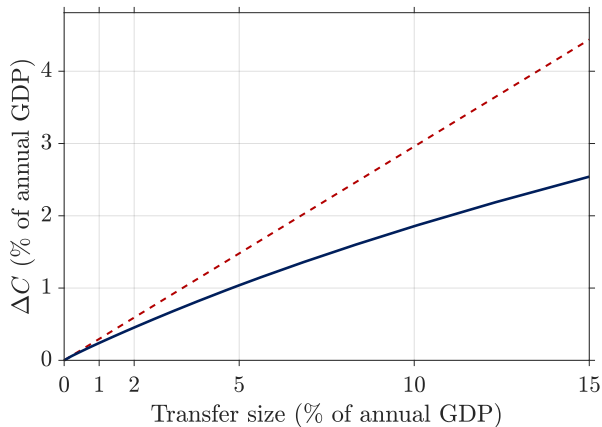
AGGREGATE CONSUMPTION RESPONSE



--- Linear
— Non-linear

- Non-linear solution is concave

AGGREGATE CONSUMPTION RESPONSE



- Linear
- Non-linear
- Non-linear solution is concave
- Linear overstates response:
 - ▶ At $\Delta = 5\%$ of GDP ($\approx \$8K$):
overstates by 42%
 - ▶ At $\Delta = 0.5\%$: overstates by 17%

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

- Concavity of the individual consumption function
- Households moving on and off the borrowing constraint

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$\mathcal{C}_0(\Delta) = \int_Y \int_{-\infty}^{\infty} c(\tilde{x} + \Delta, y) \, dG(\tilde{x} \mid y) \, dF(y)$$

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$\mathcal{C}_0(\Delta) = \int_Y \int_{-\infty}^{\infty} c(\tilde{x} + \Delta, y) \, dG(\tilde{x} \mid y) \, dF(y)$$

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$\mathcal{C}_0(\Delta) = \int_Y \int_{-\infty}^{\infty} c(\tilde{x} + \Delta, y) \, dG(\tilde{x} \mid y) \, dF(y)$$

$\downarrow$
Initial distribution (before transfers)

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$\mathcal{C}_0(\Delta) = \int_Y \int_{-\infty}^{\infty} c(\tilde{x} + \Delta, y) \, dG(\tilde{x} \mid y) \, dF(y)$$

$\downarrow$
Initial distribution (before transfers)

- Denote $\bar{x}(y)$ cash-on-hand at which the borrowing constraint stops binding
 - ▶ Household constrained if $\tilde{x} + \Delta \leq \bar{x}(y)$

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$\mathcal{C}_0(\Delta) = \int_Y \left[\underbrace{\int_{-\infty}^{\bar{x}(y)-\Delta} c(\tilde{x} + \Delta, y) \, dG(\tilde{x} \mid y)}_{\text{constrained}} + \underbrace{\int_{\bar{x}(y)-\Delta}^{\infty} c(\tilde{x} + \Delta, y) \, dG(\tilde{x} \mid y)}_{\text{unconstrained}} \right] dF(y)$$

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$C_0(\Delta) = \int_Y \left[\underbrace{\int_{-\infty}^{\bar{x}(y)-\Delta} c(\tilde{x} + \Delta, y) dG(\tilde{x} | y)}_{\text{constrained}} + \underbrace{\int_{\bar{x}(y)-\Delta}^{\infty} c(\tilde{x} + \Delta, y) dG(\tilde{x} | y)}_{\text{unconstrained}} \right] dF(y)$$

MPC has a discontinuity at $\bar{x}(y)$.

$$c_x(x, y) = \begin{cases} 1, & x < \bar{x}(y) \quad (\text{constrained}), \\ c_x^+(x, y) < 1, & x > \bar{x}(y) \quad (\text{unconstrained}). \end{cases}$$

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$C_0(\Delta) = \int_Y \left[\underbrace{\int_{-\infty}^{\bar{x}(y)-\Delta} c(\tilde{x} + \Delta, y) dG(\tilde{x} | y)}_{\text{constrained}} + \underbrace{\int_{\bar{x}(y)-\Delta}^{\infty} c(\tilde{x} + \Delta, y) dG(\tilde{x} | y)}_{\text{unconstrained}} \right] dF(y)$$

$$C'_0(0) = \bar{\Lambda} \times \underbrace{1}_{\text{MPC}=1} + (1 - \bar{\Lambda}) \times \underbrace{\mathbb{E}[c_x^+ | \tilde{x} > \bar{x}(y)]}_{\text{unconstrained MPC}}$$

where $\bar{\Lambda} \equiv \int_Y G(\bar{x}(y) | y) dF(y)$ is the steady-state constrained share;

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$C_0(\Delta) = \int_Y \left[\underbrace{\int_{-\infty}^{\bar{x}(y)-\Delta} c(\tilde{x} + \Delta, y) dG(\tilde{x} | y)}_{\text{constrained}} + \underbrace{\int_{\bar{x}(y)-\Delta}^{\infty} c(\tilde{x} + \Delta, y) dG(\tilde{x} | y)}_{\text{unconstrained}} \right] dF(y)$$

$$C'_0(0) = \bar{\Lambda} \times \underbrace{1}_{\text{MPC}=1} + (1 - \bar{\Lambda}) \times \underbrace{\mathbb{E}[c_x^+ | \tilde{x} > \bar{x}(y)]}_{\text{unconstrained MPC}}$$

Linear solution simply extrapolates steady-state aggregate MPC

- Changes in the constrained share are irrelevant to first order
 - As if constrained households maintain $\text{MPC} = 1$ regardless of Δ

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$\mathcal{C}'_0(0) = \bar{\Lambda} \times \underbrace{1}_{\text{MPC}=1} + (1 - \bar{\Lambda}) \times \underbrace{\mathbb{E}\left[c_x^+ \mid \tilde{x} > \bar{x}(y)\right]}_{\text{unconstrained MPC}}$$

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$\mathcal{C}'_0(0) = \bar{\Lambda} \times \underbrace{1}_{\text{MPC}=1} + (1 - \bar{\Lambda}) \times \underbrace{\mathbb{E}\left[c_x^+ \mid \tilde{x} > \bar{x}(y)\right]}_{\text{unconstrained MPC}}$$

$$\mathcal{C}''_0(0) = \Lambda'(0) \mathbb{E}\left[\begin{pmatrix} 1 - c_x^+ \end{pmatrix} \mid \tilde{x} = \bar{x}(y)\right] + (1 - \bar{\Lambda}) \mathbb{E}\left[c_{xx}^+ \mid \tilde{x} > \bar{x}(y)\right]$$

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$C'_0(0) = \bar{\Lambda} \times \underbrace{1}_{\text{MPC}=1} + (1 - \bar{\Lambda}) \times \underbrace{\mathbb{E}[c_x^+ \mid \tilde{x} > \bar{x}(y)]}_{\text{unconstrained MPC}}$$

$$C''_0(0) = \Lambda'(0) \mathbb{E} \left[\begin{pmatrix} 1 - c_x^+ \end{pmatrix} \mid \tilde{x} = \bar{x}(y) \right] + \underbrace{(1 - \bar{\Lambda}) \mathbb{E}[c_{xx}^+ \mid \tilde{x} > \bar{x}(y)]}_{\substack{\text{individual consumption curvature} \\ < 0}}$$

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$C'_0(0) = \bar{\Lambda} \times \underbrace{1}_{\text{MPC}=1} + (1 - \bar{\Lambda}) \times \underbrace{\mathbb{E}[c_x^+ \mid \tilde{x} > \bar{x}(y)]}_{\text{unconstrained MPC}}$$

$$C''_0(0) = \underbrace{\Lambda'(0) \mathbb{E} \left[\begin{pmatrix} 1 - c_x^+ \end{pmatrix} \mid \tilde{x} = \bar{x}(y) \right]}_{\substack{\text{movement off- the constraint} \\ <0}} + \underbrace{(1 - \bar{\Lambda}) \mathbb{E}[c_{xx}^+ \mid \tilde{x} > \bar{x}(y)]}_{\substack{\text{individual consumption curvature} \\ <0}}$$

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$C'_0(0) = \bar{\Lambda} \times \underbrace{1}_{\text{MPC}=1} + (1 - \bar{\Lambda}) \times \underbrace{\mathbb{E}[c_x^+ \mid \tilde{x} > \bar{x}(y)]}_{\text{unconstrained MPC}}$$

change in
constrained share

$$C''_0(0) = \underbrace{\Lambda'(0) \mathbb{E} \left[\begin{pmatrix} 1 - c_x^+ \end{pmatrix} \mid \tilde{x} = \bar{x}(y) \right]}_{\substack{\text{movement off- the constraint} \\ < 0}} + \underbrace{(1 - \bar{\Lambda}) \mathbb{E}[c_{xx}^+ \mid \tilde{x} > \bar{x}(y)]}_{\substack{\text{individual consumption curvature} \\ < 0}}$$

UNDERSTANDING THE GAP: TWO SOURCES OF CONCAVITY

$$C'_0(0) = \bar{\Lambda} \times \underbrace{1}_{\text{MPC}=1} + (1 - \bar{\Lambda}) \times \underbrace{\mathbb{E}[c_x^+ \mid \tilde{x} > \bar{x}(y)]}_{\text{unconstrained MPC}}$$

$$C''_0(0) = \underbrace{\Lambda'(0) \mathbb{E} \left[\left(\underbrace{1 - c_x^+}_{\text{Drop in MPC at Kink}} \right) \mid \tilde{x} = \bar{x}(y) \right]}_{\substack{\text{movement off- the constraint} \\ <0}} + \underbrace{(1 - \bar{\Lambda}) \mathbb{E}[c_{xx}^+ \mid \tilde{x} > \bar{x}(y)]}_{\substack{\text{individual consumption curvature} \\ <0}}$$

change in constrained share

The aggregate consumption response is **concave** in transfer size.

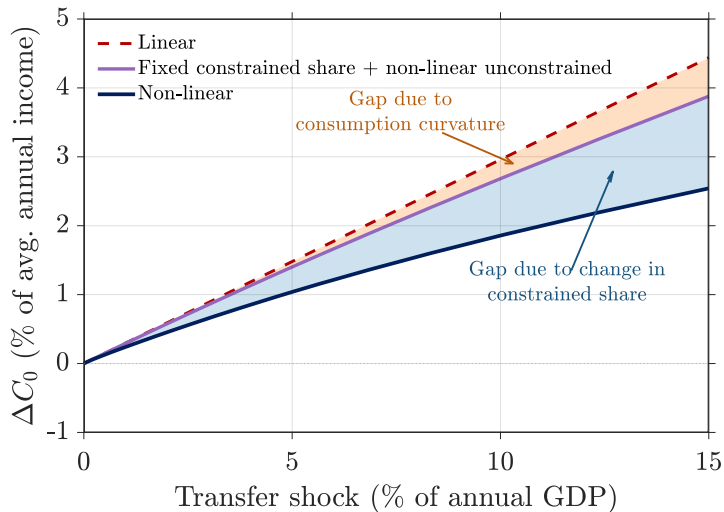
TWO SOURCES OF CONCAVITY (REFERENCE)

$$\begin{aligned}
 c_0''(0) = & \underbrace{\Lambda'(0) \mathbb{E} \left[\left(\overbrace{1 - c_x^+}^{\text{Drop in MPC at Kink}} \right) \mid \tilde{x} = \bar{x}(y) \right]}_{\substack{\text{movement off- the constraint} \\ <0}} + \underbrace{(1 - \bar{\Lambda}) \mathbb{E} [c_{xx}^+ \mid \tilde{x} > \bar{x}(y)]}_{\substack{\text{individual consumption curvature} \\ <0}}
 \end{aligned}$$

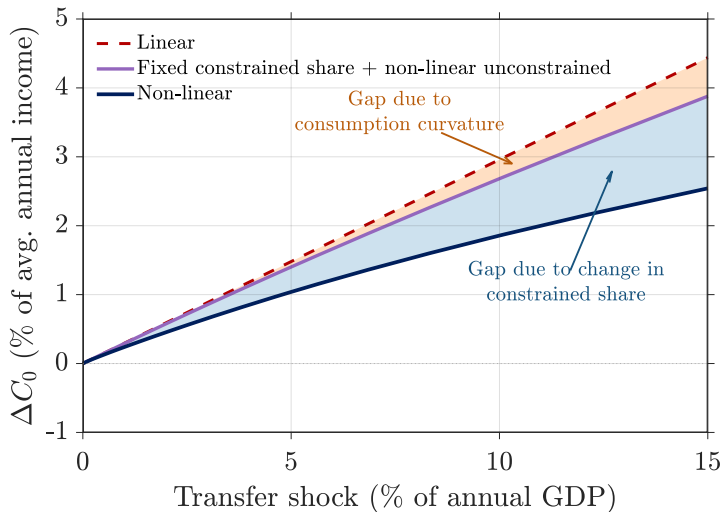
change in constrained share

The aggregate consumption response is **concave** in transfer size.

DECOMPOSING THE LINEAR-NON-LINEAR GAP



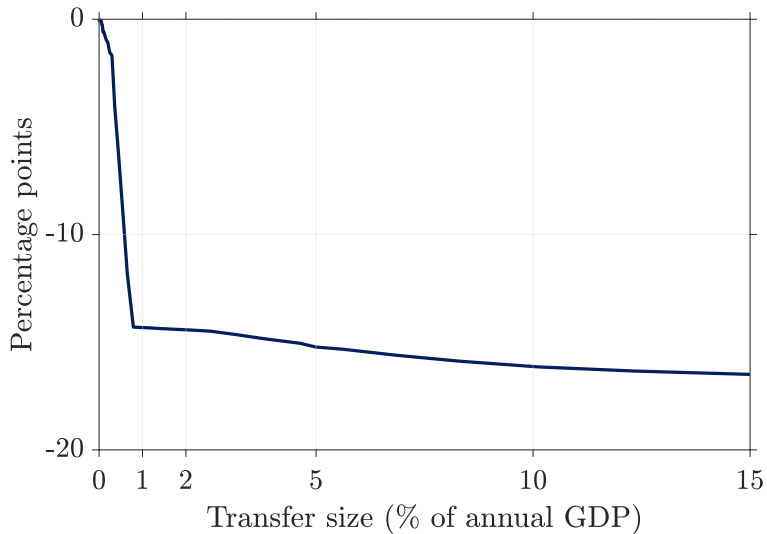
DECOMPOSING THE LINEAR-NON-LINEAR GAP



1% transfer
 Constrained share 93%
 Curvature 7%

5% transfer
 Constrained share 82%
 Curvature 18%

CHANGE IN SHARE OF CONSTRAINED HOUSEHOLDS

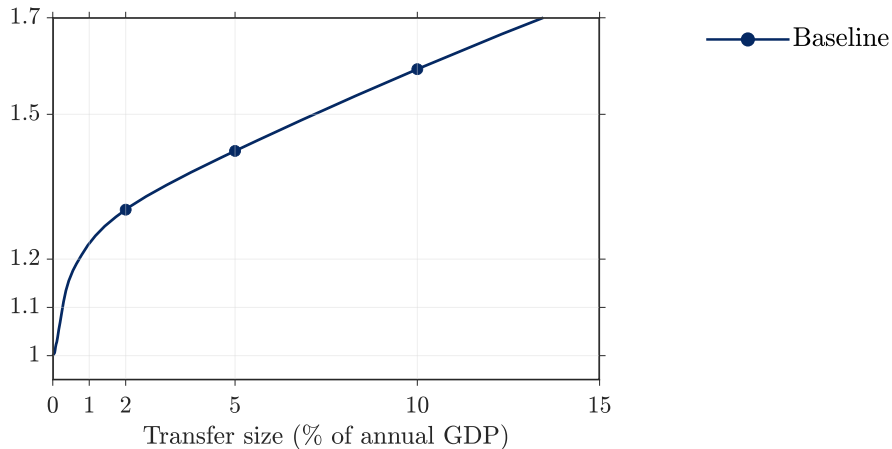


TAKING STOCK: WHY LINEAR-METHOD OVERSTATES?

- Linear solution, changes in constrained share have zero first-order effects
 - ▶ Effectively, as if constrained households permanently $MPC=1$
 - Non-linear solution is highly concave
 - ▶ Key: Drop in MPC at kink $\times$ Share of households becoming unconstrained
- $\Rightarrow$ Significant gaps between linear and non-linear solution, even for small shocks

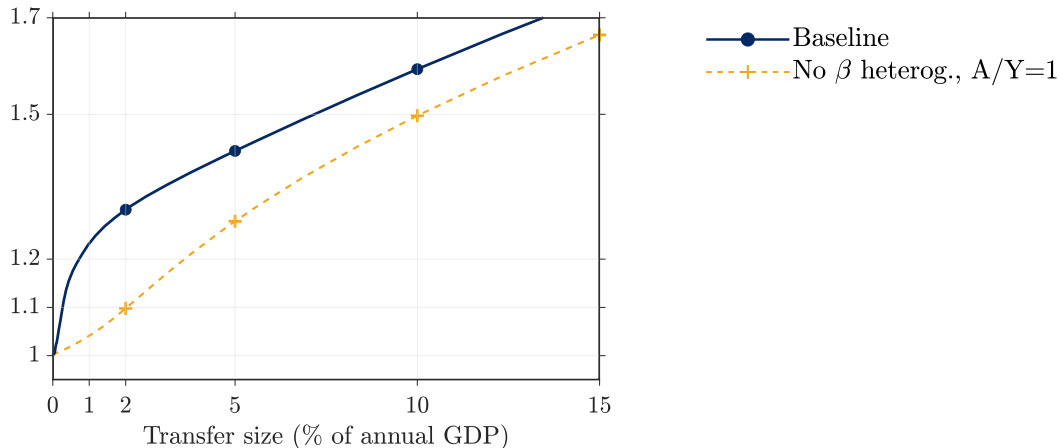
HOW SMALL IS SMALL? ROBUSTNESS OF P.E. RESULTS

Linear/Non-linear Consumption Response



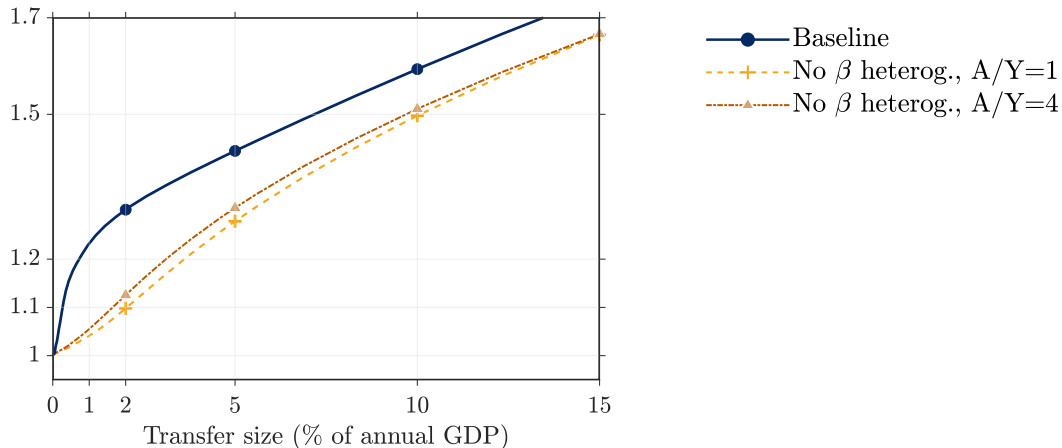
HOW SMALL IS SMALL? ROBUSTNESS OF P.E. RESULTS

Linear/Non-linear Consumption Response



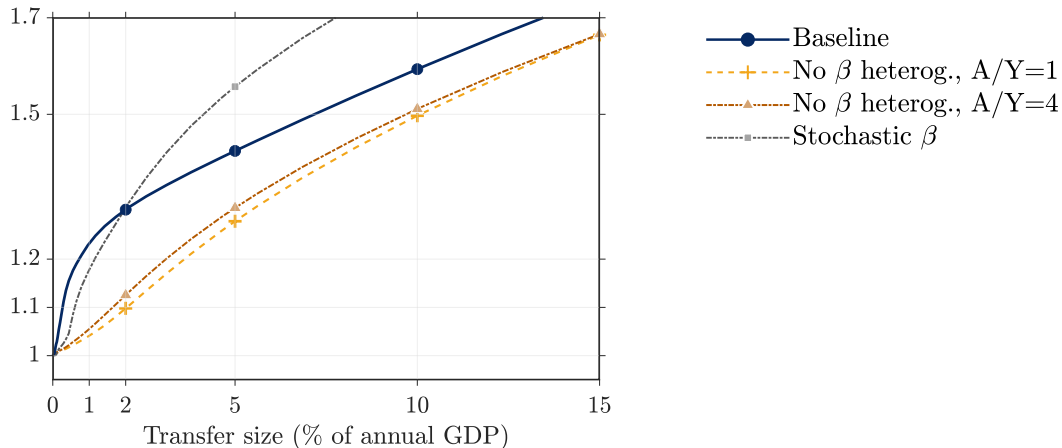
HOW SMALL IS SMALL? ROBUSTNESS OF P.E. RESULTS

Linear/Non-linear Consumption Response



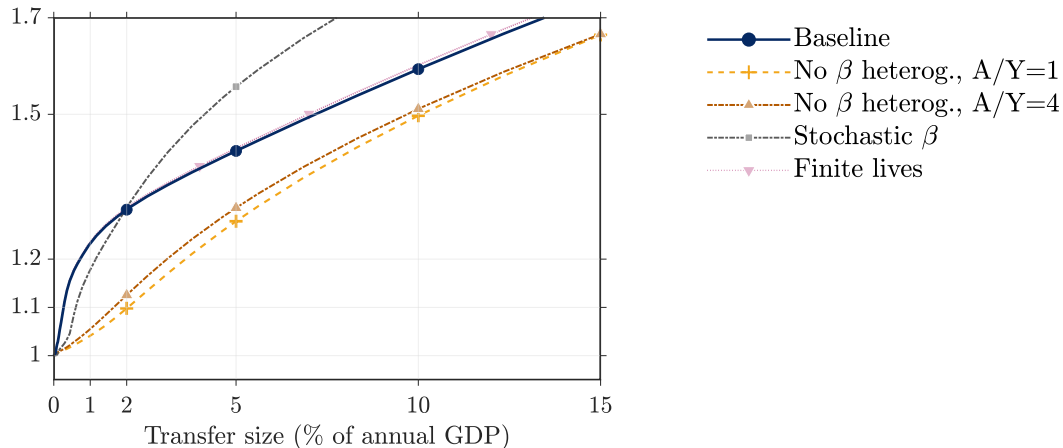
HOW SMALL IS SMALL? ROBUSTNESS OF P.E. RESULTS

Linear/Non-linear Consumption Response



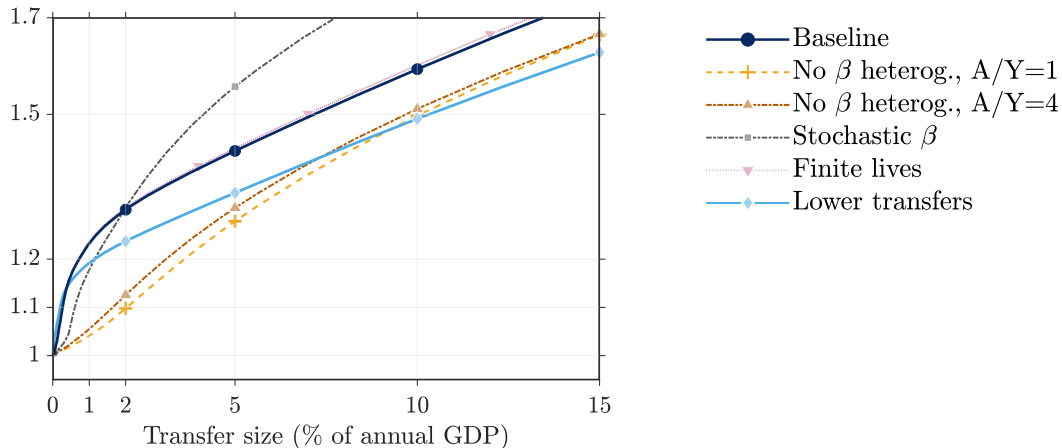
HOW SMALL IS SMALL? ROBUSTNESS OF P.E. RESULTS

Linear/Non-linear Consumption Response



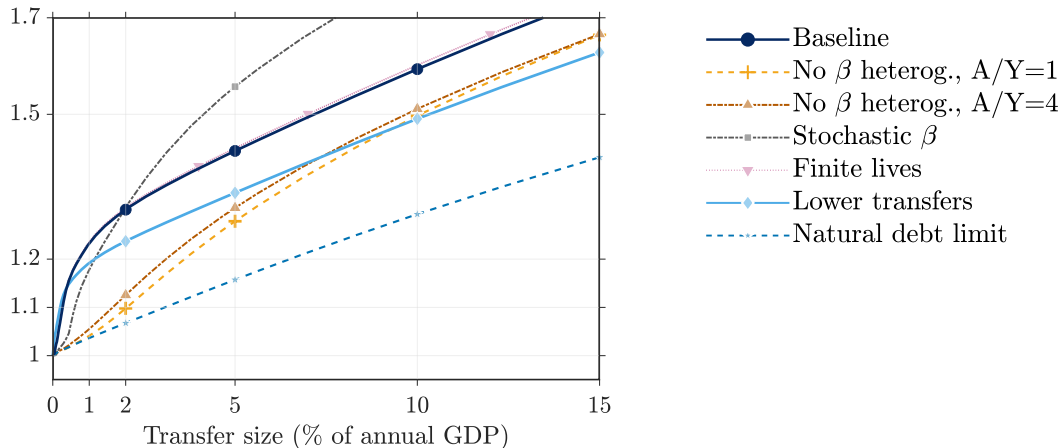
HOW SMALL IS SMALL? ROBUSTNESS OF P.E. RESULTS

Linear/Non-linear Consumption Response



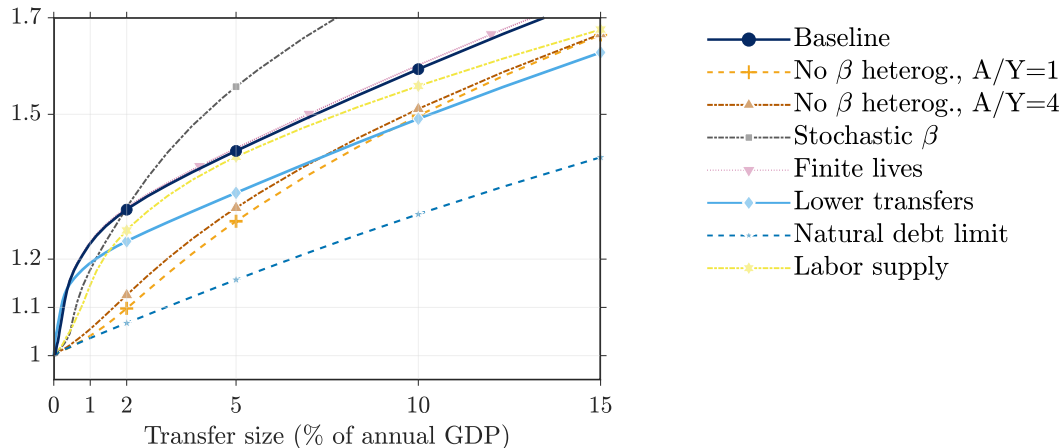
HOW SMALL IS SMALL? ROBUSTNESS OF P.E. RESULTS

Linear/Non-linear Consumption Response



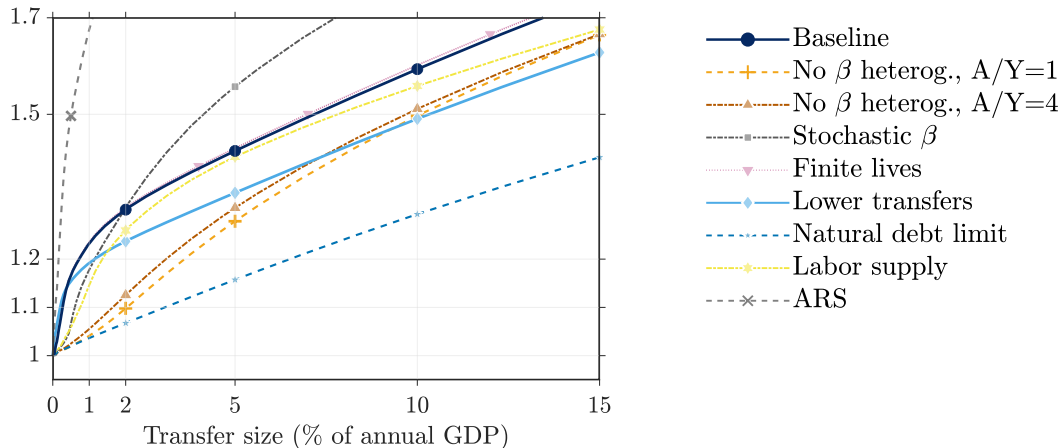
HOW SMALL IS SMALL? ROBUSTNESS OF P.E. RESULTS

Linear/Non-linear Consumption Response

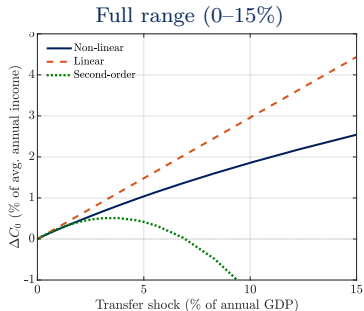
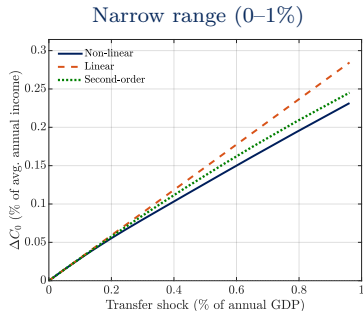


HOW SMALL IS SMALL? ROBUSTNESS OF P.E. RESULTS

Linear/Non-linear Consumption Response

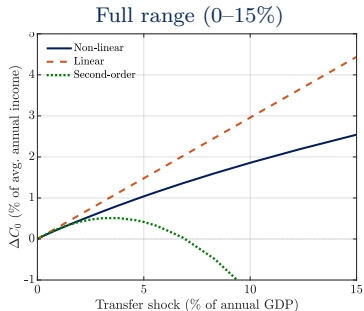
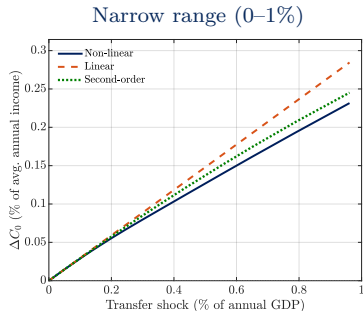


DO HIGHER-ORDER LOCAL METHODS HELP?



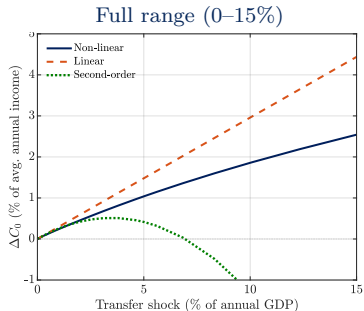
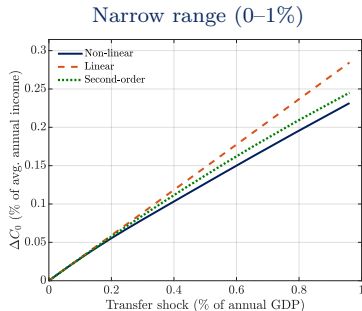
- Second-order improves for narrow shocks, but deteriorates quickly

DO HIGHER-ORDER LOCAL METHODS HELP?



- Second-order improves for narrow shocks, but deteriorates quickly

DO HIGHER-ORDER LOCAL METHODS HELP?



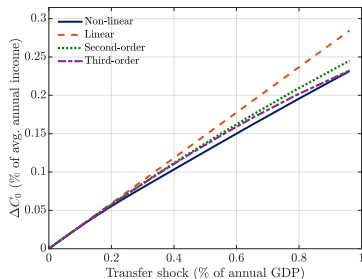
$$C_0''(0) = \underbrace{\Lambda'(0) \mathbb{E}[(1 - c_x^+) \mid \tilde{x} = \bar{x}(y)]}_{\text{movement off constraint} < 0} + \underbrace{(1 - \bar{\Lambda}) \mathbb{E}[c_{xx}^+ \mid \tilde{x} > \bar{x}(y)]}_{\text{individual curvature} < 0}$$

$$\Lambda'(0) = - \int_Y g(\bar{x}(y) \mid y) dF(y) < 0$$

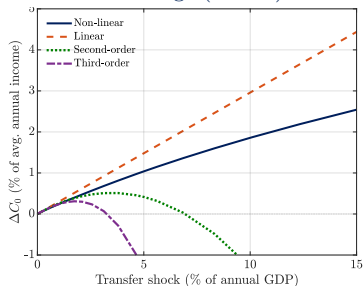
Second-order extrapolates change in constrained share at constant rate $\Rightarrow$ overshoots for small Δ , *understating* aggregate response

DO HIGHER-ORDER LOCAL METHODS HELP?

Narrow range (0–1%)



Full range (0–15%)



$$C_0'''(0) = \underbrace{\Lambda''(0)}_{\text{curvature of constrained share}} \tilde{\mathbb{E}}[(1 - c_x^+) \mid \tilde{x} = \bar{x}(y)] - \Lambda'(0) \mathbb{E}[c_{xx}^+ \mid \tilde{x} = \bar{x}(y)] + (1 - \bar{\Lambda}) \mathbb{E}[c_{xxx}^+ \mid \tilde{x} > \bar{x}(y)]$$

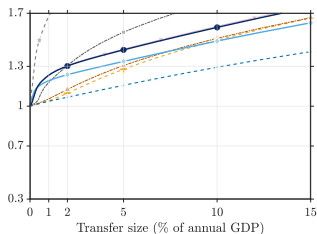
curvature of constrained share

$$\Lambda''(0) = \int_Y g'(\bar{x}(y) \mid y) dF(y) > 0$$

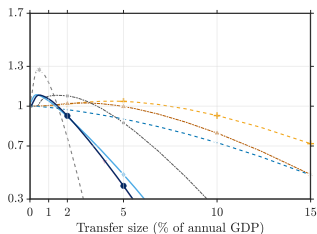
Third-order overshoots *further*

DO HIGHER-ORDER APPROXIMATIONS HELP?

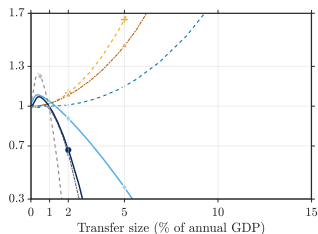
Linear / non-linear



Second order / non-linear

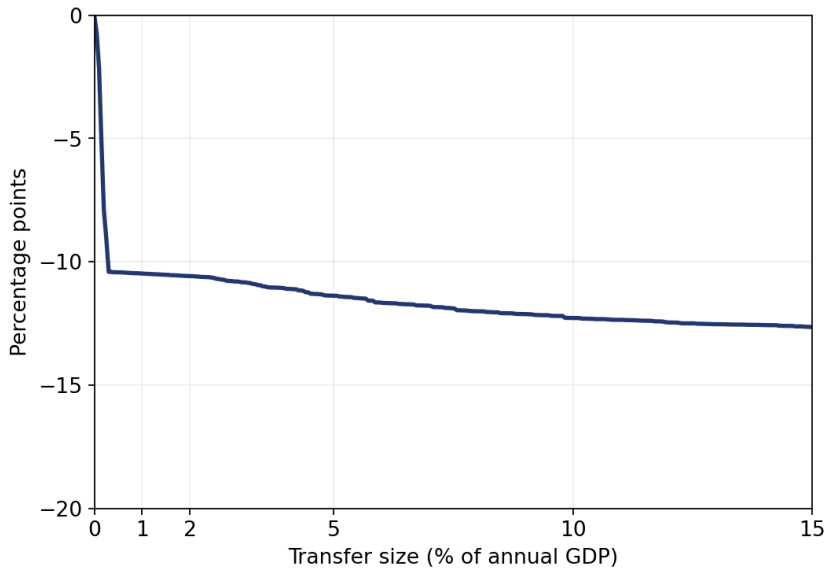


Third order / non-linear

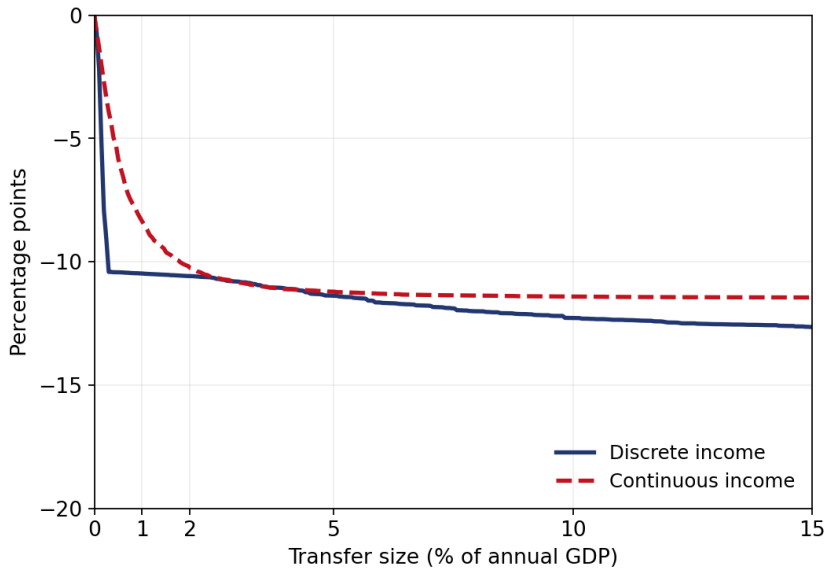


—●— Baseline - - + - - No β heterog., $A/Y=1$ - - ▲ - - No β heterog., $A/Y=4$ - - ■ - - Stochastic β
 - - ▼ - - Finite lives —◆— Lower transfers - - * - - Natural debt limit - - * - - ARS

CHANGE IN SHARE OF CONSTRAINED HOUSEHOLDS

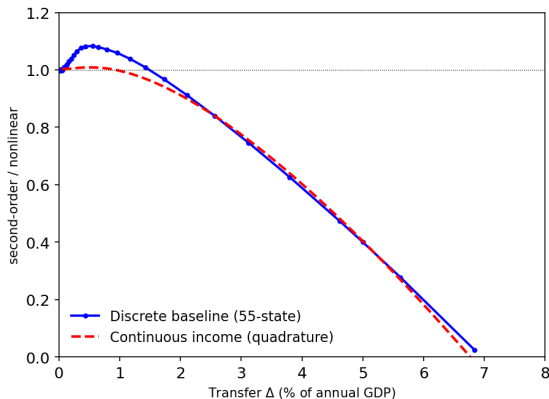


CHANGE IN SHARE OF CONSTRAINED HOUSEHOLDS



ROBUSTNESS TO DISCRETENESS OF INCOME PROCESS

Second-order/Non-Linear



A continuous income process yields a smoother higher-order approximation, but the approximation remains accurate only for small shocks.

ROADMAP

1. Partial equilibrium model and calibration
2. Fiscal stimulus in general equilibrium
3. Other shocks

GENERAL EQUILIBRIUM MODEL

PE + production, labor supply, nominal rigidities, fiscal/monetary rules.

GENERAL EQUILIBRIUM MODEL

- **Households:**

- ▶ Utility flow, $u(c_{it}) - v(n_{it})$ with $v' > 0, v'' > 0$
- ▶ Income $(1 - \tau) \left(\frac{W_t}{P_t} e_{it} n_{it} + d_{i,t} \right)$

GENERAL EQUILIBRIUM MODEL

- **Households:**

- ▶ Utility flow, $u(c_{it}) - v(n_{it})$ with $v' > 0, v'' > 0$
- ▶ Income $(1 - \tau) \left(\frac{W_t}{P_t} e_{it} n_{it} + d_{i,t} \right)$
- ▶ Labor union with sticky wages (linearized)

$$\pi_t = \kappa \left(v'(N_t) - \frac{1 - \tau}{C_t} \right) + \bar{\beta} \pi_{t+1}, \quad (\text{Phillips Curve})$$

GENERAL EQUILIBRIUM MODEL

- **Households:**

- ▶ Utility flow, $u(c_{it}) - v(n_{it})$ with $v' > 0, v'' > 0$
- ▶ Income $(1 - \tau) \left(\frac{W_t}{P_t} e_{it} n_{it} + d_{i,t} \right)$
- ▶ Labor union with sticky wages (linearized)

$$\pi_t = \kappa \left(v'(N_t) - \frac{1 - \tau}{C_t} \right) + \bar{\beta} \pi_{t+1}, \quad (\text{Phillips Curve})$$

- **Firms and Technology:**

$$\left. \begin{array}{l} \text{Production: } Y_t = N_t \\ \text{Flexible prices} \end{array} \right\} \Rightarrow W_t = P_t, \quad d_{i,t} = 0$$

GENERAL EQUILIBRIUM MODEL (CTD)

- Government budget:

$$B_t + \tau Y_t = G + T_t + (1 + r_{t-1})B_{t-1}$$

- Fiscal rule ($t > 0$)

$$T_t = T - \phi_B \bar{r}(B_{t-1} - \bar{B})$$

- Monetary policy:

$$i_t = \bar{i} + \phi_\pi(\pi_t - \bar{\pi})$$

CALIBRATION

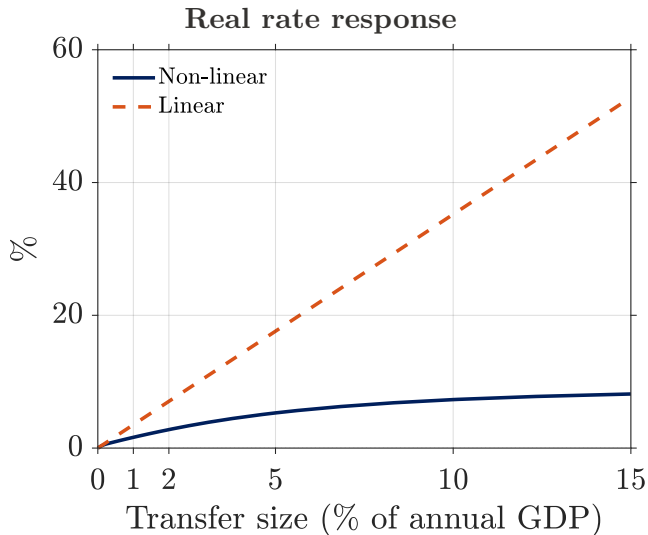
Additional Parameters for GE Model

Parameter	Value	Description
$\bar{B}$	4	Steady-state government debt=100% GDP
κ	0.01	Phillips curve slope
ν	1	Frisch elasticity of labor supply
ϕ_π	1.5	Taylor coefficient
ϕ_B	8	Fiscal rule
G	0.104	Government spending
T	0.15	Government transfers

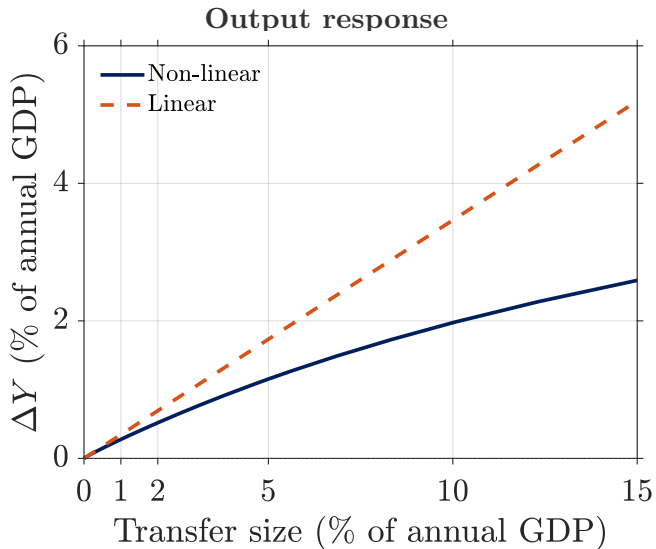
Computation:

- Linear: sequence space Jacobian (ABRS)
- Non-linear: quasi-Newton algorithm

FLEXIBLE-PRICE ALLOCATION



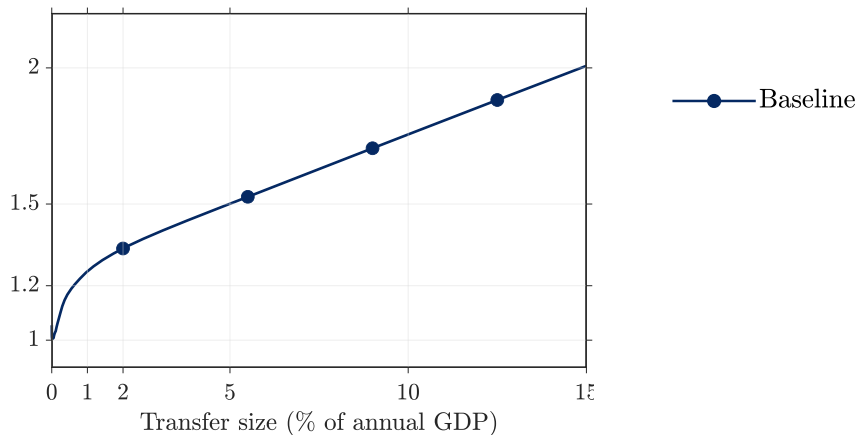
- No output effects
- Linear methods overstate increase in demand and rise in r



- At $\Delta = 2\%$ GDP: linear overstates by 34%
- At $\Delta = 5\%$ GDP: linear overstates by 50%

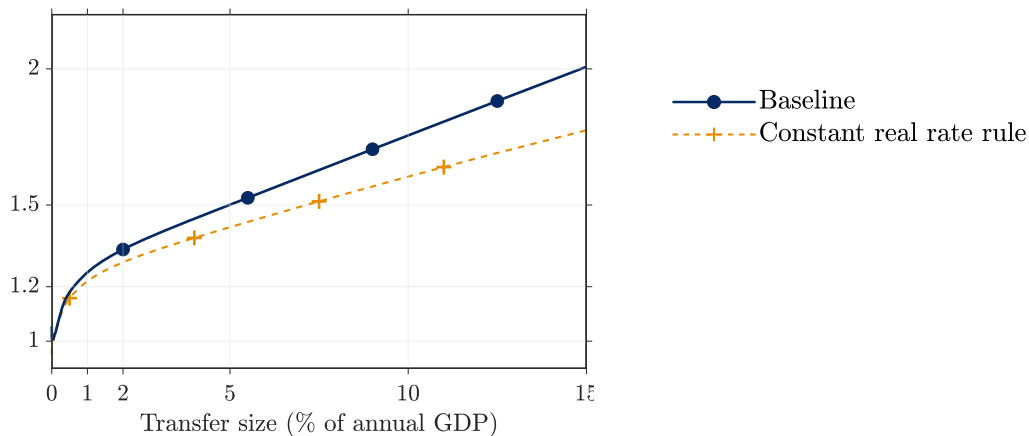
HOW SMALL IS SMALL? ROBUSTNESS IN HANK

Linear/Non-linear Output Response



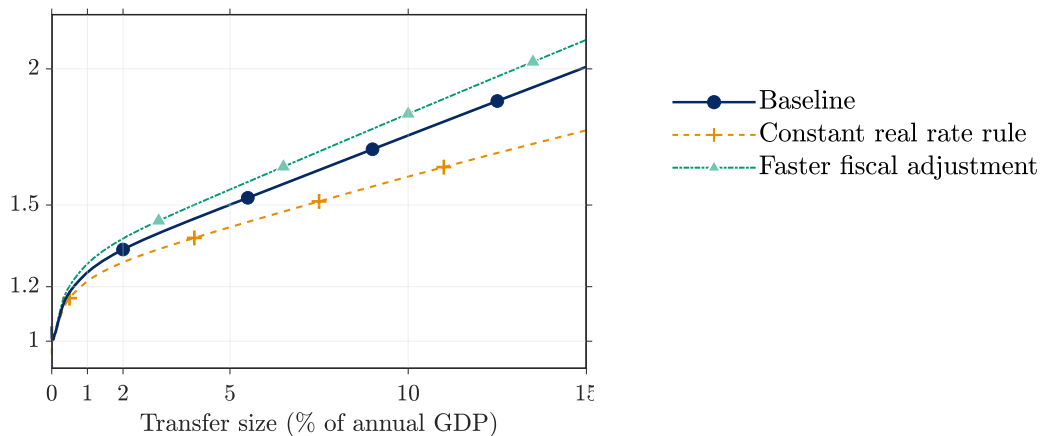
HOW SMALL IS SMALL? ROBUSTNESS IN HANK

Linear/Non-linear Output Response



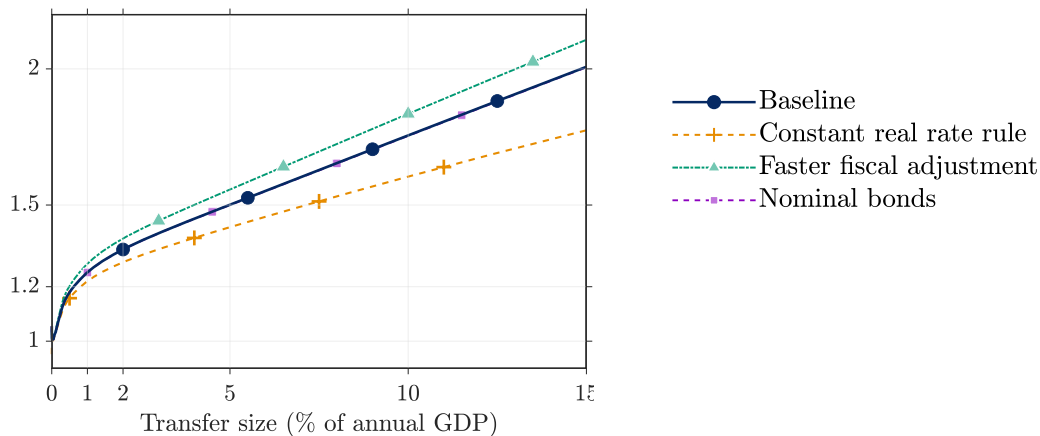
HOW SMALL IS SMALL? ROBUSTNESS IN HANK

Linear/Non-linear Output Response



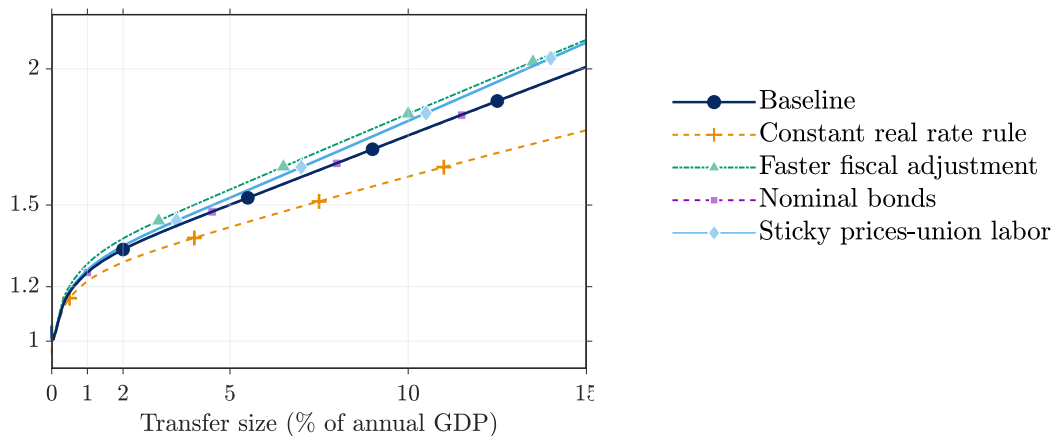
HOW SMALL IS SMALL? ROBUSTNESS IN HANK

Linear/Non-linear Output Response



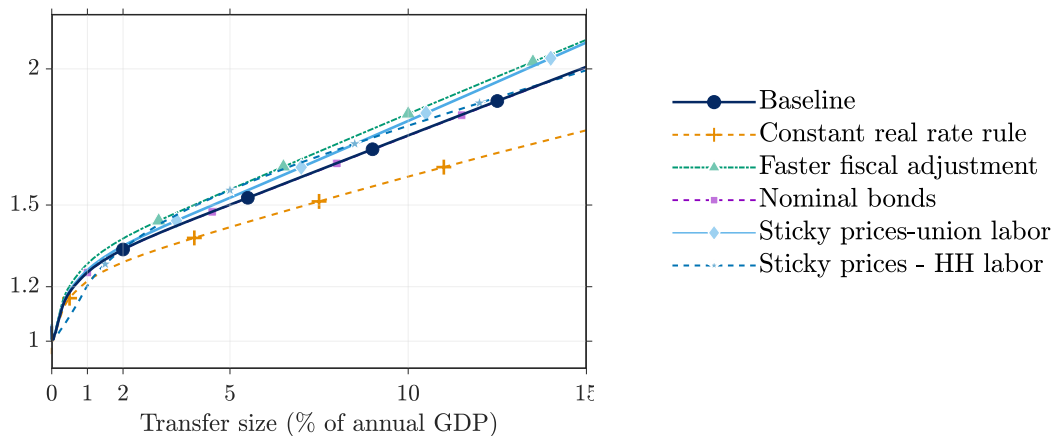
HOW SMALL IS SMALL? ROBUSTNESS IN HANK

Linear/Non-linear Output Response



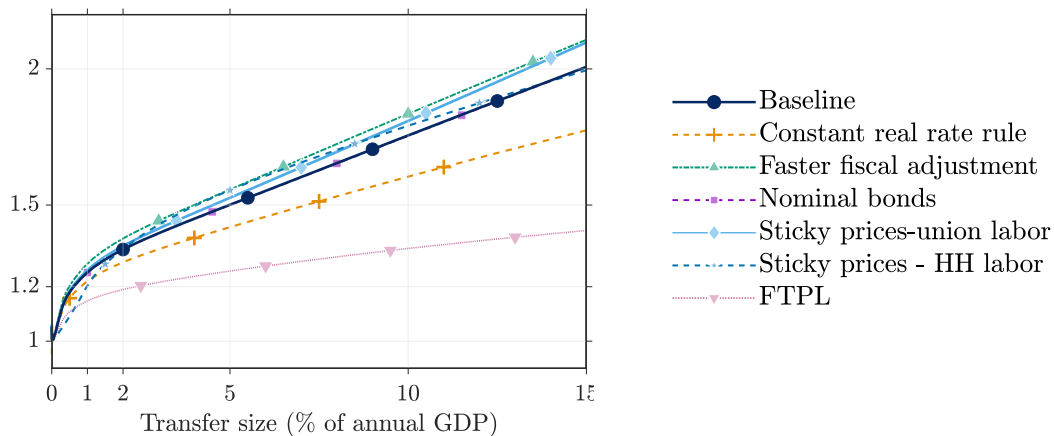
HOW SMALL IS SMALL? ROBUSTNESS IN HANK

Linear/Non-linear Output Response



HOW SMALL IS SMALL? ROBUSTNESS IN HANK

Linear/Non-linear Output Response



OTHER RESULTS

1. Fiscal contractions vs. expansions

go →

2. Gap in persistence

go →

3. Targeted transfers

go →

4. Anticipated transfers

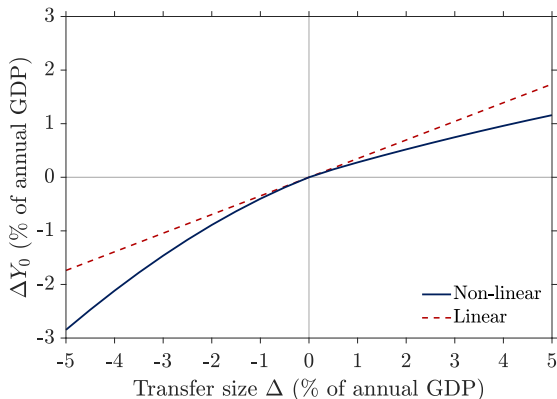
go →

5. Other shocks

go →

FISCAL CONTRACTIONS VS. EXPANSIONS

Output response

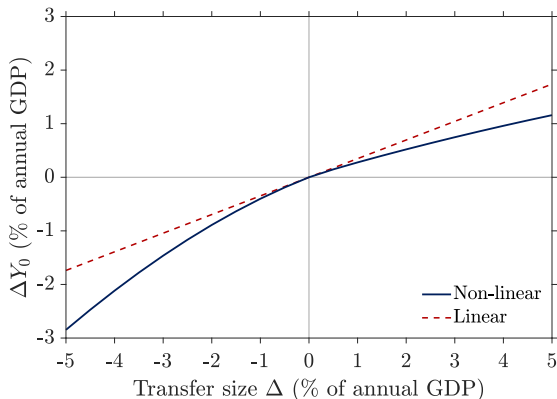


Concavity: linear **overstates** expansions, **understates** contractions.

Note: Baseline HANK model

FISCAL CONTRACTIONS VS. EXPANSIONS

Output response

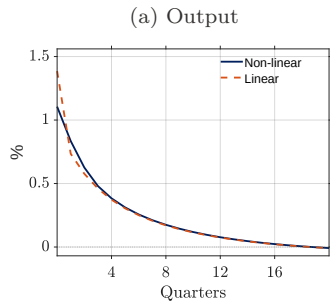


Concavity: linear **overstates** expansions, **understates** contractions.

Non-linearity slightly larger for contractions:

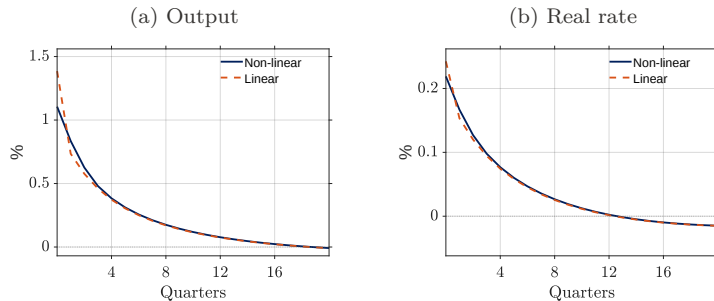
Note: Baseline HANK model

LINEAR METHOD UNDERSTATES PERSISTENCE



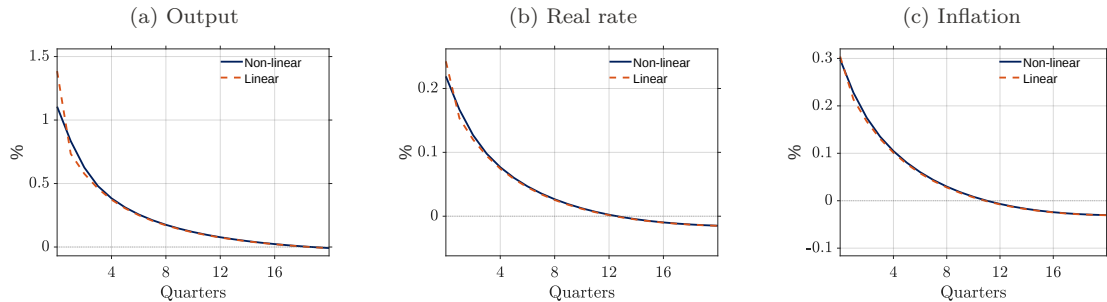
Note: Impulse responses in the HANK to a 1% of annual GDP transfer.

LINEAR METHOD UNDERSTATES PERSISTENCE



Note: Impulse responses in the HANK to a 1% of annual GDP transfer.

LINEAR METHOD UNDERSTATES PERSISTENCE

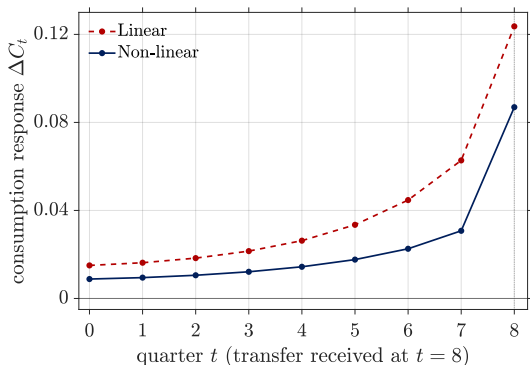


Note: Impulse responses in the HANK to a 1% of annual GDP transfer.

LINEAR OVERSTATES RESPONSE TO ANTICIPATED TRANSFERS

LINEAR OVERSTATES RESPONSE TO ANTICIPATED TRANSFERS

Transfer of 1% of annual GDP received at $t = 8$, anticipated at $t = 0$



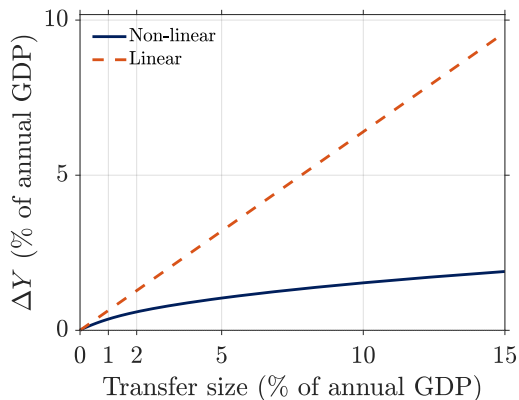
- Unconstrained households consume more in anticipation of income
- Linear solution ignores the constraint $\Rightarrow$ again overstating the response

◀ Other results

TARGETED TRANSFERS VS. UNTARGETED TRANSFERS

◀ OTHER RESULTS

(a) Output response (bottom 20%)

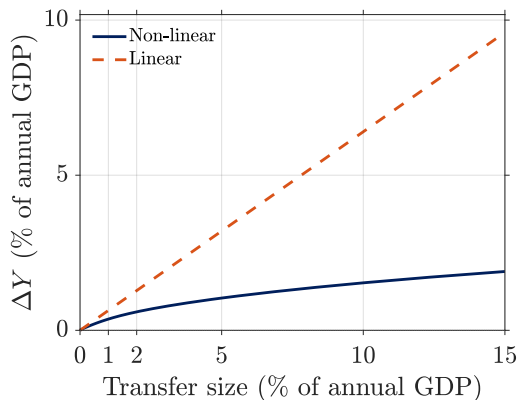


(b) Linear / non-linear

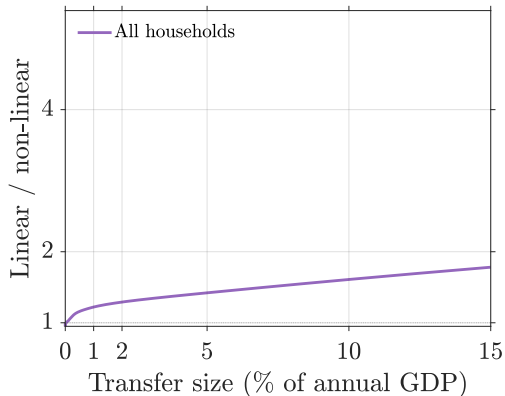
TARGETED TRANSFERS VS. UNTARGETED TRANSFERS

◀ OTHER RESULTS

(a) Output response (bottom 20%)



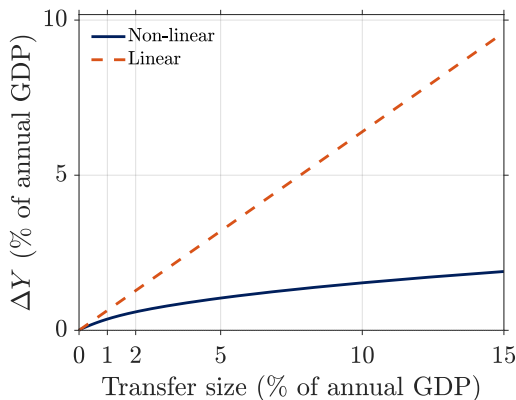
(b) Linear / non-linear



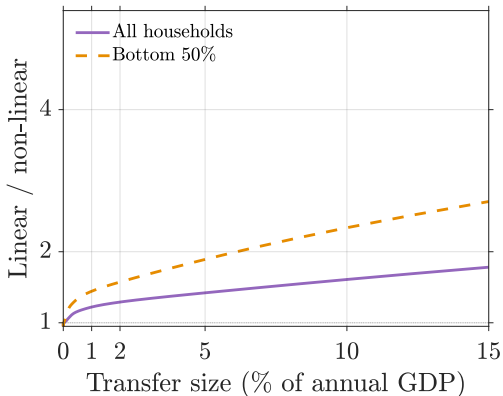
TARGETED TRANSFERS VS. UNTARGETED TRANSFERS

◀ OTHER RESULTS

(a) Output response (bottom 20%)



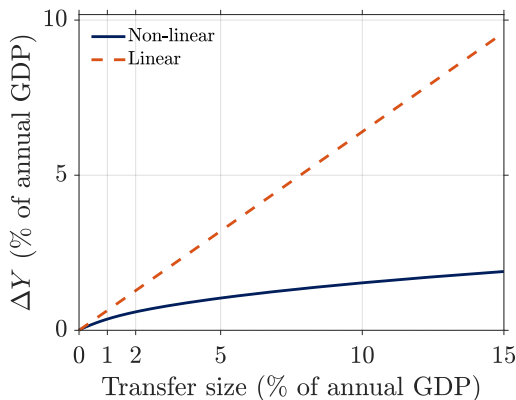
(b) Linear / non-linear



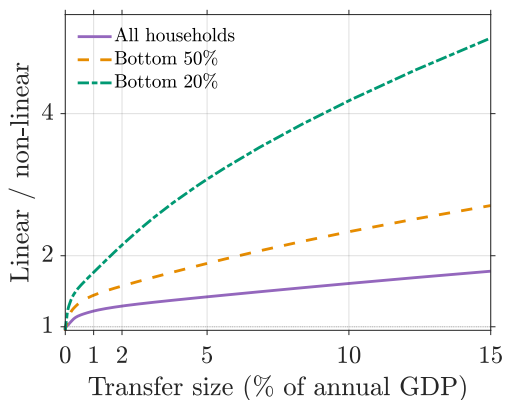
TARGETED TRANSFERS VS. UNTARGETED TRANSFERS

◀ OTHER RESULTS

(a) Output response (bottom 20%)



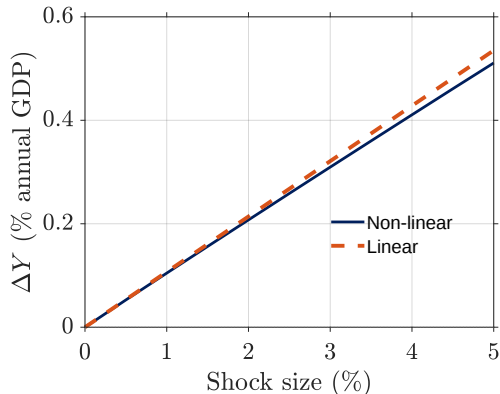
(b) Linear / non-linear



NON-LINEARITIES FOR MONETARY POLICY AND TFP

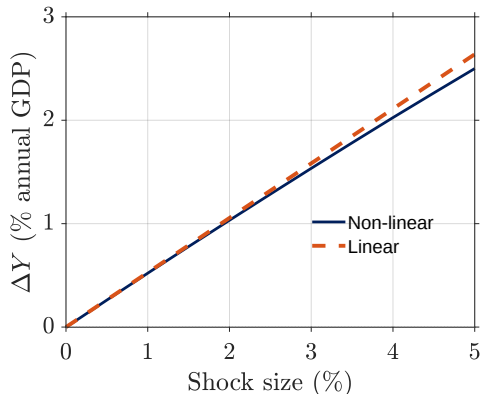
◀ OTHER RESULTS

(a) Monetary (rate cut)



▶ Tax-rate cut

(b) TFP



▶ Discount-factor (β) shock

CONCLUSIONS

- First-order methods significantly overstate the effects of fiscal transfers
 - ▶ Key: higher transfers shift households out of the borrowing constraint, so the aggregate MPC falls with transfer size

CONCLUSIONS

- First-order methods significantly overstate the effects of fiscal transfers
 - ▶ Key: higher transfers shift households out of the borrowing constraint, so the aggregate MPC falls with transfer size
- For monetary policy, linear methods are accurate. But $HA \approx RA$

CONCLUSIONS

- First-order methods significantly overstate the effects of fiscal transfers
 - ▶ Key: higher transfers shift households out of the borrowing constraint, so the aggregate MPC falls with transfer size
- For monetary policy, linear methods are accurate. But $HA \approx RA$
- When transmission works through departures from Ricardian equivalence, first-order methods are quantitatively unreliable

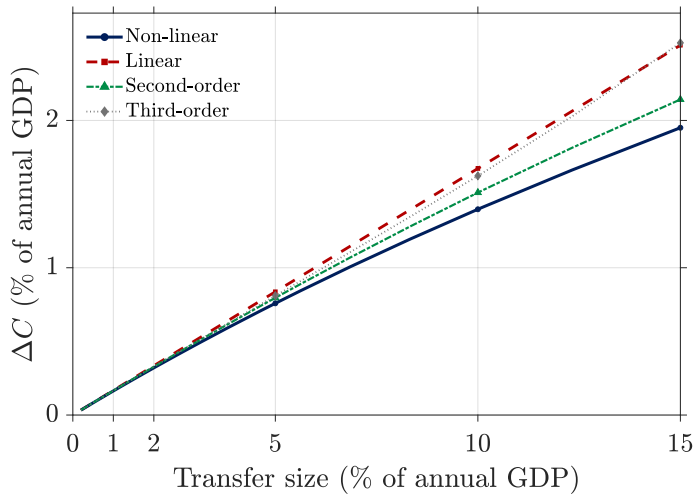
CONCLUSIONS

- First-order methods significantly overstate the effects of fiscal transfers
 - ▶ Key: higher transfers shift households out of the borrowing constraint, so the aggregate MPC falls with transfer size
- For monetary policy, linear methods are accurate. But $HA \approx RA$
- When transmission works through departures from Ricardian equivalence, first-order methods are quantitatively unreliable
 - ▶ Higher-order local approx. improves for narrow shocks, but deteriorate quickly

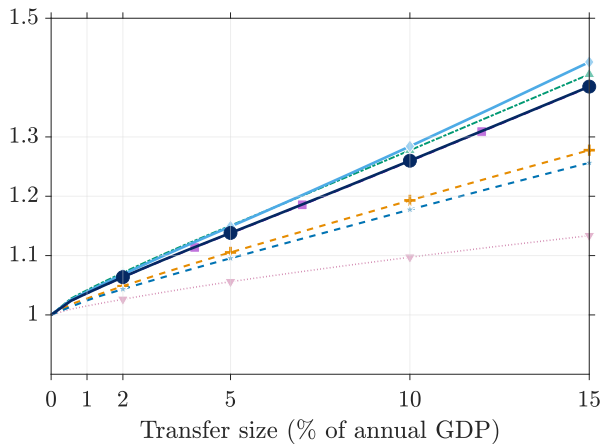
APPENDIX

Additional figures

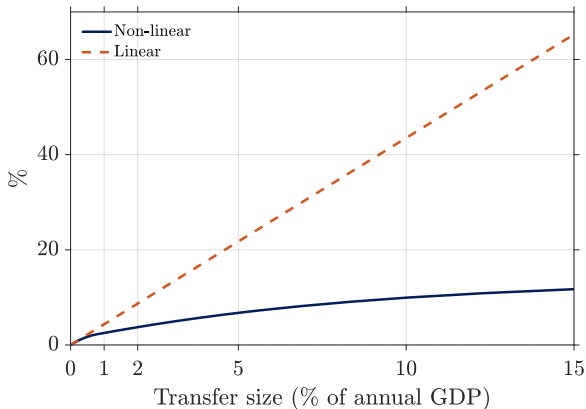
UNCONSTRAINED: HIGHER-ORDER LOCAL APPROX.



GE: FIRST-YEAR LINEAR-TO-NON-LINEAR RATIO



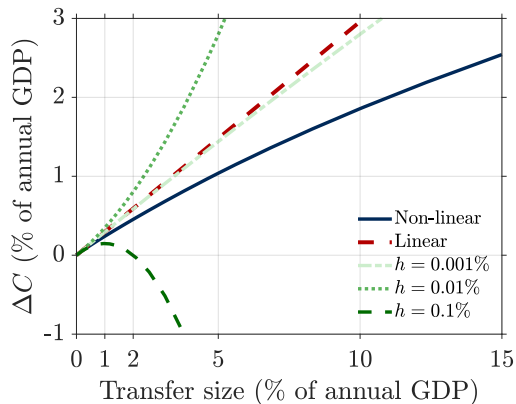
GE: FLEXIBLE PRICES WITH HOUSEHOLD LABOR CHOICE



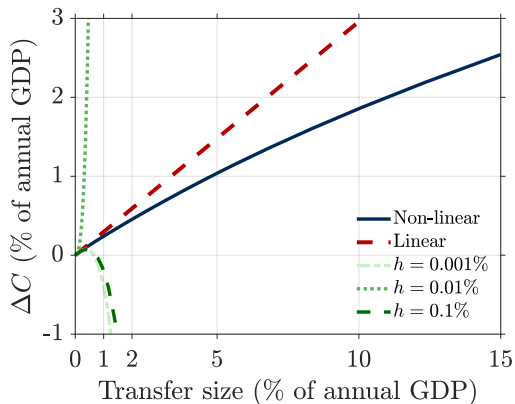
- Households choose labor freely (no union)
- Output effects via labor-supply incentives
- Real-rate response remains highly **concave**

HIGHER-ORDER WITH FINITE DIFFERENCES

(a) Second order



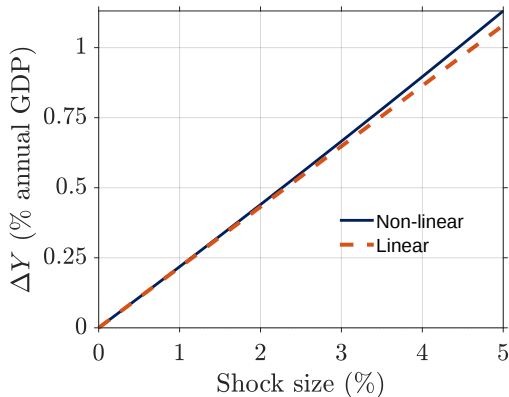
(b) Third order



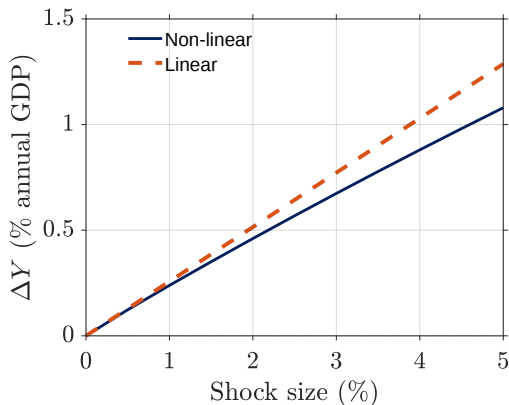
Extremely sensitive to step size and poorly behaved even for small shocks.

OTHER SHOCKS: DISCOUNT FACTOR AND LABOR TAX

(a) Discount-factor shock

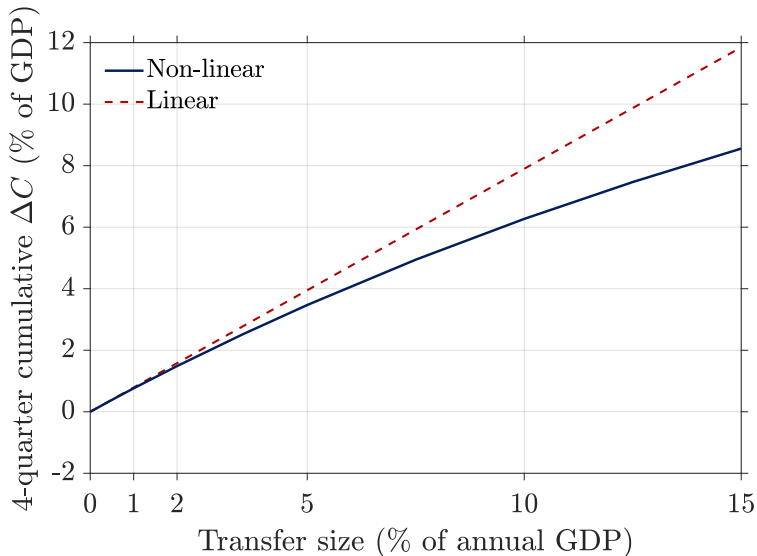


(b) Labor-tax cut

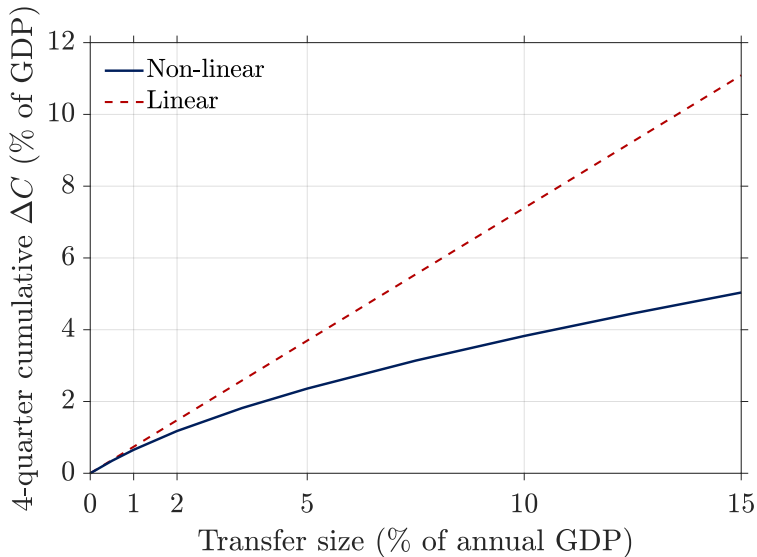


A τ cut shifts fewer resources to low-income, constrained households.

4-QUARTER CUMULATIVE RESPONSE (BASELINE HANK)

[← BACK](#)

4-QUARTER CUM. RESPONSE (BOTTOM-20% TARGETED)

[← BACK](#)

ALTERNATIVE DECOMPOSITION

[◀ BACK](#)

Exact non-linear **constrained** share + first-order Taylor for the **unconstrained**:

$$C_0^{\text{hyb}}(\Delta) = \int_Y \left[\underbrace{\int_{-\infty}^{\bar{x}(y)-\Delta} c(\tilde{x} + \Delta, y) dG(\tilde{x} | y)}_{\text{exact constrained share, MPC=1}} + \underbrace{\int_{\bar{x}(y)-\Delta}^{\infty} [c(\tilde{x}, y) + c_x^+(\tilde{x}, y) \Delta] dG(\tilde{x} | y)}_{\text{unconstrained: first-order Taylor}} \right] dF(y)$$

