# How Small is Small? Nonlinearities in Heterogeneous-Agent Models*

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**Abstract**

In heterogeneous-agent models, marginal propensities to consume decrease with wealth, falling sharply when households move off their borrowing constraints. We decompose the concavity of aggregate consumption responses to transfers into contributions from changes in the share of constrained households and curvature of individual consumption policies. The former accounts for most of the nonlinearity. First-order approximations miss this movement and are accurate only up to a few hundred dollars. Second-order approximations are accurate over a larger range but deteriorate rapidly beyond it. Third- and higher-order approximations perform worse than linear above several hundred dollars. We extend the analysis to other shocks.

**Keywords:** Heterogeneous agents, nonlinearities, HANK

**JEL classification:** C63, D15, D31, D52, E21, E62, G51

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# 1 Introduction

Heterogeneous-agent models with incomplete markets have become a workhorse framework for studying fiscal and monetary interventions. These models are appealing because they capture salient features of household spending and saving behavior. In particular, suitably calibrated versions can match both the average level and distribution of marginal propensities to consume (MPCs) in the data.

These features are especially relevant for studying fiscal transfers. In this class of models, borrowing constraints limit consumption smoothing and so are a key source of both high MPCs and departures from Ricardian equivalence. Lump-sum transfers can therefore raise current consumption even when financed by offsetting future lump-sum taxes, and the level and distribution of MPCs shape the size of this response. In the standard representative-agent model, by contrast, such transfers leave consumption unchanged.

A large part of the recent literature has analyzed fiscal and monetary policies in heterogeneous-agent models using various forms of first-order methods. The most common approach, labeled "sequence-space Jacobian" (SSJ) by Auclert, Bardóczy, Rognlie and Straub (2021), computes equilibrium transition paths by linearizing aggregate consumption with respect to aggregate prices, shocks and policy variables around the steady state. It is well understood that such methods approximate nonlinear equilibria well when the policy innovations and exogenous disturbances being studied are small. But how small is small? We argue that for fiscal interventions, small can be much smaller than many researchers working with these models might realize.

For one-time transfers, we find that first-order approximation methods accurately capture the consumption response only for transfers of at most a few hundred dollars per household. In our baseline calibration, the linear solution overstates the aggregate consumption response by about 16% for a transfer of \$500 and by about 44% for an aggregate transfer of 1% of annual GDP (around \$1,700 per household). Second-order approximations improve accuracy over a wider range of transfers, but deteriorate for larger transfers, eventually understating the aggregate consumption response. Third- and fourth-order approximations become less accurate than the linear approximation for transfers above several hundred dollars.

We trace these patterns to the kinks that borrowing constraints generate in household consumption policies. Two mechanisms make aggregate consumption concave in the size of the transfer. The first is the familiar concavity of individual consumption functions for unconstrained households (e.g., Carroll and Kimball, 1996). The second is the movement of households off their borrowing constraints. As the transfer size increases, some households

become unconstrained and their MPCs fall discontinuously when they cross the kink. We provide an analytical and quantitative decomposition of these two sources of concavity and use it to characterize what local approximations of different orders capture. We find that movement off the constraint accounts for the vast majority of the concavity in the aggregate response.

The first-order approximation abstracts from both sources of concavity. It extrapolates the aggregate consumption response using the average MPC at steady state, without accounting for the decline in MPCs as households move off the borrowing constraint. By continuing to apply households' initially high MPCs even after their constraints cease to bind, it increasingly overstates the aggregate consumption response as the size of the transfer grows.

A second-order approximation incorporates the local response of the share of constrained households to the transfer, as well as the curvature of consumption policies for unconstrained households. This improves accuracy over a moderate range of transfers. However, the share of constrained households is itself highly nonlinear in the size of the transfer. It initially declines rapidly but then flattens near zero as fewer households leave the constraint for each additional dollar transferred, producing an approximately L-shaped relationship. The accuracy of the second-order approximation deteriorates for larger transfers because it effectively extrapolates the initial decline in this share of constrained households at a constant rate.

A third-order approximation additionally captures how the rate of change in the share of constrained households varies with transfer size. It improves on the second-order approximation over a narrow range of transfers, but rapidly loses accuracy as transfers grow. It becomes less accurate than the linear approximation for transfers above about 0.5% of annual GDP (around $900 per household). The fourth-order approximation becomes less accurate than the linear approximation at still smaller transfer sizes.

In the remainder of the paper we explore the robustness of these properties of local approximations to a wide variety of model features and calibration choices, as well as the role of nonlinearities in general-equilibrium versions of the model under different assumptions about pricing, production and policies. In our baseline model with sticky wages, the linear solution overstates the first-quarter output response to a transfer of 1% of annual GDP by about 50%. Substantial nonlinearities remain with sticky prices, alternative labor-supply arrangements, and nominal government debt. They become stronger with faster fiscal adjustment and weaker when the real interest rate is held fixed. For an unfunded fiscal stimulus, whose expansionary effects rely less on high-MPC households and departures from Ricardian equivalence, the output response is much closer to linear.

For more general shocks and policies, we show that nonlinearities depend on the extent of movement across kinks in the consumption function together with the difference in marginal consumption responses to the shock on either side of the kink. Linear approximations perform poorly when substantial movement is accompanied by a large response gap, for example with transfers targeted toward low-income households. They perform much better when the effects of movement across the constraint are weak, either because fewer households cross it or because their marginal responses change little upon crossing. We find this to be the case for output responses to productivity, monetary-policy, and discount-factor shocks, which are close to linear over the ranges we explore. We note that these are all shocks for which heterogeneity in MPCs typically matters little for the aggregate effects.

**Related literature.** Our paper is related to the large literature on heterogeneous-agent models, and in particular to applications using first-order methods to study fiscal and monetary interventions in heterogeneous-agent New Keynesian (HANK) models.[1] In addition to Auclert, Bardóczy, Rognlie and Straub (2021), see Boppart, Krusell and Mitman (2018) for a related approach in sequence space using finite differences, and Reiter (2009) for a linearization around the steady-state cross-sectional distribution of households.

Studies that allow for nonlinear effects of fiscal policy in incomplete markets include Carroll, Crawley, Slacalek and White (2021), Brinca et al. (2025), Mongey and Waugh (2025), Hagedorn, Manovskii and Mitman (2019), and Beraja and Zorzi (2024). Closer to us are Hagedorn, Manovskii and Mitman (2019), who solve for nonlinear transition paths following fiscal expansions, and Beraja and Zorzi (2024), who show how the marginal propensity to spend on durables declines with the size of stimulus checks. Boppart, Krusell and Mitman (2018) find that, for the range of productivity shocks studied, the nonlinear solution is approximately proportional to the size of the shock. None of these papers systematically compares linear and higher-order local approximations with nonlinear solutions, or disentangles the different sources of concavity.

Our analysis of higher-order approximations draws on recent advances in perturbation methods for heterogeneous-agent models developed by Bhandari, Bourany, Evans and Golosov (2023), Bilal (2023), and Auclert, Rigato, Rognlie and Straub (2026). Motivated by our findings, Bayer, Briglia, Luetticke, Weiss and Winkelmann (2026) develop a state-space perturbation algorithm that yields highly accurate first-order approximations

[1] See for example Aggarwal, Auclert, Rognlie and Straub (2023), Bardóczy, Sim and Tischbirek (2024), Bayer, Born, Luetticke and Müller (2023), Rachel and Summers (2019), Hänsel (2024), Campos, Fernández-Villaverde, Nuño and Paz (2024), Eichenbaum, Guerreiro and Obradovic (2026), Angeletos, Lian and Wolf (2024a), and Angeletos, Lian and Wolf (2024b).

when the approximation point is adjusted to account for the post-shock share of constrained households.

Our results also clarify the relationship between models with limited heterogeneity, such as those with a representative saver and a representative hand-to-mouth household (e.g., Galí, López-Salido and Vallés, 2007; Bilbiie, 2008; Debortoli and Galí, 2024), and heterogeneous-agent models in the spirit of Bewley (1986) and Aiyagari (1994). In the former, the share of constrained households is fixed exogenously. In the latter, the share of constrained households changes in response to aggregate shocks. However, when solutions are computed as a first-order approximation around the steady-state distribution, the share of constrained households is treated as if it were fixed.

**Roadmap.** We start in Section 2 with a partial-equilibrium exercise that highlights the key mechanism behind the nonlinearity and illustrates its quantitative significance. In Section 3, we then analyze various general-equilibrium models with different assumptions about production, nominal rigidities, fiscal financing and monetary policy. In Section 4, we derive a general expression for the curvature of the aggregate consumption function and give examples of shocks and other policy interventions where linear solutions approximate nonlinear solutions much better, and others, such as transfers targeted to low-income households, where they perform worse.

# 2 Partial Equilibrium

## 2.1 Model

**Demographics.** We consider a discrete-time, infinite-horizon economy, $t = 0, 1, \ldots$. The economy is populated by a measure-one continuum of households, indexed by $i$. We assume the path for aggregate disturbances is known at $t = 0$, so there is no aggregate uncertainty.

**Preferences.** Households derive utility from consumption, with $u(c_{i,t})$ strictly increasing and concave, and discount future utility with discount factor $\beta$. We introduce discount-factor heterogeneity in the calibrated model in Section 2.3.

**Income and taxes.** Each period households receive earnings $y_{i,t}$ whose realizations are independent across households. Earnings combine a persistent component, denoted by $y^P_{i,t}$, which follows a Markov process, with transitory components. We assume that the income

process is such that the cross-sectional distribution of income has a smooth density and does not contain mass points. We describe the calibration of the income process in Section 2.3. Households pay a proportional tax at rate $\tau$ on their earnings and receive a constant lump-sum transfer $T$ from the government.

**Assets.** Households can save in a one-period risk-free real asset that pays an interest rate $r$. The amount saved in period $t$ is $a_{i,t}$, which yields $(1+r)a_{i,t}$ in period $t+1$. They face the borrowing constraint $a_{i,t} \geq 0$.

**Household problem.** Let $x_{i,t} \equiv (1+r)a_{i,t-1} + (1-\tau)y_{i,t} + T$ denote household $i$'s cash-on-hand at the beginning of period $t$. The household problem is

$$\begin{aligned} \max_{\{c_{i,t},a_{i,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \\ \text{subject to} \\ c_{i,t} + a_{i,t} &\leq x_{i,t}, \\ x_{i,t+1} &= (1+r)a_{i,t} + (1-\tau)y_{i,t+1} + T, \\ a_{i,t} &\geq 0. \end{aligned}$$

We restrict attention to parameter configurations such that the economy admits a unique stationary distribution of assets and assume that each household's initial assets $a_{i,-1}$ are drawn from that stationary distribution.

The first-order condition of this problem yields the standard intertemporal Euler equation,

$$u'(c_{i,t}) \geq \beta(1+r)\mathbb{E}_t u'(c_{i,t+1}),$$

with equality if $a_{i,t} > 0$. We refer to households that set $a_{i,t} = 0$ as *constrained* and those that set $a_{i,t} > 0$ as *unconstrained*.

**Marginal propensities to consume.** The solution to the household problem yields a consumption policy function $c(x, y^P)$ whose arguments are cash-on-hand $x$ and the persistent components of income $y^P$. We define the marginal propensity to consume (MPC) for a household with state $(x, y^P)$, which we denote by $\mathfrak{m}(x, y^P)$, as the right derivative of $c(x, y^P)$.[2]

[2]When we use the unqualified term MPC or MPC function, we are referring to the local MPC, or the MPC out of an infinitesimal transfer. When we use the term MPC "out of an amount $\Delta$", we are referring to $\mathfrak{m}^{\Delta}(x, y^P) \equiv \frac{c(x+\Delta, y^P) - c(x, y^P)}{\Delta}$.

For a household with persistent income $y^P$, we denote the level of cash-on-hand $x$ at which the borrowing constraint stops binding by $\bar{x}(y^P)$. A constrained household consumes all its cash-on-hand and hence $c(x, y^P) = x$ for $x \leq \bar{x}(y^P)$. An unconstrained household has $c(x, y^P) < x$. In Appendix B.4, we show that the consumption function $c(x, y^P)$ is differentiable in $x$ for $x > \bar{x}(y^P)$ and is continuous but not differentiable at $x = \bar{x}(y^P)$. For $x > \bar{x}(y^P)$, it is also strictly concave in $x$ (see Carroll and Kimball, 1996). The MPC function is hence given by

$$\mathfrak{m}(x, y^P) = \begin{cases} 1, & x < \bar{x}(y^P), \\ c_x^+(x, y^P), & x \geq \bar{x}(y^P), \end{cases} \tag{1}$$

where $c_x^+(x, y^P)$ is the right derivative of $c(x, y^P)$. In Appendix B.4 we also show that $c_x^+(\bar{x}(y^P), y^P) < 1$, implying that the MPC drops discontinuously at $x = \bar{x}(y^P)$, at which point the consumption function is kinked. Constrained households below the kink consume one-for-one out of additional cash and have an MPC equal to 1. Unconstrained households have an MPC strictly less than 1. Figure 1a illustrates this discontinuity. It plots the MPC function $\mathfrak{m}(x, y^P)$ in our calibrated economy for households in the bottom and top 10% of the persistent component of earnings. Both lines are equal to 1 up to the point where the borrowing constraint stops binding (about \$15K and \$54K of cash-on-hand, respectively) and then drop discontinuously.

**Aggregation.** The steady state of the model is a joint distribution of households across cash-on-hand $x$ and the persistent components of income $y^P$. We describe the distribution in terms of the cumulative distribution function of persistent income, $F(y^P)$, and the cumulative distribution of cash-on-hand conditional on the persistent components of income, $G(x \mid y^P)$.

**Policy experiment.** Starting from the stationary distribution at $t = 0$, we study an unexpected one-time increase of $\Delta$ in the lump-sum transfer to all households. The transfer equals $T + \Delta$ at $t = 0$ and $T$ for $t > 0$.[3] For a transfer of size $\Delta$, we denote aggregate consumption at time $t$ by $\mathcal{C}_t(\Delta)$. Since this is a one-time transfer in partial equilibrium, the household policy function $c(x, y^P)$ is not affected by the transfer, and so aggregate consumption in the initial period is given by

$$\mathcal{C}_0(\Delta) = \int_{Y^P} \int_0^\infty c(x + \Delta, y^P) \, \mathrm{d}G(x \mid y^P) \, \mathrm{d}F(y^P),$$

[3]Note this is not a zero-probability event from the perspective of an individual household because the income process includes a transitory component.

where $Y^P$ is the space of possible realizations of $y^P$.

## 2.2 Inspecting Nonlinearity

**Sources of nonlinearity.** The main point of our paper is that $\mathcal{C}_0(\Delta)$ is highly concave in $\Delta$. There are two reasons for the concavity of the aggregate consumption function. First, for unconstrained households the consumption policy function $c(x, y^P)$ is concave in cash-on-hand $x$ (see Carroll and Kimball, 1996). Second, a transfer moves some constrained households off the borrowing constraint, lowering their MPC out of additional transfers.

To illustrate how these sources of nonlinearity are captured by local approximations of $\mathcal{C}_0(\Delta)$ around $\Delta = 0$, we decompose $\mathcal{C}_0(\Delta)$ into contributions from constrained and unconstrained households, as follows:

$$\mathcal{C}_0(\Delta) = \int_{Y^P} \left[ \underbrace{\int_0^{\bar{x}(y^P)-\Delta} c(x+\Delta, y^P)\, \mathrm{d}G(x \mid y^P)}_{\substack{\text{constrained} \\ \text{households}}} + \underbrace{\int_{\bar{x}(y^P)-\Delta}^{\infty} c(x+\Delta, y^P)\, \mathrm{d}G(x \mid y^P)}_{\substack{\text{unconstrained} \\ \text{households}}} \right] \mathrm{d}F(y^P). \tag{2}$$

After receiving a transfer of size $\Delta$, a household is constrained if and only if its post-transfer cash-on-hand is less than $\bar{x}(y^P)$, which is the case if $x < \bar{x}(y^P) - \Delta$. This is reflected in the limits on the integrals in equation (2). As $\Delta$ rises, the mass of households for which the constraint binds falls. We let $\Lambda(\Delta) = \int_{Y^P} G(\bar{x}(y^P) - \Delta \mid y^P)\, \mathrm{d}F(y^P)$ denote the share of households that remain constrained after receiving a transfer $\Delta$.

**First-order derivative.** Differentiating (2) with respect to $\Delta$ and evaluating at $\Delta = 0$ gives[4]

$$\mathcal{C}_0'(0) = \bar{\Lambda} \times \underbrace{1}_{\substack{\text{MPC constrained} \\ \text{households}}} + (1 - \bar{\Lambda}) \underbrace{\mathbb{E}\big[c_x^+ \mid x \geq \bar{x}(y^P)\big]}_{\substack{\text{MPC unconstrained} \\ \text{households}}}, \tag{3}$$

where $\bar{\Lambda} \equiv \Lambda(0)$ is the mass of constrained households in the stationary distribution.

The derivative of the aggregate consumption function $\mathcal{C}_0'(0)$ (which we refer to as the aggregate MPC) is a weighted average of the MPCs of constrained and unconstrained households, with weights given by their steady-state population shares. The expectation in the second term is taken over the joint stationary distribution of cash-on-hand and persistent income, conditional on the household being unconstrained. The transfer does affect the share of constrained households, which can be seen by the presence of $\Delta$ in the integration limits

[4] See Appendix B for derivations of the results in this section.

in (2). However, because the consumption function is continuous at $\bar{x}(y^P)$, this change in the share of constrained households does not appear in the first derivative of $\mathcal{C}_0(\Delta)$.

An implication of (3) is that the aggregate MPC at the steady state equals a weighted average of the mean MPCs of constrained and unconstrained households, with weights given by their initial population shares. Thus, a first-order approximation of aggregate consumption after a transfer of size $\Delta$, which we denote by $\mathcal{C}_0^L(\Delta)$

$$\mathcal{C}_0^L(\Delta) = \mathcal{C}_0(0) + \mathcal{C}_0'(0)\Delta \tag{4}$$

does not capture the effect of households moving off the constraint as a result of the transfer, or the curvature of the consumption function for unconstrained households. See Appendix B.4 for the corresponding first-order responses at later dates, which are the intertemporal MPCs of Auclert et al. (2024).

**Second-order derivative.** The second derivative of $\mathcal{C}_0(\Delta)$ evaluated at $\Delta = 0$ is

$$\mathcal{C}_0''(0) = \underbrace{\Lambda'(0)\,\mathbb{E}\left[\left(1 - c_x^+\right) \mid x = \bar{x}(y^P)\right]}_{\substack{\text{MPC drop for households} \\ \text{moving off the constraint}}} + \underbrace{(1-\bar{\Lambda})\,\mathbb{E}\left[c_{xx}^+ \mid x \geq \bar{x}(y^P)\right]}_{\substack{\text{curvature of consumption for} \\ \text{unconstrained households}}}. \tag{5}$$

The term $\Lambda'(0) = -\int_{Y^P} g(\bar{x}(y^P) \mid y^P)\,\mathrm{d}F(y^P) < 0$ is the marginal change in the mass of constrained households, where $g(\cdot \mid y^P)$ is the density of $G(\cdot \mid y^P)$. Its magnitude is determined by the stationary density of households at the kinks of the consumption function.

The two terms in equation (5), both of which are negative, reflect the two sources of nonlinearity. The first term is the effect of households moving off the constraint as a result of the transfer. A marginally larger transfer moves a mass of households $-\Lambda'(0)$ into the unconstrained region, where their MPC drops discontinuously from 1 to $c_x^+(\bar{x}(y^P), y^P) < 1$, lowering the aggregate consumption response. The second term is the familiar concavity of individual consumption policies for unconstrained households in incomplete-markets models. As in Carroll and Kimball (1996), a higher transfer reduces the MPC for unconstrained households.

A second-order approximation of aggregate consumption after a transfer of size $\Delta$, which we denote by $\mathcal{C}_0^Q(\Delta)$,

$$\mathcal{C}_0^Q(\Delta) = \mathcal{C}_0(0) + \mathcal{C}_0'(0)\Delta + \frac{1}{2}\mathcal{C}_0''(0)\Delta^2$$

therefore captures both sources of nonlinearity, but in a restricted way. For initially uncon-

strained households, it assumes that the consumption function is approximately quadratic over the range $x$ to $x + \Delta$. For initially constrained households, it implicitly assumes that the mass who remain constrained declines linearly in the size of the transfer $\Delta$.

**Third-order derivative.** The third derivative of $\mathcal{C}_0(\Delta)$ at $\Delta = 0$ can be expressed as

$$\mathcal{C}_0'''(0) = \underbrace{\Lambda''(0)\,\mathbb{E}\big[1 - c_x^+ \mid x = \bar{x}(y^P)\big] - \Lambda'(0)\,\mathrm{Cov}\left(1 - c_x^+,\, \frac{g'}{g}\,\middle|\, x = \bar{x}(y^P)\right)}_{\text{refinement to MPC drop for households moving off the constraint}} \tag{6}$$

$$\underbrace{-\Lambda'(0)\,\mathbb{E}\big[c_{xx}^+ \mid x = \bar{x}(y^P)\big]}_{\text{curvature of consumption for households moved off the constraint}} + \underbrace{(1 - \bar{\Lambda})\,\mathbb{E}\big[c_{xxx}^+ \mid x \geq \bar{x}(y^P)\big]}_{\text{third-order correction for unconstrained households}},$$

where $g'(\cdot \mid y^P)$ is the derivative of the density of cash-on-hand.

The first three terms in equation (6) refine the consumption response for initially constrained households, relative to the second-order approximation. The first term accounts for the fact that as $\Delta$ increases, the marginal share of households pushed off the constraint changes. This is captured by the local curvature of the constrained share $\Lambda''(0)$. The covariance in the second term accounts for the fact that, as $\Delta$ rises, the mix of persistent-income types crossing the kink changes, reflected in the term $g'/g$, and the drop in the MPC differs across these households. The third term, on the second line, captures the curvature of the consumption function for initially constrained households once they have left the constraint. The final term refines the consumption response for initially unconstrained households by allowing for third-order curvature in the consumption policy function.

## 2.3 Steady-State Calibration

**Demographics.** A period represents a quarter. We express model outputs in terms of annual GDP per household, which was around $167,500 in 2019.

**Income process.** We model log earnings, $\log y_{i,t}$, as the sum of three orthogonal components. The first two are an AR(1) persistent component $y_{i,t}^P$ and an *iid* transitory component $y_{i,t}^T$. Innovations to both components arrive infrequently according to independent Poisson processes. The five parameters of the earnings process are taken from the estimated-arrivals specification of Kaplan and Violante (2022), who estimate them using second and fourth moments of annual earnings changes from the Panel Study of Income Dynamics. The estimated quarterly arrival probabilities are 0.237 for persistent innovations and 0.073 for

transitory shocks, corresponding to average intervals of about one year and three and a half years, respectively. Appendix A describes the parameterization and the discretization of the process.

The third component is an *iid* noise shock $y_{i,t}^N$ which arrives every quarter and is drawn from a Gaussian distribution with parameters $\big(-\frac{\sigma_N^2}{2}, \sigma_N^2\big)$. We include a noise shock of this type following the ideas and arguments in Bhandari et al. (2023) and Auclert et al. (2026), who explain the importance of a smooth income distribution for computing second- and higher-order approximations. Including this component ensures that the cross-sectional distributions of income and cash-on-hand are smooth, as assumed in Section 2. We set the standard deviation of the noise shock to $\sigma_N = 0.005$. We choose this value because it is large enough to ensure that higher-order derivatives are accurate, while being small enough to not have a meaningful effect on either the nonlinear or linear solutions. See Table C.1 and Appendix C.5 for comparisons and further discussion.

**Preferences.** We assume $u(c) = \log(c)$. In our baseline calibration, we introduce discount-factor heterogeneity. We assume the discount factor $\beta_i$ is fixed over time for each household, and is uniformly distributed across five values on an equally spaced grid. We calibrate the location and range of the grid for $\beta_i$ so that the ratio of mean wealth to annual GDP is 100% ($167,500) and the average of the quarterly MPC out of a one-time $1,000 windfall, $\mathfrak{m}^{\$1,000}(x, y^P)$, is 0.25. The resulting discount factors range from 0.960 to 0.992.

**Tax and transfers.** We set the proportional tax rate $\tau = 0.274$ to match total government revenue of 27.4% of GDP in 2019. We set the lump-sum transfer $T = 0.14$ (equivalent to around $23,500 per household per year) to match federal social benefits to domestic households.[5]

**Assets.** We set the interest rate to 2% p.a. and the borrowing limit to $\underline{a} = 0$. The positive lump-sum transfer $T$ implies that the natural borrowing limit is strictly negative despite the lower bound on income being 0.

**Cash-on-hand and MPC distributions.** Figure 1a shows the stationary distribution of cash-on-hand overlaid with the MPC function $\mathfrak{m}(x, y^P)$, for households with low and high

[5]Government social benefits to persons, FRED series B087RC1Q027SBEA (https://fred.stlouisfed.org/series/B087RC1Q027SBEA), which excludes transfers to persons abroad.

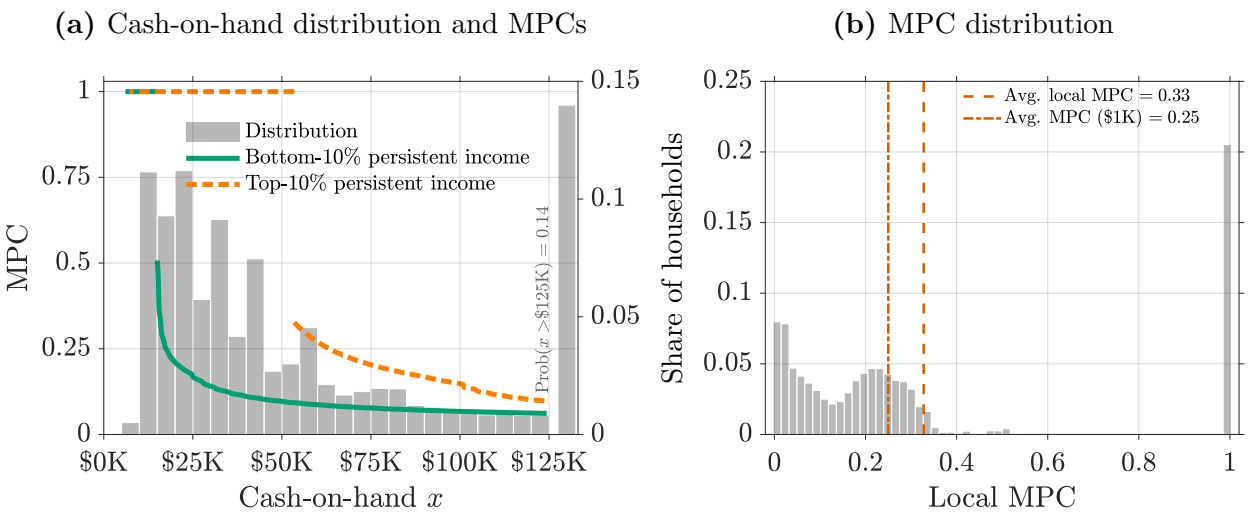



**Figure 1:** Steady-state cash-on-hand distribution and MPCs

*Note:* The left axis of panel (a) plots the local MPC $\mathfrak{m}(x, y^P)$ as a function of cash-on-hand $x$ for two households with the median $\beta_i$ that differ in the persistent component of earnings: one in the bottom 10% (the persistent state that contains the 10th percentile) and one in the top 10% (the persistent state that contains the 90th percentile). The right axis of panel (a) presents the distribution of cash-on-hand among households with the median $\beta_i$, in \$5K bins up to \$125K; the bar on the right shows the share of these households with cash-on-hand above \$125K. Panel (b) presents the distribution of local MPCs. The dashed and dash-dotted vertical lines mark the average local MPC and the average MPC out of \$1K, respectively.

persistent income.[6] Figure 1b shows a histogram of local MPCs, with the average local MPC and the average MPC out of a \$1,000 windfall shown for comparison by the vertical dashed and dash-dotted lines, respectively.

Two features of MPCs in this class of models are apparent. First, the MPC is declining in cash-on-hand. For each level of persistent income, it equals 1 up to the point where the borrowing constraint stops binding and then drops discontinuously, after which it declines smoothly as cash-on-hand rises. The level of cash-on-hand at which the constraint stops binding is higher for households with higher persistent income. Households with higher persistent income expect higher future income and hence want to consume more today, so they remain constrained for higher levels of cash-on-hand, and their MPC stays higher once they are unconstrained.

Second, the distribution of MPCs is highly dispersed and bimodal. The subset of constrained households has an MPC of 100%, while the remaining unconstrained households

[6]We plot the MPC out of an infinitesimal windfall because this is the MPC that is relevant for the linear vs nonlinear comparisons that are the focus of this paper. We calibrate to the MPC out of a \$1,000 windfall because that is more consistent with the empirical evidence we target.

have much lower MPCs, ranging from close to 0% to around 50%.

## 2.4 Aggregate Transfers in Partial Equilibrium

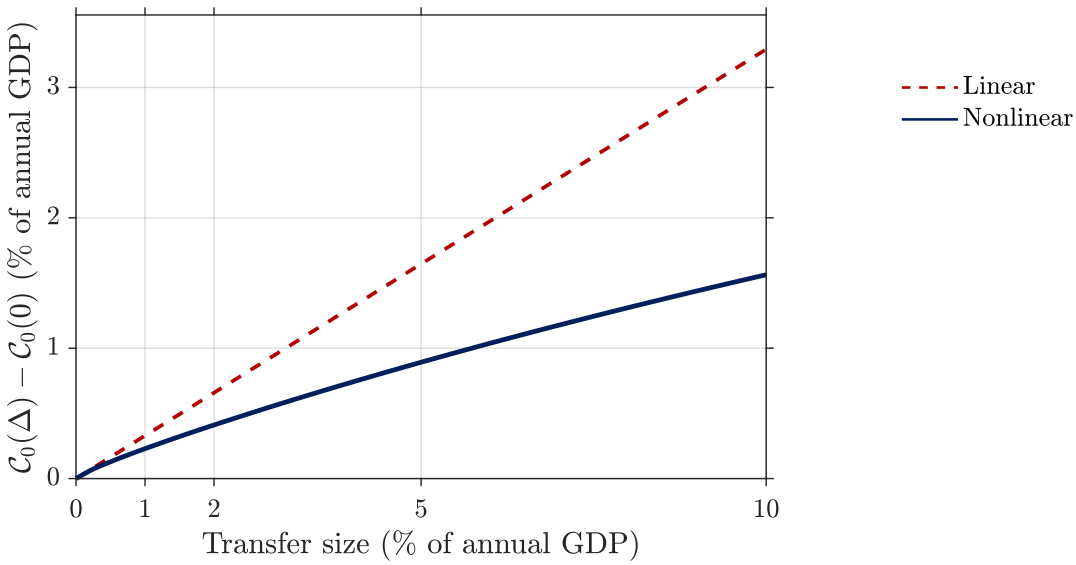
**Computation.** We compute steady-state decision rules using the method of endogenous grid points (Carroll, 2006). Since we are studying a one-time transfer at $t = 0$ in partial equilibrium, we can compute the nonlinear response to the transfer by propagating forward the household wealth distribution using the steady-state decision rules, starting from the steady-state distribution with each household's cash-on-hand shifted up by $\Delta$. We compute the linear approximation using the formula in equation (4). See Appendix C for computational details, including computation of the derivative $\mathcal{C}_0'(0)$.

**Accuracy of linear approximation.** Figure 2 contains the main results of the partial-equilibrium exercise. Figure 2a shows the first-quarter increase in aggregate consumption $\mathcal{C}_0(\Delta) - \mathcal{C}_0(0)$ as a function of the size of the transfer $\Delta$, both expressed as a percentage of annual GDP per household ($167,500). The blue solid line labeled "Nonlinear" shows the results computed using the nonlinear solution and the red dashed line labeled "Linear" shows the results from the linear approximation in (4). For small transfers, the linear and nonlinear solutions produce similar aggregate consumption responses. But for larger transfers, the linear solution substantially overstates the consumption response. For a transfer of 0.5% of annual GDP (around $840 per household), the linear solution overstates the nonlinear solution by about 28%, and for a transfer of 1% of annual GDP (around $1,680), by about 44%.

**Historical stimulus episodes and nonlinearities.** To place the size of transfers and nonlinearities in context, consider a few fiscal stimulus packages in U.S. history. The Tax Reduction Act of 1975, signed by President Gerald Ford, provided a one-time rebate of 1974 federal income taxes. The rebate payments generally ranged from $100 to $200 per tax return, equivalent to about 0.4–0.8% of annual GDP per household. The rebate amounts under the Economic Growth and Tax Relief Reconciliation Act of 2001 corresponded to approximately 0.3–0.6% of annual GDP per household.[7] The Economic Stimulus Act of 2008 provided larger rebates: the maximum basic payments corresponded to around 0.5% of

[7]The maximum rebates were $300 for single filers, $500 for heads of household, and $600 for married couples filing jointly.

**(a)** Aggregate effects of a one-time transfer in partial equilibrium




**(b)** Ratio of linear to nonlinear aggregate consumption response

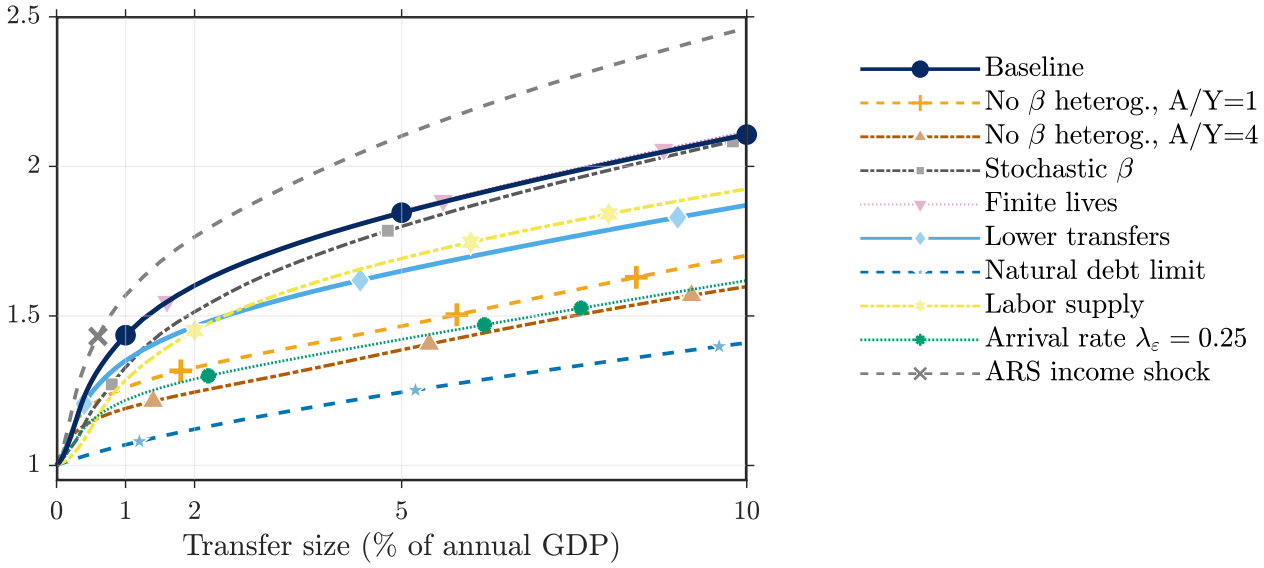



**Figure 2:** Partial-equilibrium effects of a one-time transfer

*Notes:* Panel (a) shows the first-quarter consumption response, $\mathcal{C}_0(\Delta) - \mathcal{C}_0(0)$, and the transfer $\Delta$, both as a percentage of annual GDP. Panel (b) presents the ratio of the linear to the nonlinear consumption response for different model specifications. See Table A.3 in Appendix A.3 for more details.

annual GDP per household for single filers and 1% for married couples filing jointly.[8] During the COVID-19 pandemic, the cumulative amount of fiscal transfers paid to households was around 12.5% of annual GDP, spread over several quarters.[9] In practice, stimulus transfers are often directed toward lower-income households, which increases the degree of nonlinearity (see Section 4).

**Alternative calibrations.** Figure 2b shows that the concavity of $\mathcal{C}_0(\Delta)$ is a robust feature of this class of models and is not specific to our particular calibration and choice of model features. The figure shows the ratio of the linear consumption response to the nonlinear response for transfers of different sizes, for a range of alternative calibrations and versions of the model. We report the results in terms of the ratio of the linear to the nonlinear consumption responses because the size of the response differs across models. Figure G.5 in Appendix G reports the corresponding four-quarter ratios.

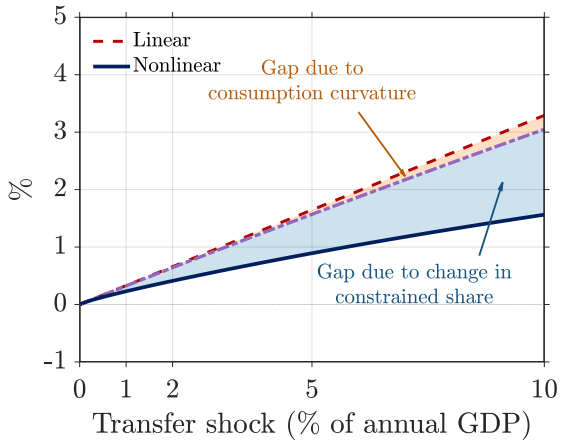
The figure includes versions of the model without discount-factor heterogeneity; with stochastic discount factors; with different levels of aggregate wealth; a life-cycle version with finite lives; with lower lump-sum transfers; with a looser borrowing limit set equal to the natural borrowing limit; with endogenous labor supply; with a version of our baseline earnings process in which transitory shocks arrive on average once per year; and with the idiosyncratic earnings process from Auclert et al. (2024). In all these versions, and many others we have experimented with, the linear solution overstates the aggregate consumption response by meaningful amounts. For a transfer of 1% of annual GDP ($1,680 per household), the overstatement ranges from 7% (natural borrowing limit) to 57% (the earnings process of Auclert et al., 2024).

**Decomposition of nonlinearity.** In Figure 3a we decompose the difference between the linear and nonlinear response in the baseline calibration into the two sources of concavity

[8]The stimulus also included supplements for qualifying children and income-based phaseouts. The maximum basic payments were $600 for single filers and $1,200 for married couples filing jointly, plus $300 per qualifying child under 17.

[9]The stimulus included (i) the CARES Act (March 2020, approximately $2.2 trillion, including direct payments to households, expanded unemployment insurance, and forgivable loans to firms under the Paycheck Protection Program); (ii) the December 2020 relief package (approximately $0.9 trillion); and (iii) the American Rescue Plan (March 2021, approximately $1.9 trillion). The federal deficit was roughly $1.6 trillion above trend, equivalent to around $12,000 per household, or around 7.5% of annual GDP.

**(a)** Decomposition of the linear–nonlinear gap




**(b)** Share of constrained households

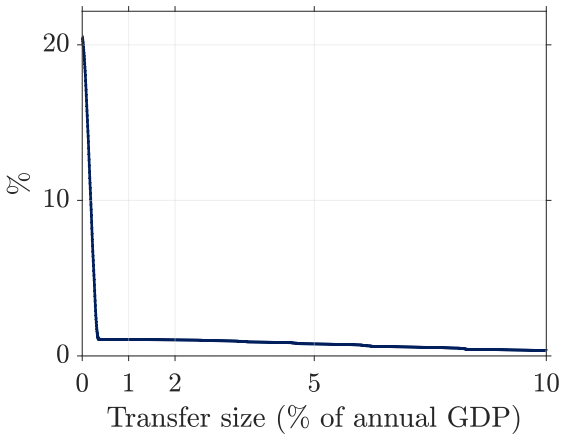



**Figure 3:** Decomposition of the linear–nonlinear gap and the constrained share

*Note:* In panel (a), the dash-dotted line ("fixed-constrained-share series") assigns consumption $x + \Delta$ to initially constrained households and uses the fully nonlinear consumption response of initially unconstrained households. Panel (b) presents the share of constrained households at $t = 0$ as a function of $\Delta$.

described in Section 2.2. In Appendix B.3 we show that this difference can be expressed as

$$\mathcal{C}_0^L(\Delta) - \mathcal{C}_0(\Delta) = \underbrace{\int_{Y^P} \int_{\bar{x}(y^P)-\Delta}^{\bar{x}(y^P)} \left[x + \Delta - c(x + \Delta, y^P)\right] \, \mathrm{d}G(x \mid y^P) \, \mathrm{d}F(y^P)}_{\text{households who cross the constraint}}$$
$$+ \underbrace{\int_{Y^P} \int_{\bar{x}(y^P)}^{\infty} \left[c(x, y^P) + c_x(x, y^P)\Delta - c(x + \Delta, y^P)\right] \, \mathrm{d}G(x \mid y^P) \, \mathrm{d}F(y^P)}_{\text{initially unconstrained households}}. \tag{7}$$

The first term is the difference between linear and nonlinear consumption for initially constrained households who move off the constraint following the transfer. The linear approximation continues to apply their initial MPC of 1 and hence predicts consumption of $x + \Delta$, whereas in the nonlinear solution their consumption is $c(x+\Delta, y^P) < x+\Delta$. The second term captures the gap due to curvature in the consumption functions of initially unconstrained households. The figure shows that the vast majority of the nonlinearity in the aggregate consumption response comes from households who move off the constraint, rather than from curvature among initially unconstrained households. At a transfer of 1% of annual GDP, households that move off the borrowing constraint account for about 95% of the gap between the linear and nonlinear impact responses (Figure E.4 in Appendix E reports this share by transfer size).

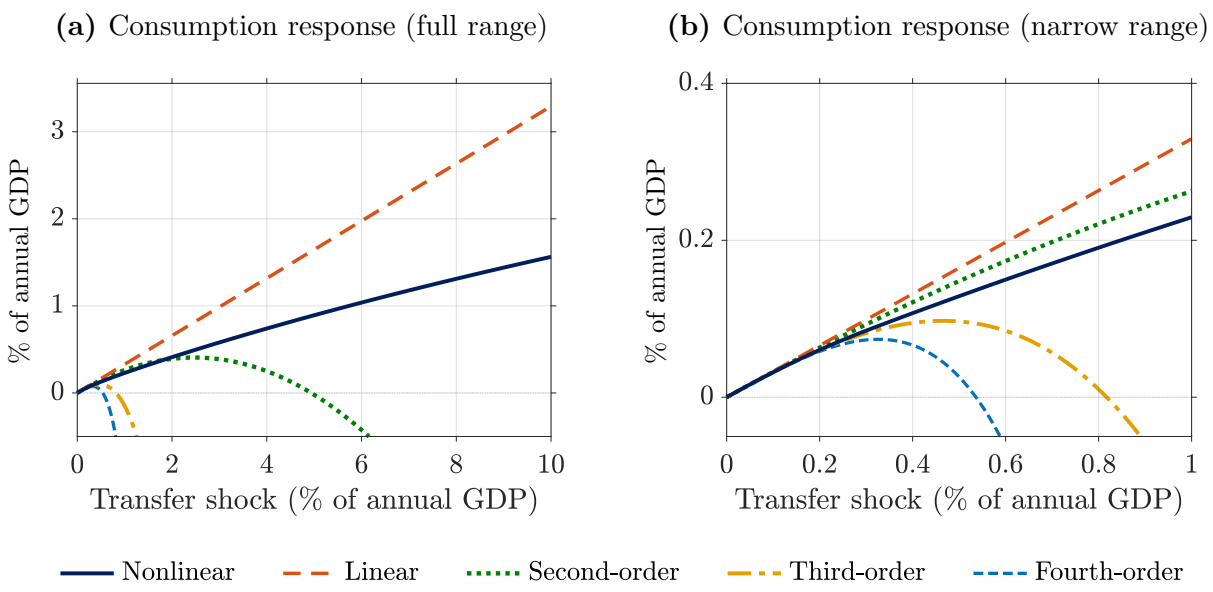



**Figure 4:** Local higher-order approximations to the impact response of aggregate consumption

*Note:* Impact response of aggregate consumption, $\mathcal{C}_0(\Delta) - \mathcal{C}_0(0)$, to a one-time transfer $\Delta$ in the baseline partial-equilibrium model. The linear and second- to fourth-order lines are Taylor expansions of $\mathcal{C}_0(\Delta)$ around $\Delta = 0$. The nonlinear response and the linear, second- and third-order approximations use the baseline grid with 800 points; only the fourth-order approximation uses 10,000 points. See Appendix C.4 for computational details.

**Higher-order approximations.** Figure 4 reports higher-order Taylor approximations of $\mathcal{C}_0(\Delta)$ around $\Delta = 0$. These approximations require $\mathcal{C}_0(\Delta)$ to be sufficiently smooth, and we thus follow Bhandari et al. (2023) and Auclert et al. (2026) in allowing for a noise shock, as described in Section 2.3.[10] Appendix C.4 describes how we compute the approximations, and provides several robustness analyses.

Figure 4a compares the nonlinear solution with the linear and higher-order approximations over the full range of shock sizes up to 10% of annual GDP, and Figure 4b zooms in on shocks up to 1% of annual GDP. For small shocks, the higher-order approximations improve on the linear approximation. The second-order approximation starts to understate the aggregate response for shocks of about 2% of annual GDP, but remains more accurate than the linear approximation for shocks up to around 4% of annual GDP. The third-order approximation, by contrast, is more accurate than the linear approximation only for transfers below about 0.5% of annual GDP (around $900 per household). The fourth-order approxi-

[10] Bhandari et al. (2023) assume idiosyncratic shocks with a density to guarantee differentiability. In the leading example of Bilal (2023), income follows a diffusion, so the distribution has a density except at the borrowing limit, where there is a mass point.

mation becomes less accurate than the linear approximation for transfers above about 0.3% of annual GDP (around $550 per household).

**Change in the share of constrained households.** To understand these findings, recall from Figure 3a that the majority of the nonlinearity in $\mathcal{C}_0(\Delta)$ arises because the mass of constrained households falls as the size of the transfer increases. Equations (3), (5) and (6) show how each order of approximation implicitly captures this movement of households off the constraint. The first-order approximation uses information only on the steady-state share, $\bar{\Lambda}$, the second-order approximation adds information on the slope $\Lambda'(0)$, the third-order approximation adds information on the curvature $\Lambda''(0)$, and so on. Intuitively, the reason for the poor approximation of $\mathcal{C}_0(\Delta)$ for larger values of $\Delta$ is that the share of constrained households in the nonlinear solution, $\Lambda(\Delta)$, is not captured well by these derivatives at $\Delta = 0$. This can be seen in Figure 3b, which plots $\Lambda(\Delta)$ in our baseline calibration. $\Lambda(\Delta)$ initially falls rapidly as more households are pushed off the borrowing constraint as $\Delta$ increases, but for shock sizes above 0.35% of annual GDP there are almost no more households to become unconstrained.[11]

# 3 General Equilibrium Model

In this section, we study the extent to which the nonlinearity of the aggregate consumption function impacts the general equilibrium effects of one-time fiscal stimulus programs. We extend the model from the previous section to include production, labor supply, asset market clearing and nominal rigidities, as in canonical HANK models (e.g., Kaplan et al., 2018; Auclert et al., 2023).

[11] In Appendix F we describe a model without idiosyncratic shocks but with heterogeneity in initial assets to illustrate why Taylor approximations around zero can fail to recover the nonlinear response at a fixed transfer, regardless of how high the approximation order is. In that model, all aggregate nonlinearity comes from households moving off the borrowing constraint. Once the constrained share reaches zero, aggregate consumption becomes exactly affine, with a slope that is less than the aggregate MPC. The Taylor series around zero therefore cannot represent the consumption response over an interval that extends past this threshold. See Proposition 1 in Appendix F.

## 3.1 Model

**Preferences.** Households have utility over real consumption $c_{i,t}$ and hours worked $n_{i,t}$:

$$\mathbb{E}_0 \sum_{t \geq 0} \beta_i^t \left[ u(c_{i,t}) - v(n_{i,t}) \right],$$

where $v(\cdot)$ is a strictly increasing, convex function of hours worked.

**Income and assets.** Real household income consists of earnings $e_{i,t} \frac{W_t}{P_t} n_{i,t}$ and dividend income $d_{i,t}$, where $e_{i,t}$ is an idiosyncratic productivity shock, normalized so that the cross-sectional mean of efficiency units equals one, $\int_i e_{i,t}\, di = 1$. In this section, we denote total non-asset income as $y_{i,t} = e_{i,t} \frac{W_t}{P_t} n_{i,t} + d_{i,t}$. We denote the interest rate earned on assets held between $t$ and $t+1$ by $r_t$. With this notation, the borrowing and budget constraints are the same as in Section 2.

**Labor supply.** In our baseline model we assume flexible prices and sticky wages. The aggregate effective labor input $N_t = \int_i n_{i,t} e_{i,t} di$ is chosen by a labor union and is allocated across households so that each household works the same number of hours, implying that $n_{i,t} = N_t$. In Section 3.4 we also report results from a version of the model with sticky prices, flexible wages, and a labor supply choice at the household level.

**Production.** A representative firm produces the final good $Y_t$ with a linear production function that uses effective labor $N_t$ as its only factor of production. Equilibrium therefore implies a real wage of $\frac{W_t}{P_t} = 1$ per effective unit of labor and zero profits so that $d_{i,t} = 0$.

**Wage Phillips curve.** We adopt the wage-setting model in Auclert et al. (2024), in which monopolistically competitive labor unions set nominal wages subject to quadratic adjustment costs to maximize the welfare of a household with aggregate consumption $C_t = \int_i c_{i,t} di$ and hours $N_t$. This leads to the wage Phillips curve

$$\pi_t = \kappa \left( v'(N_t) - \frac{1-\tau}{C_t} \right) + \bar{\beta}\, \pi_{t+1}, \tag{8}$$

where $\pi_t$ denotes wage (and price) inflation, $\kappa$ depends on the wage adjustment cost and $\bar{\beta}$ is the average discount factor. This is the same linearized wage Phillips curve that arises in an analogous representative-agent economy (e.g., Erceg et al., 2000). We choose to work

directly with the linearized Phillips curve so as to focus entirely on the nonlinearity of the consumption function stemming from the borrowing constraint.[12]

**Fiscal policy.** We allow for the government to set a time-varying lump-sum transfer $T_t$. The government issues real bonds $B_t$ subject to the budget constraint[13]

$$B_t - B_{t-1} = G + T_t - \tau Y_t + r_{t-1} B_{t-1}, \tag{9}$$

where $G$ is a fixed level of government consumption. Outside the steady state, the government adjusts the lump-sum transfer component of the tax-and-transfer schedule according to the rule

$$T_t = T - \phi_B \bar{r}(B_{t-1} - \bar{B}), \tag{10}$$

where $\phi_B > 1$ and $\bar{r}$ and $\bar{B}$ are the steady-state levels of the interest rate and government debt. This restriction implies that transfers are set so as to ensure that debt always returns to its steady-state level. Hence, fiscal policy is passive in the language of Leeper (1991).

**Monetary policy.** The central bank sets the nominal interest rate according to the Taylor rule

$$i_t = \bar{i} + \phi_\pi(\pi_t - \bar{\pi}), \tag{11}$$

where $\bar{i}$ is the steady-state nominal rate, $\bar{\pi}$ is steady-state inflation and $\phi_\pi > 1$. Hence, monetary policy is active in the language of Leeper (1991). A standard Fisher equation connects real and nominal rates, $1 + i_t = (1 + r_t)(1 + \pi_{t+1})$ for $t \geq 0$.

**Equilibrium.** Given fiscal and monetary policy rules (10) and (11), and an initial distribution of households over assets, productivity and discount factors, $\Gamma_0$, an equilibrium is a sequence of household policy functions $\{c_t(a, e, \beta), a'_t(a, e, \beta)\}_{t=0}^{\infty}$ and associated distributions $\{\Gamma_t(a, e, \beta)\}_{t=1}^{\infty}$, aggregate quantities $\{Y_t, C_t, B_t, N_t, d_t\}_{t=0}^{\infty}$ and prices $\{P_t, W_t, \pi_t, i_t, r_t\}_{t=0}^{\infty}$ such that (i) households and firms optimize; (ii) the wage Phillips curve (8) is satisfied, $P_t = W_t$ and $d_t = 0$; (iii) the government budget constraint (9) holds; (iv) the asset market clears

$$B_t = \int a'_t(a, e, \beta)\, \mathrm{d}\Gamma_t(a, e, \beta) \quad \text{for } t \geq 0;$$

[12] Kaplan and Miyahara (2026) examine the nonlinearities from endogenous price adjustment in a menu cost model. Other examples of studies on the importance of nonlinearities in different contexts with incomplete markets include Cao and Nie (2017) and de Groot, Durdu and Mendoza (2025). See Eggertsson and Singh (2019) for nonlinearities arising from the zero lower bound (ZLB).

[13] We present results for an economy in which the government issues nominal bonds in Section 3.4.

and (v) aggregate consumption is given by

$$C_t = \int c_t(a, e, \beta)\, \mathrm{d}\Gamma_t(a, e, \beta) \quad \text{for } t \geq 0.$$

Walras' law implies that the goods-market-clearing condition $C_t + G = Y_t$ holds.

## 3.2 Calibration

The calibration follows closely the parameterization of the partial equilibrium model in Section 2. Below we describe the features that are specific to the general equilibrium version. See Table A.4 in Appendix A.3 for further details.

**Preferences.** We assume the following functional form for the disutility of labor

$$v(n) = \chi \frac{n^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}},$$

where $\nu$ is the Frisch elasticity of labor supply, which we set equal to 1. We normalize $\chi$ so that quarterly output equals one in steady state. As in Section 2, we choose the set of discount factors to match a ratio of average wealth to annual GDP of 100% and an average quarterly MPC of 25% out of a one-time \$1,000 windfall.

**Idiosyncratic income process.** The stochastic process for individual efficiency units of labor $e_{i,t}$ is the same as the process for household labor income in Section 2.

**Wage setting.** We set the slope of the wage Phillips curve $\kappa = 0.01$, as in Auclert et al. (2024).

**Fiscal policy.** We set government consumption $G$ to 11.4% of GDP. The steady-state tax rate and lump-sum transfer are the same as in Section 2. Hence the primary surplus as a percentage of GDP is $\tau - T - G = (0.274 - 0.14) - 0.114 = 2\%$. With a debt-to-GDP ratio of 100%, this implies a steady-state interest rate of 2% p.a. We set the parameter governing the speed of repayment outside the steady state to $\phi_B = 8$. This implies that the half-life of an increase in government debt is roughly 5 years.

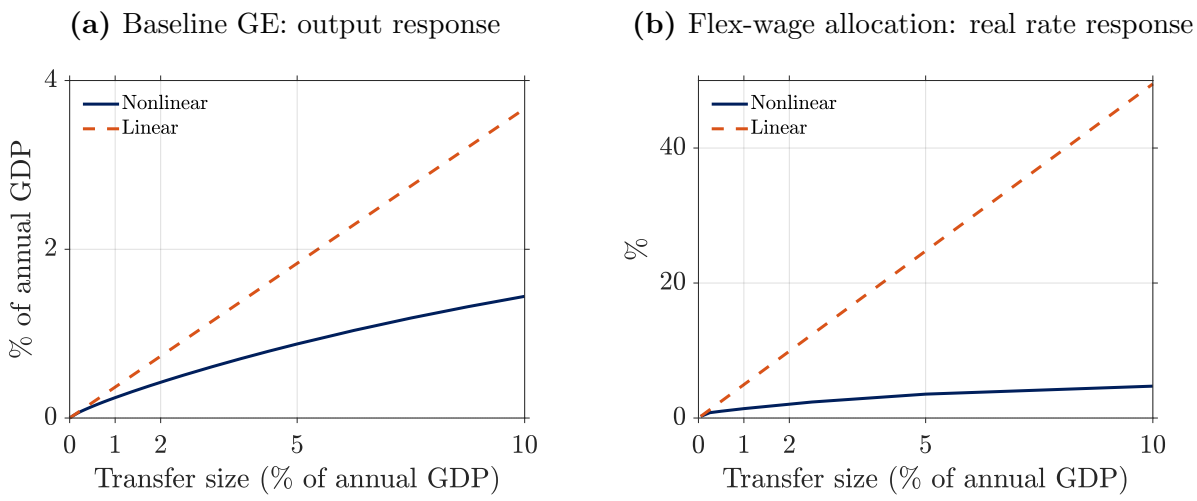



**Figure 5:** Aggregate effects of a one-time transfer in general equilibrium

*Notes:* The left panel presents the first-quarter output response with sticky wages. The right panel presents the first-quarter real interest-rate response with flexible wages. Rate deviations are annualized and expressed in percentage points.

**Monetary policy.** We set steady-state inflation to $\bar{\pi} = 0$, so that the steady-state nominal rate is $\bar{\imath} = 2\%$ p.a. In our baseline experiments we set the Taylor rule coefficient $\phi_\pi = 1.5$.

## 3.3 Aggregate Transfers in General Equilibrium

**Experiment.** As in Section 2, we study an unexpected one-time increase in the lump-sum transfer to all households at $t = 0$, which we implement by setting $T_0 = T + \Delta$. In our baseline experiment we assume that the stimulus is financed by lowering future lump-sum transfers for $t > 0$ according to the fiscal rule (10).

**Computation.** We obtain the linear solutions using the SSJ method, following Auclert et al. (2021). We compute nonlinear solutions using a quasi-Newton algorithm, as described in Appendix D.

**Baseline HANK results.** Figure 5a shows the aggregate output response for different values of $\Delta$ for the linear and nonlinear solution methods. The nonlinear solution is concave in the size of the shock. For a stimulus that is 0.5% of annual steady-state GDP (around \$840 per household), the first-quarter output response is around 0.14% of annual GDP according to the nonlinear solution and around 0.18% according to the linear solution. The error in the

linear solution is thus around 33%. For a stimulus of size 1% of annual steady-state GDP (around $1,680 per household), the first-quarter error in the linear solution is around 52%.[14]

To understand where the errors in the linear solution stem from, it is useful to recap the mechanism by which a one-time transfer raises output in this model. Because of a failure of Ricardian equivalence, the combination of a higher transfer at $t = 0$ and lower future transfers leads households to want to increase their consumption, which leads to a rise in the natural interest rate. In the presence of nominal rigidities, this leads to an inflationary output boom as long as the central bank does not raise the nominal rate enough to fully offset the rise in the natural rate.[15]

The strength of this mechanism depends on the initial upward pressure on household consumption, which is determined entirely by the departure from Ricardian equivalence, as reflected in the subset of households with MPCs above the permanent-income-hypothesis (PIH) benchmark. Recall that in a representative-agent version of this experiment there would be no effect on output for any size of stimulus. But as we saw in Figure 1, the high-MPC households are those who are on or close to the borrowing constraint and so have a consumption response that is highly nonlinear in the size of the transfer. Thus the same forces that drive the concave response of consumption in the partial equilibrium experiments in Section 2 also lead to concavity in the output response in general equilibrium.

**Dynamic response.** Figure 6 shows the impulse responses of output, the real interest rate, and inflation. The linear and nonlinear solutions display different dynamics for output. The larger first-quarter response in the linear solution is partly offset by a smaller response in the subsequent quarters, so that the linear solution understates the persistence of output relative to the nonlinear solution. This greater persistence reflects saving by some households that are initially constrained but move off the constraint after receiving the transfer, leading to a higher level of aggregate demand in subsequent quarters. It follows that the linear solution provides a closer approximation to the first-year output response (see Figure G.5 in Appendix G). On the one hand, one might interpret these findings as suggesting that the linear solution provides an accurate approximation if one focuses on the first-year rather than the first-quarter response. On the other hand, a main reason for studying a quarterly model is to connect with quarterly business cycle data and estimate the stochastic properties

[14] In Figure G.3 in Appendix G we report analogous figures for unexpected reductions in transfers ($\Delta < 0$), which illustrate that the linear solution understates the fall in output.

[15] The precise mechanism by which output rises depends on the particular model of nominal rigidities. In this example, which features a monopolistic labor union, nominal wage rigidity, and a fixed real wage, the union trades off the cost of wage adjustments with the utility cost of supplying labor in excess of the utility-maximizing level.

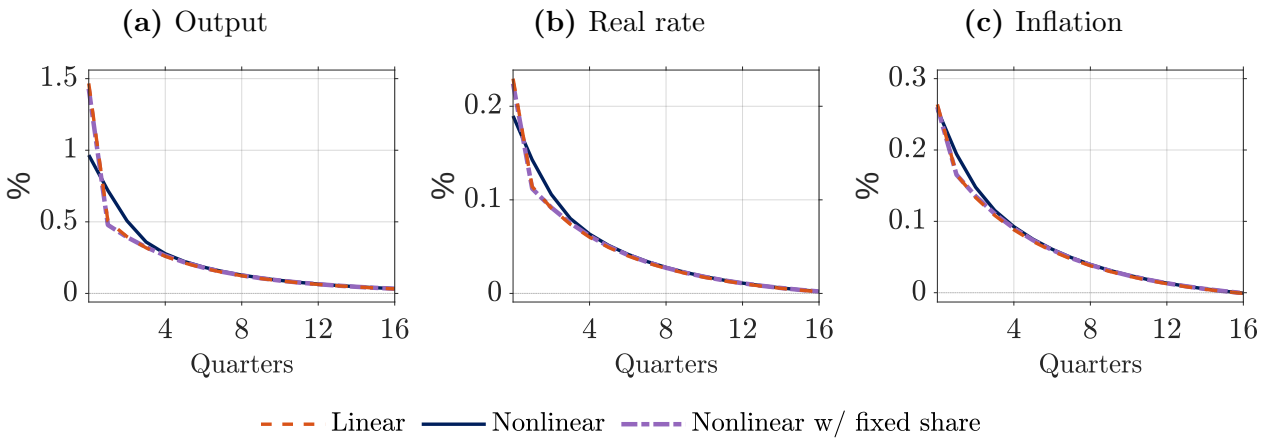



**Figure 6:** Dynamic responses

*Notes:* Impulse responses to a one-time transfer of 1% of annual GDP at $t = 0$ in the baseline general-equilibrium model. The purple dashed line is the nonlinear solution when households that are borrowing constrained in the steady state consume all of their cash-on-hand at $t = 0$. Output is in percent of steady-state quarterly output; the real rate and inflation are annualized and in percentage points.

of structural disturbances, for which the dynamic responses play an essential role. These inferences depend on the timing and persistence of the model's responses, which can differ even when first-year responses are similar.

The different dynamics are driven by households moving off the borrowing constraint. To illustrate this, Figure 6 shows results from a hypothetical economy in which initially constrained households are assumed to consume all of their cash-on-hand in the period of the shock ($t = 0$), including both the exogenous transfer and any general equilibrium change in their income. From $t = 1$ onward, these households follow their nonlinear policies, as do initially unconstrained households in all periods. This is a general-equilibrium counterpart of the decomposition in Figure 3a. The figure shows that this one change in behavior leads to responses of output, real rate, and inflation that are almost identical to those in the linear solution.

These findings are closely connected to the role of intertemporal MPCs emphasized by Auclert et al. (2024). In two-agent (TANK) models, the spending response to a debt-financed transfer is concentrated in the impact period, because hand-to-mouth households spend their entire transfer immediately, while Ricardian households do not respond (Galí et al., 2007; Bilbiie, 2008). Heterogeneous-agent models differ from two-agent models because both (i) low-wealth households who are not hand-to-mouth and (ii) hand-to-mouth households who are moved off their constraint have high MPCs that decline only gradually over time, so that

spending out of a transfer is spread over several quarters. In the linear solution, constrained households behave like the hand-to-mouth agents in two-agent models, in that they spend the entire transfer on impact and nothing afterwards. Nonlinearity thus matters not only for the size of the output response on impact, but also for its persistence.

**Interest rate response with flexible wages.** Figure 5b shows the interest rate response when there are no wage adjustment costs. The red dashed and blue solid lines show the linear and nonlinear first-quarter responses when unions continue to choose the hours on behalf of households. In this economy, the fiscal stimulus policy has no effect on employment, aggregate output, or aggregate consumption. However, it does lead to movements in the real interest rate. The funded stimulus program is a mechanism to relieve the effects of borrowing constraints. Constrained households with high MPCs increase their consumption in response to the shock. Since output is unaffected, the interest rate must rise to induce higher-wealth households to cut their consumption. The linear approximation overstates the movement in the interest rate by a very large amount. It follows that the implied movements in "r-star" in response to a fiscal stimulus shock, and hence the need for monetary tightening, are overstated in linearized HANK economies relative to the nonlinear solution.

Figure G.4 in Appendix G shows the interest rate response in an analogous version of the model without a labor union, in which individual households can choose labor supply freely. In this model there are small output effects because labor-supply incentives differ across households with different productivity and asset levels. But since the extent of nonlinearity is ultimately tied to the share of constrained households, the aggregate effect looks similar to the version with a union.

## 3.4 Other Versions of the HANK Model

The baseline general equilibrium model in the previous section features sticky wages, flexible prices, real government debt and an active-monetary/passive-fiscal policy configuration. In this section we investigate whether the extent of nonlinearity in the effects of fiscal stimulus is affected by changes in these model features. The findings are summarized in Figure 7.

**Faster fiscal adjustment.** The green dash-dot line labeled "Faster fiscal adjustment" in Figure 7 shows that the nonlinearity is more pronounced when the fiscal authority stabilizes debt more rapidly with a fiscal rule that sets $\phi_B = 16$ rather than $\phi_B = 8$ in the baseline. Since future transfers are lowered sooner, consumption is affected primarily among

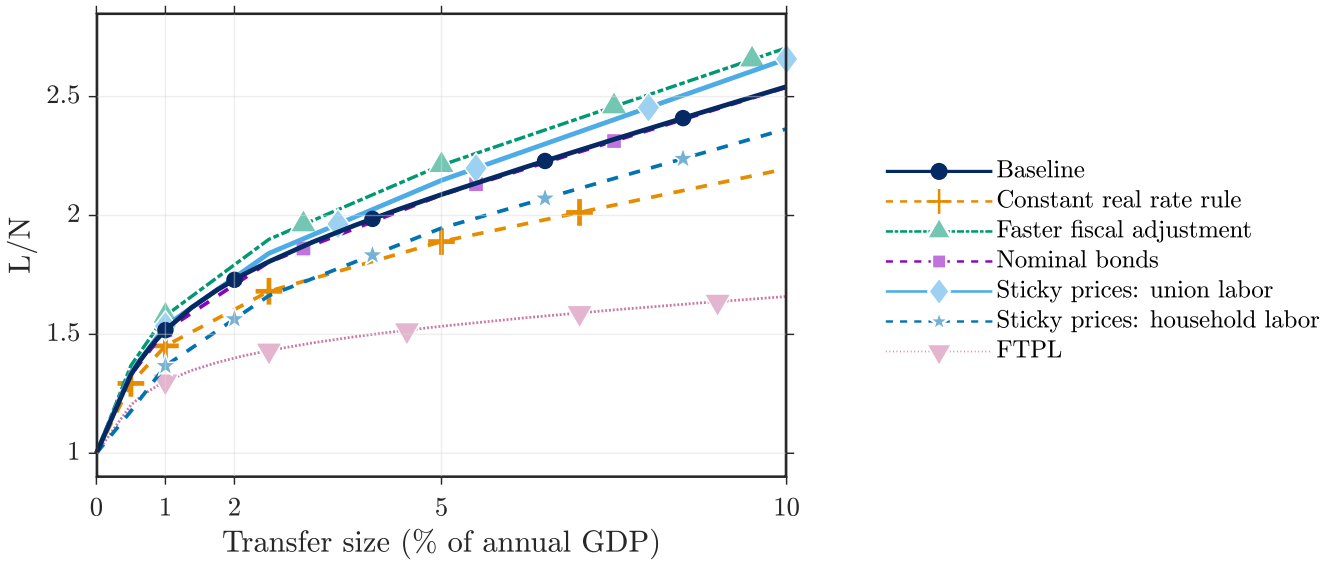



**Figure 7:** Sensitivity in HANK

*Notes:* The figure presents the ratio of the linear to the nonlinear first-quarter output response for different model specifications.

the most constrained households, who contribute most to the nonlinearity of the aggregate consumption response.

**Fixed real rate.** The orange dashed line labeled "Constant real rate rule" in Figure 7 shows results from an economy where the central bank keeps the real rate constant. In the nonlinear solution, some initially constrained households save part of the transfer and spend more in subsequent quarters. Under the baseline Taylor rule, the resulting persistence in aggregate demand keeps real rates elevated, strengthening the incentive to save on impact. With the real rate held constant, this incentive is weaker, so fewer households move off the borrowing constraint. Households that are already unconstrained also postpone less consumption. The ratio of the linear to the nonlinear impact response is therefore smaller and closer to its partial equilibrium value.

**Sticky prices and flexible wages.** An alternative model of nominal rigidities is one with sticky prices and flexible wages. We model sticky prices by introducing an intermediate-goods sector with a CES aggregator, monopolistic competition, and quadratic price adjustment costs as in Rotemberg (1982). This version of the model features a standard New Keynesian

Phillips curve of the form

$$\pi_t = \kappa \left( \frac{W_t}{P_t} - 1 \right) + \bar{\beta} \pi_{t+1}.$$

Figure 7 displays results for two versions, one in which the union chooses a common number of hours for all households (light-blue solid line), and one in which households freely choose how much labor to supply (blue dashed line).[16] Both versions exhibit a degree of nonlinearity similar to that in the baseline model.

**Unfunded fiscal stimulus.** We also consider a version of the model in which future inflation, rather than future primary surpluses, pays for the stimulus, as in the fiscal theory of the price level (FTPL). To implement this version, we make two changes to the baseline model. First, we assume that the government issues nominal rather than real debt. Unexpected inflation at $t = 0$ following the stimulus lowers the real value of outstanding debt and hence the future primary surpluses needed to service it. However, this has a negligible effect on the extent of nonlinearity, as illustrated by the purple dashed line labeled "Nominal bonds" in Figure 7.

Second, we alter the fiscal and monetary rules so that $\phi_B = \phi_\pi = 0$. This has the effect of pegging the nominal rate at $i_t = \bar{i}$ and holding real transfers fixed at their steady-state level $T_t = T$ for $t > 0$. Hence, in the language of Leeper (1991), fiscal policy is active and monetary policy is passive.

The ratio of the linear to nonlinear first-quarter output response is shown with the pink dotted line labeled "FTPL" in Figure 7. The figure shows that for an unfunded stimulus, the extent of nonlinearity is much less severe than for the other versions of the model. The reason is that the output effect of the unfunded stimulus is substantially larger than that of the funded stimulus (see Figure G.7 in Appendix G), and a large part of the output effect of an unfunded stimulus does not rely on a failure of Ricardian equivalence or on households having MPCs above the PIH benchmark. Even in a representative-agent model, an unfunded stimulus of this type has an expansionary effect on output. The upward pressure on consumption arises because households seek to exchange nominal government debt for real consumption, as explained in Cochrane (2023). Redistribution toward high-MPC households plays a role as well, as explained by Kaplan, Nikolakoudis and Violante (2024), but it is a small part of the overall effect on output.

[16] When households choose labor supply, they set $n_{i,t}$ so that $\frac{1}{c_{i,t}} e_{i,t}(1-\tau)\frac{W_t}{P_t} = v'(n_{i,t})$. When the labor union chooses labor supply, it sets $N_t$ so that $\frac{1}{C_t}\frac{W_t}{P_t}(1-\tau) = v'(N_t)$. We assume that profits are distributed in proportion to efficiency units.

# 4 Other Shocks

We have focused on a particular policy experiment, a one-time lump-sum fiscal transfer. In this section we show that the decomposition of the sources of nonlinearity in Section 2.2 is a special case of a more general decomposition that we use as an organizing framework to study a range of other shocks and policies. The general lesson is that the degree of nonlinearity for a given shock depends on (i) the extent to which it shifts households across the kink in their consumption function, (ii) the difference in the sensitivity of consumption to the shock on either side of the kink, and (iii) the curvature of individual consumption policies in the shock.

## 4.1 General Formulation of Nonlinearities

We consider an unexpected change to a perturbation variable $\omega$ that is revealed at $t = 0$, and normalize the steady-state value of $\omega$ to 0. This could include, for example, a current or future change in transfers or total factor productivity (TFP). We focus on the impact response at $t = 0$ and suppress the time subscript in this section. Households are indexed by their pre-shock state $(x, y)$, where $x$ is their cash-on-hand, and $y$ collects the relevant idiosyncratic states (the persistent income component $y^P$ in Section 2). We assume that the consumption policy function $c(x, y, \omega)$ is continuous in all arguments but for each $y$ has a kink at $x = \bar{x}(y, \omega)$, whose location depends on $\omega$. We denote the fraction of households below the kink, $x \leq \bar{x}(y, \omega)$, immediately after the change in $\omega$ as $\Lambda(\omega) \equiv \int_Y G(\bar{x}(y, \omega) \mid y)\, \mathrm{d}F(y)$.

Let $c^-(x, y, \omega)$ and $c^+(x, y, \omega)$ denote the consumption policy function below and above the kink, respectively. We write $c^-_\omega(x, y)$ and $c^+_\omega(x, y)$ for the partial derivatives with respect to $\omega$, evaluated at $\omega = 0$ at a given pre-shock state $(x, y)$, on either side of the kink $\bar{x}(y)$. We denote by $\delta(y)$ the difference in these derivatives on either side of the kink,

$$\delta(y) \equiv c^-_\omega(\bar{x}(y), y) - c^+_\omega(\bar{x}(y), y), \tag{12}$$

where, with a slight abuse of notation, we use $\bar{x}(y)$ to denote $\bar{x}(y, 0)$. Similarly, we will use $\bar{x}_\omega(y)$ to denote the derivative of the location of the kink with respect to $\omega$, evaluated at $\omega = 0$. The gap $\delta(y)$ may be positive or negative, depending on the nature of the shock. To map this more general notation to the partial-equilibrium transfer experiment of Section 2.2, set $\omega = \Delta$ (the size of the transfer) and note that $c^-_\omega(x, y) = 1$ (constrained households have an MPC $= 1$), $c^+_\omega(x, y) = c^+_x(x, y) < 1$ (unconstrained households have an MPC $< 1$), and $\delta(y) = 1 - c^+_x(\bar{x}(y), y)$.

Differentiating the aggregate consumption function $\mathcal{C}(\omega) = \int_Y \int c(x, y, \omega)\, \mathrm{d}G(x \mid y)\, \mathrm{d}F(y)$ and evaluating at $\omega = 0$ gives[17]

$$\mathcal{C}'(0) = \underbrace{\Lambda(0)\, \mathbb{E}\big[c_\omega^-(x, y) \mid x < \bar{x}(y)\big]}_{\text{households below the kink}} + \underbrace{[1 - \Lambda(0)]\, \mathbb{E}\big[c_\omega^+(x, y) \mid x > \bar{x}(y)\big]}_{\text{households above the kink}}. \tag{13}$$

As in the transfer experiment in Section 2.2, continuity of the consumption function at the kinks means that the first-order aggregate response is the average marginal response for households on either side of the kink, and movement across kinks does not contribute to the first-order effect.[18]

Differentiating a second time gives

$$\begin{aligned} \mathcal{C}''(0) = {} & \underbrace{\Lambda'(0)\, \frac{\mathbb{E}[\delta(y)\bar{x}_\omega(y) \mid x = \bar{x}(y)]}{\mathbb{E}[\bar{x}_\omega(y) \mid x = \bar{x}(y)]}}_{\text{movement across the kink}} + \underbrace{[1 - \Lambda(0)]\, \mathbb{E}\big[c_{\omega\omega}^+(x, y) \mid x > \bar{x}(y)\big]}_{\text{individual curvature: households above the kink}} \\ & + \underbrace{\Lambda(0)\, \mathbb{E}\big[c_{\omega\omega}^-(x, y) \mid x < \bar{x}(y)\big]}_{\text{individual curvature: households below the kink}}. \end{aligned} \tag{14}$$

The first term captures movement across the kink. $\Lambda'(0)$ is the marginal change in the share of households below the kink induced by a change in $\omega$. The ratio of expectations in the first term is the average drop (or jump) in the marginal consumption response at the kink, $\delta(y)$, weighted by how fast each household's kink moves in response to the shock, $\bar{x}_\omega(y)$,

$$\frac{\mathbb{E}[\delta(y)\bar{x}_\omega(y) \mid x = \bar{x}(y)]}{\mathbb{E}[\bar{x}_\omega(y) \mid x = \bar{x}(y)]} = \frac{\displaystyle\int_Y \delta(y)\, g(\bar{x}(y) \mid y)\, \bar{x}_\omega(y)\, dF(y)}{\displaystyle\int_Y g(\bar{x}(y) \mid y)\, \bar{x}_\omega(y)\, dF(y)}. \tag{15}$$

For the transfer experiment, $\bar{x}_\omega(y) = -1$ for all $y$, so the movement term reduces to $\Lambda'(0)\mathbb{E}[1 - c_x^+ \mid x = \bar{x}(y)]$.

The second term is the curvature for households initially above the kink. The third term allows for the possibility that, depending on the nature of the shock $\omega$, the consumption of households initially below the kink may also be nonlinear in $\omega$.

[17] See Appendix B.2 for the derivations of (13) and (14).

[18] This is also true in general equilibrium. Households may move across a kink as a direct effect of the shock $\omega$, and also because equilibrium prices and taxes may move. Continuity eliminates the first-order effect of households crossing the kink for either reason.

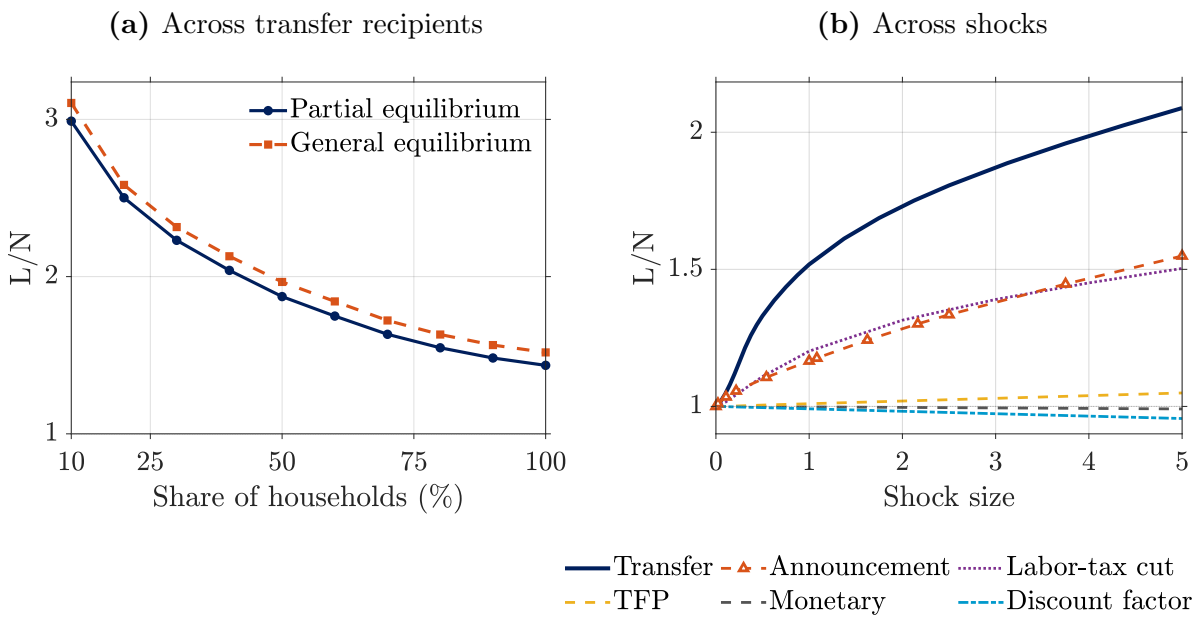



**Figure 8:** Nonlinearities across targeted transfers and across shocks

*Note:* Both panels report the ratio of the linear to the nonlinear response (L/N). Panel (a) holds the transfer fixed at 1% of annual GDP and gives it only to the lowest-income households; the horizontal axis shows the share of households that receive the transfer. Panel (b) shows the output response in general equilibrium to six transitory shocks, each measured in its own units on the horizontal axis: the baseline transfer and the transfer announcement (a transfer announced at $t = 0$ and paid at $t = 1$) in percent of annual GDP; the monetary shock $\varepsilon_i$ in annualized percentage points, such that $i_t = \bar{i} + \phi_\pi(\pi_t - \bar{\pi}) - \varepsilon_i$; the productivity shock $100(Z_0 - 1)$, and the discount-factor shock $\tilde{\beta}_{i0} = \beta_i(1 - \varepsilon_\beta)$, measured by $100\varepsilon_\beta$, both in percent; and the one-quarter labor-tax cut, measured by its ex-ante revenue loss $\varepsilon_\tau$ in percent of annual GDP ($\tau_0 = \tau - 4\varepsilon_\tau$).

## 4.2 Specific Shocks

We use these formulas to interpret the aggregate consumption responses to several shocks in general equilibrium.

**Targeted transfers.** A targeted transfer pays $s(y)\omega$, where $s(y) \geq 0$ and $\int_Y s(y)\, \mathrm{d}F(y) = 1$, so $\omega$ indexes the overall size of the transfer. A uniform transfer is a special case with $s(y) = 1$. Since the shock is a transfer that directly augments cash-on-hand, $\delta(y) = s(y)(1 - c_x^+)$, and $\bar{x}_\omega(y) = -s(y)$. The change in the share of households below the kink is given by $\Lambda'(0) = -\int_Y s(y) g(\bar{x}(y) \mid y)\, \mathrm{d}F(y)$, and hence the second-order contribution of households crossing the kink to the change in aggregate consumption (first term in equation (14)) is

$$-\int_Y s(y)^2 \big[1 - c_x^+(\bar{x}(y), y)\big] g(\bar{x}(y) \mid y)\, \mathrm{d}F(y).$$

The factor $s(y)^2$ reflects both the size of the drop in the consumption response and the movement in the location of the kink, while $g(\bar{x}(y) \mid y)$ measures the density near the kink within each income group.

Since low-income households are more likely to be at the constraint, a transfer program that is targeted toward these households implies $s(y) > 1$ for constrained households. And since the movement-across-the-constraint term accounts for most of the nonlinearity and scales with $s(y)^2$, targeting low-income households leads to more nonlinearity. Figure 8a shows the ratio of the linear to the nonlinear response for a targeted transfer program with an aggregate transfer budget of 1% of annual GDP, which pays a single transfer amount to households in the bottom $q$ percent of the income distribution, for different values of $q$. This ratio is approximately 1.5 for a uniform transfer, and rises to around 3 when targeted to the bottom 10%.[19]

**TFP shocks.** A positive TFP shock raises each household's income in proportion to its productivity, so it acts like a targeted lump-sum transfer weighted toward high-income households. Since high-income households are disproportionately at high levels of wealth, it delivers fewer resources to the low-income households at the kink than to the average household, so it is akin to a targeted transfer program with $s(y) < 1$ for constrained households. Since the contribution to the movement-off-constraint term scales with $s(y)^2$, there is much less nonlinearity than with a lump-sum transfer that is the same size as the aggregate increase in income. This can be seen in Figure 8b.[20]

**Tax cuts.** A cut in the proportional labor-income tax has a similar distributional effect. Holding pretax earnings fixed, it raises disposable income in proportion to earnings, and hence $s(y)$ is proportional to the expected pretax earnings of households of type $y$. Relative to a lump-sum transfer of the same aggregate size, it provides fewer additional resources to low-income households near the kink, reducing the degree of nonlinearity. This is reflected in Figure 8b (see also Figure H.1d in Appendix H).

**Monetary policy.** We consider a monetary policy shock that leads to a lower real rate for one period, and let $\omega = r_0 - \bar{r}$ denote the change in this rate. For constrained households,

[19] See also Figures G.1 and G.2 in Appendix G, and the partial-equilibrium counterparts, Figures E.2 and E.3 in Appendix E.

[20] Constrained households account for only 7.9% of aggregate earnings against a 20.5% population share. Thus, for the same aggregate increase in resources, a proportional shock delivers less than two-fifths as much to constrained households as a lump-sum transfer. See also Figure H.1b in Appendix H.

the direct effect is zero, $c^-_\omega(x,y)=0$. Unconstrained households increase their consumption in response to the rate cut, $c^+_\omega(x,y)<0$, because of intertemporal substitution. When some of these initially unconstrained households reach the borrowing constraint, their direct consumption response to further rate cuts falls to zero. As a result, the movement-across-the-kink margin contributes to concavity. However, Figure 8b shows that the resulting nonlinearities are very weak, consistent with the simulations in Auclert et al. (2021), which show that the linear SSJ solution tracks the nonlinear response very accurately.[21]

**Transfer announcement.** A transfer $\omega$ announced at $t=0$ and paid at $t=1$ leaves current cash-on-hand unchanged. Constrained households therefore do not respond at the announcement date, $c^-_\omega(x,y)=0$, while unconstrained households increase consumption, $c^+_\omega(x,y)>0$. Consequently,

$$\delta(y)=-c^+_\omega<0, \qquad \bar{x}_\omega(y)=\frac{c^+_\omega(\bar{x}(y),y)}{1-c^+_x(\bar{x}(y),y)}>0.$$

Some households consume enough in response to the future transfer to run down their assets until the borrowing constraint binds, which increases the constrained share. Compared with a current transfer, both the response gap $\delta(y)$ and the threshold response $\bar{x}_\omega(y)$ change sign, leaving their product negative. This movement onto the constraint again contributes to concavity, and Figure 8b shows that the degree of nonlinearity is much smaller than for an immediate transfer.

**Discount-factor shocks.** We consider a temporary impatience shock, $\widetilde{\beta}_{i0}=\beta_i(1-\varepsilon_\beta)$, that raises current consumption among unconstrained households while leaving constrained households unchanged. Unlike for transfers, the response is mildly convex and the linear solution understates the nonlinear response (Figure 8b). This reflects two forces that work in opposite directions, corresponding to the first two terms in (14) (the third is zero because constrained households do not respond). The first is movement across the kink, which contributes to concavity. As with a transfer announcement, some households reduce saving until the borrowing constraint binds. Once constrained, they stop responding, so each further increment of impatience raises consumption among fewer households. This force is much weaker than for transfers because households near the kink respond little to impatience. The marginal consumption responses on either side of the kink are therefore similar, and $\delta(y)$ is

[21] Werning (2015) and Kaplan et al. (2018) provide conditions under which incomplete markets leave the aggregate response to interest rates unchanged, even with substantial MPC heterogeneity. They show that the distribution of MPCs affects the transmission mechanism of monetary policy and its decomposition of the response into direct and indirect effects.

small. The second force is the individual curvature of households that remain unconstrained, which contributes to convexity. Impatience lowers the value of saving, so these households raise consumption until its marginal utility falls in proportion to the shock. Because marginal utility flattens as consumption rises, each further increment of impatience requires a larger increase in consumption. On balance, this second force dominates slightly.

# 5 Conclusions

Heterogeneous-agent models are useful for studying the effects of fiscal stimulus because the interaction between borrowing constraints and incomplete markets for idiosyncratic risk leads to a failure of Ricardian equivalence. The resulting MPCs are highly nonlinear in household wealth, so the aggregate effects of transfers to high-MPC households decline rapidly with transfer size.

We show that the most important source of the nonlinearity in the aggregate response is the movement of initially constrained households across kinks in their consumption functions. Using first-order methods to approximate the aggregate consumption response to a one-time transfer effectively extrapolates the average MPC and does not take into account this movement of households on or off the borrowing constraint. By capturing the local movement off the constraint, higher-order methods can improve accuracy and extend the range of shock sizes for which local methods are accurate, but may rapidly become unreliable for larger transfers.

Our paper has abstracted from aggregate risk. As computational methods for the nonlinear global solutions of heterogeneous-agent economies with aggregate risk continue to advance, future work should evaluate the accuracy of linear and higher-order local solutions in those economies.[22] For now that remains an open question.

[22] Examples include Auclert et al. (2026) and Yang, Wang, Schaab and Moll (2025).

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# Online Appendix to "How Small is Small? Nonlinearities in Heterogeneous-Agent Models"

Javier Bianchi and Greg Kaplan

# A Calibration

This appendix describes the income process used in our quantitative analysis and the numerical implementation.

## A.1 Panel Study of Income Dynamics

The empirical moments and parameter estimates are taken from Kaplan and Violante (2022), henceforth KV. Their PSID sample covers total annual household labor income for households whose heads were aged 25 to 65 from 1968 to 2008. They exclude annual labor income below \$7,250 in 2016 dollars, corresponding to 1,000 hours per year at \$7.25 per hour. KV describe a first-stage regression on age and year dummies before constructing income moments. We reproduce their reported moments in Table A.1; our baseline implementation does not repeat this empirical estimation.

Let $\log y_t^{\text{ann}}$ denote the log of annual labor income in year $t$ (in this subsection, $t$ indexes years, not quarters), and let annual income growth at lag $d$ be

$$\Delta_d \log y_t^{\text{ann}} = \begin{cases} \log y_t^{\text{ann}} & \text{if } d = 0, \\ \log y_{t+d}^{\text{ann}} - \log y_t^{\text{ann}} & \text{if } d > 0. \end{cases}$$

Define cross-sectional central moments of annual income growth of order $j$ at lag $d$ as

$$m_{j,d} = \mathbb{E}\left[\left(\Delta_d \log y_t^{\text{ann}} - \mathbb{E}[\Delta_d \log y_t^{\text{ann}}]\right)^j\right],$$

and the kurtosis of income growth at lag $d$ as

$$K_d = \frac{m_{4,d}}{(m_{2,d})^2}.$$

**Table A.1:** Empirical moments of annual income (lag 0) and income growth (lags 1–5). Source: PSID 1968–2008.

| Lag ($d$) | $m_{2,d}$ | $m_{4,d}$ | $K_d$ |
|---|---|---|---|
| 0 | 0.504 | 0.930 | 3.65 |
| 1 | 0.142 | 0.220 | 10.90 |
| 2 | 0.207 | 0.369 | 8.57 |
| 3 | 0.235 | 0.410 | 7.42 |
| 4 | 0.280 | 0.544 | 6.96 |
| 5 | 0.295 | 0.557 | 6.39 |

## A.2 Income Process and Discretization

Let $y_t^0$ denote quarterly earnings before the independent, always-on noise component. We implement the two KV components using zero-mean log innovations:

$$\log y_t^0 = \begin{cases} z_t + \varepsilon_t - \mu_y & \text{with probability } \lambda_\varepsilon, \qquad \varepsilon_t \sim \mathcal{N}(0,\, \sigma_\varepsilon^2)\,, \\ z_t - \mu_y & \text{with probability } 1-\lambda_\varepsilon, \end{cases}$$

$$z_t = \begin{cases} \phi_z\, z_{t-1} + \eta_t & \text{with probability } \lambda_\eta, \qquad \eta_t \sim \mathcal{N}\big(0,\, \sigma_\eta^2\big)\,, \\ \phi_z\, z_{t-1} & \text{with probability } 1-\lambda_\eta, \end{cases} \tag{A.1}$$

where $\mu_y$ is such that $\mathbb{E}[y_t^0] = 1$.[23]

The innovations and arrival events are independent across components and over time. In the notation of Section 2.3, $z_t$ is the persistent component $y_t^P$, and the transitory term, equal to $\varepsilon_t$ with probability $\lambda_\varepsilon$ and to zero otherwise, is $y_t^T$. Full earnings are $y_t = y_t^0 e^{y_t^N}$, where $y_t^N$ is the independent log-noise shock described in Section 2.3, with distribution $\mathcal{N}(-\sigma_N^2/2, \sigma_N^2)$ and $\sigma_N = 0.005$. The noise shocks are independent over time and independent of the persistent and the transitory components. Since $\mathbb{E}[e^{y_t^N}] = 1$, full earnings also satisfy $\mathbb{E}[y_t] = 1$. Annual income is the sum of the four quarterly earnings realizations $y_t$ within the year.

KV estimate the five parameters jointly using the variances at lags zero, one, and five and

[23] The process above implies

$$\mu_y = \log\Big(1 - \lambda_\varepsilon + \lambda_\varepsilon e^{\sigma_\varepsilon^2/2}\Big) + \sum_{j=0}^{\infty} \log\Big(1 - \lambda_\eta + \lambda_\eta e^{\phi_z^{2j}\sigma_\eta^2/2}\Big)\,.$$

the kurtoses at lags one and five. We use these values, presented in Table A.2. The persistent component has persistence parameter $\phi_z = 0.987$ and innovation variance conditional on arrival $\sigma_\eta^2 = 0.0516$; the transitory shock has variance conditional on arrival $\sigma_\varepsilon^2 = 1.6243$, implying large but infrequent income fluctuations.

We use an equally spaced grid with $n_z = 11$ points for the persistent state. Let $\Theta$ be the transition matrix conditional on the arrival of an innovation. The unconditional transition matrix is

$$\Pi_z = \lambda_\eta \Theta + (1 - \lambda_\eta) D_{\phi_z},$$

where $D_{\phi_z}$ gives the transition probabilities when no innovation arrives. If $\phi_z z$ falls between two grid points, we assign probabilities to those points so that the expected value of next period's state is $\phi_z z$, as in (A.1). We choose the grid width so that the stationary variance under $\Pi_z$ matches $\lambda_\eta \sigma_\eta^2 / (1 - \phi_z^2)$.

For the transitory component, we approximate $\mathcal{N}(0, \sigma_\varepsilon^2)$ using Gauss–Hermite quadrature with $n_\varepsilon = 5$ points. We multiply the quadrature weights by $\lambda_\varepsilon$ and add the probability of no innovation, $1 - \lambda_\varepsilon$, to the weight on the zero node. We then shift all five nodes by the same amount so that $\mathbb{E}[e^{\hat{\varepsilon}}] = 1$. This normalization leaves the central moments of log transitory income unchanged. The resulting nodes are $\{-3.73, -1.82, -0.09, 1.64, 3.55\}$, with corresponding probabilities $\{0.001, 0.016, 0.966, 0.016, 0.001\}$, rounded to two and three decimal places, respectively. The joint grid for the persistent and transitory components thus has $11 \times 5 = 55$ points. We integrate over the noise shock $y_t^N$ using Gauss–Legendre quadrature, as described in Appendix C.

**Table A.2:** Imported KV income parameters and numerical discretization

| Parameter | Description | Symbol | Value |
|---|---|---|---|
| *A. Structural parameters (KV)* | | | |
| AR(1) persistence | Persistent component | $\phi_z$ | 0.987 |
| Persistent shock variance | Conditional variance | $\sigma_\eta^2$ | 0.0516 |
| Transitory shock variance | Conditional variance | $\sigma_\varepsilon^2$ | 1.6243 |
| Persistent arrival rate | Quarterly probability | $\lambda_\eta$ | 0.237 |
| Transitory arrival rate | Quarterly probability | $\lambda_\varepsilon$ | 0.073 |
| *B. Discretization* | | | |
| Persistent states | Tauchen grid | $n_z$ | 11 |
| Transitory states | Gauss–Hermite mixture | $n_\varepsilon$ | 5 |
| Discrete income nodes | | $n_z \times n_\varepsilon$ | 55 |

## A.3 Further Details on Calibration

**Table A.3:** Calibration: parameter values and steady-state moments across PE scenarios

| | Parameters | | Moments | |
|---|---|---|---|---|
| Scenario | $\bar{\beta}$ | $\sigma_\beta$ | Avg. MPC | $A/Y$ |
| Baseline | 0.976 | 0.008 | 0.25 | 1.00 |
| $\lambda_\varepsilon = 0.25$ | 0.972 | 0.010 | 0.25 | 1.00 |
| No $\beta$ heterog., $A/Y = 1$ | 0.987 | 0 | 0.12 | 1.00 |
| No $\beta$ heterog., $A/Y = 4$ | 0.992 | 0 | 0.05 | 4.00 |
| Stochastic $\beta$ | 0.983 | 0.029 | 0.25 | 1.00 |
| Finite lives | 0.982 | 0.008 | 0.25 | 1.00 |
| Lower transfers | 0.971 | 0.011 | 0.25 | 1.00 |
| Natural debt limit | 0.963 | 0.015 | 0.25 | 1.00 |
| Endogenous labor supply | 0.964 | 0.009 | 0.25 | 1.00 |
| ARS income process | 0.976 | 0.006 | 0.25 | 1.00 |

*Note:* The versions of the model are as follows: $\lambda_\varepsilon = 0.25$: transitory shocks arrive on average once per year ($\lambda_\varepsilon = 0.25$ instead of 0.073), and $\sigma_\varepsilon^2$ is lowered to keep the variance of the transitory component, $\lambda_\varepsilon \sigma_\varepsilon^2$, at its estimated value. *No $\beta$ heterog. ($A/Y = 1$)* and *No $\beta$ heterog. ($A/Y = 4$)* are versions without discount-factor heterogeneity, calibrated to match aggregate wealth-to-annual-GDP ratios of 1 and 4, respectively. *Stochastic $\beta$*: discount factors follow an idiosyncratic AR(1) process with autocorrelation $\rho_\beta = 0.990$. *Finite lives*: a life-cycle version with overlapping generations and an annual mortality probability of 2%. *Lower transfers*: steady-state lump-sum transfers are set to half of the baseline value. *Natural debt limit*: households face a 0.1% quarterly probability of transitioning to a zero-income state, and their lump-sum transfers are set to zero. *Endogenous labor supply*: households choose hours worked endogenously with isoelastic disutility from labor and unitary Frisch elasticity. *ARS income process*: replaces the baseline idiosyncratic earnings process with the one used in Auclert et al. (2024).

**Table A.4:** Summary of Parameters for the General Equilibrium Model

| Parameter | Value | Description |
|---|---|---|
| $\bar{B}$ | 4 | Steady-state government debt (100% of annual GDP) |
| $\bar{r}$ | 0.02 | Steady-state real rate (annualized) |
| $\bar{\pi}$ | 0 | Steady-state inflation |
| $\kappa$ | 0.01 | Wage Phillips curve slope |
| $\nu$ | 1 | Frisch elasticity of labor supply |
| $\phi_\pi$ | 1.5 | Taylor coefficient |
| $\phi_B$ | 8 | Fiscal rule coefficient on debt |
| $G$ | 0.114 | Government spending (share of GDP) |
| $T$ | 0.14 | Steady-state lump-sum transfers (share of GDP) |
| $\tau$ | 0.274 | Proportional tax rate |
| $\chi$ | 0.82 | Labor disutility |

# B Derivatives of the Aggregate Consumption Function

This appendix derives equations (3), (5) and (6) of Section 2.2 by differentiating (2) with respect to $\Delta$, and equations (13) and (14) of Section 4.1 for a general shock $\omega$. It also derives the decomposition (7) of the gap between the linear and nonlinear responses and the derivatives $c_x^+$, $c_{xx}^+$ and $c_{xxx}^+$ of the individual consumption policy.

## B.1 Transfers

**First derivative.** We differentiate each of the two integrals in (2). Below the kink, $c(x+\Delta, y^P) = x + \Delta$; $g(\cdot \mid y^P)$ denotes the density of $G(\cdot \mid y^P)$.

$$\frac{\mathrm{d}}{\mathrm{d}\Delta}\int_0^{\bar{x}(y^P)-\Delta} c(x+\Delta, y^P)\,\mathrm{d}G(x \mid y^P) = G\big(\bar{x}(y^P)-\Delta \mid y^P\big) - \bar{x}(y^P)\,g\big(\bar{x}(y^P)-\Delta \mid y^P\big), \tag{B.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}\Delta}\int_{\bar{x}(y^P)-\Delta}^{\infty} c(x+\Delta, y^P)\,\mathrm{d}G(x \mid y^P) = \int_{\bar{x}(y^P)-\Delta}^{\infty} c_x^+(x+\Delta, y^P)\,\mathrm{d}G(x \mid y^P) + c\big(\bar{x}(y^P), y^P\big)\,g\big(\bar{x}(y^P)-\Delta \mid y^P\big). \tag{B.2}$$

The two boundary terms cancel because consumption is continuous at the kink, $c(\bar{x}(y^P), y^P) = \bar{x}(y^P)$. Adding (B.1) and (B.2) and integrating over $F$,

$$\mathcal{C}_0'(\Delta) = \int_{Y^P}\left[G\big(\bar{x}(y^P)-\Delta \mid y^P\big) + \int_{\bar{x}(y^P)-\Delta}^{\infty} c_x^+(x+\Delta, y^P)\,\mathrm{d}G(x \mid y^P)\right]\mathrm{d}F(y^P). \tag{B.3}$$

At $\Delta = 0$, integrating the first term in brackets over $F$ gives $\bar{\Lambda}$, while the double integral equals $(1-\bar{\Lambda})\,\mathbb{E}[c_x^+ \mid x \geq \bar{x}(y^P)]$, where

$$\mathbb{E}[c_x^+ \mid x \geq \bar{x}(y^P)] \equiv (1-\bar{\Lambda})^{-1}\int_{Y^P}\int_{\bar{x}(y^P)}^{\infty} c_x^+(x, y^P)\,\mathrm{d}G(x \mid y^P)\,\mathrm{d}F(y^P).$$

This is equation (3).

**Second derivative.** Differentiating (B.3) again with respect to $\Delta$ gives:

$$\mathcal{C}_0''(\Delta) = \int_{Y^P} \left[ -\Big(1 - c_x^+\big(\bar{x}(y^P), y^P\big)\Big)\, g\big(\bar{x}(y^P) - \Delta \mid y^P\big) + \int_{\bar{x}(y^P)-\Delta}^{\infty} c_{xx}^+(x+\Delta, y^P)\, \mathrm{d}G(x \mid y^P) \right] \mathrm{d}F(y^P). \tag{B.4}$$

Define the density-weighted average at the kink

$$\mathbb{E}\big[1 - c_x^+ \mid x = \bar{x}(y^P)\big] \equiv \frac{\displaystyle\int_{Y^P} \big[1 - c_x^+\big(\bar{x}(y^P), y^P\big)\big]\, g\big(\bar{x}(y^P) \mid y^P\big)\ \mathrm{d}F(y^P)}{\displaystyle\int_{Y^P} g\big(\bar{x}(y^P) \mid y^P\big)\ \mathrm{d}F(y^P)} \tag{B.5}$$

and note that, since $\Lambda(\Delta) = \int_{Y^P} G\big(\bar{x}(y^P) - \Delta \mid y^P\big)\, \mathrm{d}F(y^P)$,

$$\Lambda'(0) = -\int_{Y^P} g\big(\bar{x}(y^P) \mid y^P\big)\, \mathrm{d}F(y^P). \tag{B.6}$$

Evaluating (B.4) at $\Delta = 0$ and substituting these two expressions, we obtain equation (5), where $\mathbb{E}[c_{xx}^+ \mid x \geq \bar{x}(y^P)]$ is defined analogously to $\mathbb{E}[c_x^+ \mid x \geq \bar{x}(y^P)]$.

**Third derivative.** Since $c_x^+(\bar{x}(y^P), y^P)$ is independent of $\Delta$, differentiating (B.4) gives

$$\begin{aligned}\mathcal{C}_0'''(\Delta) = \int_{Y^P} \Bigg[ &\Big(1 - c_x^+\big(\bar{x}(y^P), y^P\big)\Big)\, g'\big(\bar{x}(y^P) - \Delta \mid y^P\big) + c_{xx}^+\big(\bar{x}(y^P), y^P\big)\, g\big(\bar{x}(y^P) - \Delta \mid y^P\big) \\ &+ \int_{\bar{x}(y^P)-\Delta}^{\infty} c_{xxx}^+(x+\Delta, y^P)\, \mathrm{d}G(x \mid y^P) \Bigg] \mathrm{d}F(y^P).\end{aligned} \tag{B.7}$$

Here $g'(x \mid y^P) \equiv \partial g(x \mid y^P)/\partial x$. We have that

$$\Lambda''(0) = \int_{Y^P} g'\big(\bar{x}(y^P) \mid y^P\big)\, \mathrm{d}F(y^P) \tag{B.8}$$

and, writing $g' = (g'/g)\, g$ and using (B.6), (B.8) and the density-weighted average in (B.5) and its covariance,

$$\begin{aligned}\int_{Y^P} \big(1 - c_x^+\big)\, g'\, \mathrm{d}F(y^P) = \Lambda''(0)\, \mathbb{E}\big[1 - c_x^+ \mid x = \bar{x}(y^P)\big] \\ - \Lambda'(0)\, \mathrm{Cov}\left(1 - c_x^+, \frac{g'}{g} \,\middle|\, x = \bar{x}(y^P)\right),\end{aligned}$$

where $c_x^+$, $g$ and $g'$ are evaluated at the kink and the covariance uses the same weights as (B.5), $g(\bar{x}(y^P) \mid y^P)\,\mathrm{d}F(y^P)/\int_{Y^P} g(\bar{x}(y^P) \mid y^P)\,\mathrm{d}F(y^P)$. Evaluating (B.7) at $\Delta = 0$ and substituting these two expressions, together with $\int_{Y^P} c_{xx}^+\, g\,\mathrm{d}F(y^P) = -\Lambda'(0)\,\mathbb{E}[c_{xx}^+ \mid x = \bar{x}(y^P)]$, we obtain equation (6), with $\mathbb{E}[c_{xxx}^+ \mid x \geq \bar{x}(y^P)]$ defined analogously.

## B.2 General Shocks

We now derive equations (13) and (14) of Section 4.1. Aggregate consumption is

$$\mathcal{C}(\omega) = \int_Y \left[ \int_{-\infty}^{\bar{x}(y,\omega)} c^-(x,y,\omega)\,\mathrm{d}G(x \mid y) + \int_{\bar{x}(y,\omega)}^{\infty} c^+(x,y,\omega)\,\mathrm{d}G(x \mid y) \right] \mathrm{d}F(y), \tag{B.9}$$

where $c^-$ and $c^+$ denote the consumption policy below and above the kink $\bar{x}(y,\omega)$, and the constrained share is $\Lambda(\omega) = \int_Y G(\bar{x}(y,\omega) \mid y)\,\mathrm{d}F(y)$.

**First derivative.** Differentiating (B.9) with respect to $\omega$, we obtain

$$\begin{aligned}\mathcal{C}'(\omega) = \int_Y \Bigg[ &\Big( c^-\big(\bar{x}(y,\omega),y,\omega\big) - c^+\big(\bar{x}(y,\omega),y,\omega\big) \Big)\, g\big(\bar{x}(y,\omega) \mid y\big)\, \bar{x}_\omega(y,\omega) \\ &+ \int_{-\infty}^{\bar{x}(y,\omega)} c_\omega^-(x,y,\omega)\,\mathrm{d}G(x \mid y) + \int_{\bar{x}(y,\omega)}^{\infty} c_\omega^+(x,y,\omega)\,\mathrm{d}G(x \mid y) \Bigg] \mathrm{d}F(y).\end{aligned}$$

Because consumption is continuous at the kink, the boundary term vanishes and

$$\mathcal{C}'(\omega) = \int_Y \left[ \int_{-\infty}^{\bar{x}(y,\omega)} c_\omega^-(x,y,\omega)\,\mathrm{d}G(x \mid y) + \int_{\bar{x}(y,\omega)}^{\infty} c_\omega^+(x,y,\omega)\,\mathrm{d}G(x \mid y) \right] \mathrm{d}F(y). \tag{B.10}$$

At $\omega = 0$, the two terms equal $\Lambda(0)\,\mathbb{E}[c_\omega^- \mid x < \bar{x}(y)]$ and $[1-\Lambda(0)]\,\mathbb{E}[c_\omega^+ \mid x > \bar{x}(y)]$, where the expectations are averages over constrained and unconstrained households,

$$\begin{aligned}\mathbb{E}[c_\omega^- \mid x < \bar{x}(y)] &\equiv \frac{\int_Y \int_{-\infty}^{\bar{x}(y)} c_\omega^-\,\mathrm{d}G(x \mid y)\,\mathrm{d}F(y)}{\Lambda(0)}, \\ \mathbb{E}[c_\omega^+ \mid x > \bar{x}(y)] &\equiv \frac{\int_Y \int_{\bar{x}(y)}^{\infty} c_\omega^+\,\mathrm{d}G(x \mid y)\,\mathrm{d}F(y)}{1-\Lambda(0)}.\end{aligned}$$

This is equation (13).

**Second derivative.** Differentiating (B.10) again with respect to $\omega$ gives

$$\mathcal{C}''(\omega) = \int_Y \Bigg[ \delta(y,\omega)\, g\big(\bar{x}(y,\omega) \mid y\big)\, \bar{x}_\omega(y,\omega) + \int_{-\infty}^{\bar{x}(y,\omega)} c^-_{\omega\omega}(x,y,\omega)\, \mathrm{d}G(x \mid y) + \int_{\bar{x}(y,\omega)}^{\infty} c^+_{\omega\omega}(x,y,\omega)\, \mathrm{d}G(x \mid y) \Bigg] \mathrm{d}F(y).$$

Here $\delta(y,\omega) \equiv c^-_\omega\big(\bar{x}(y,\omega), y, \omega\big) - c^+_\omega\big(\bar{x}(y,\omega), y, \omega\big)$, so that $\delta(y,0) = \delta(y)$ as defined in (12). Differentiating $\Lambda(\omega)$ gives $\Lambda'(0) = \int_Y g(\bar{x}(y) \mid y)\, \bar{x}_\omega(y)\, \mathrm{d}F(y)$, so the first term at $\omega = 0$ equals $\Lambda'(0)$ times the ratio of expectations in (15). The last two terms equal $\Lambda(0)\, \mathbb{E}[c^-_{\omega\omega} \mid x < \bar{x}(y)]$ and $[1 - \Lambda(0)]\, \mathbb{E}[c^+_{\omega\omega} \mid x > \bar{x}(y)]$, with the expectations defined as above. This gives equation (14).

## B.3 Decomposition: Derivation of (7)

For a given transfer $\Delta$, we split households into those who remain constrained, those who move off the constraint, and those who are initially unconstrained. The difference between linear and nonlinear consumption for a household with initial state $(x, y^P)$ is

$$c(x,y^P) + \mathfrak{m}(x,y^P)\Delta - c(x+\Delta, y^P) = \begin{cases} 0, & x < \bar{x}(y^P) - \Delta, \\ x + \Delta - c(x+\Delta, y^P), & \bar{x}(y^P) - \Delta < x < \bar{x}(y^P), \\ c(x,y^P) + c_x(x,y^P)\Delta - c(x+\Delta, y^P), & x > \bar{x}(y^P). \end{cases}$$

Note that since the distribution is continuous, it does not matter which group includes households exactly at the thresholds. Households who remain constrained consume $x + \Delta$ under both solutions and therefore contribute nothing to the gap between the linear and nonlinear solutions. The switchers' contribution is nonnegative because consumption cannot exceed cash-on-hand, and the initially unconstrained households' contribution is nonnegative by concavity of their consumption policy. Integrating these two contributions over the initial distribution yields equation (7).

## B.4 Individual MPCs and Higher-Order Derivatives

This appendix derives analytical first- and higher-order derivatives of the individual consumption function. On the unconstrained branch, these are the objects denoted $c^+_x$, $c^+_{xx}$, and $c^+_{xxx}$ in the main text.

We retain the notation $c(x, y^P)$ from the main text, where $y^P$ contains the persistent components of income.[24] Saving is $a' = x - c(x, y^P)$, so next-period cash-on-hand is

$$x' = (1 + r)\big[x - c(x, y^P)\big] + (1 - \tau)y' + T. \tag{B.11}$$

On the unconstrained branch, $x > \bar{x}(y^P)$, substituting this expression into the Euler equation gives

$$u'\big(c(x, y^P)\big) = \beta(1 + r)\mathbb{E}\left[u'\left(c\left((1 + r)\big[x - c(x, y^P)\big] + (1 - \tau)y' + T,\ y^{P\prime}\right)\right) \,\middle|\, y^P\right]. \tag{B.12}$$

On the constrained branch, for $x < \bar{x}(y^P)$, the borrowing constraint binds, saving is zero, and $c(x, y^P) = x$. Hence

$$c_x(x, y^P) = 1, \qquad c_{xx}(x, y^P) = 0, \qquad c_{xxx}(x, y^P) = 0. \tag{B.13}$$

On the unconstrained branch, differentiation of (B.12) with respect to $x$ gives

$$u''(c)c_x = \beta(1 + r)^2(1 - c_x)\mathbb{E}\left[u''\big(c(x', y^{P\prime})\big)c_x(x', y^{P\prime}) \,\middle|\, y^P\right], \tag{B.14}$$

where we used that (B.11) implies

$$\frac{\mathrm{d}x'}{\mathrm{d}x} = (1 + r)(1 - c_x).$$

Here $c_x(x', y^{P\prime})$ is the derivative with respect to the first argument of the future consumption policy. The factor $(1 + r)(1 - c_x)$ accounts for how that argument changes with current $x$. Rearranging,

$$c_x(x, y^P) = \frac{\beta(1 + r)^2\mathbb{E}\left[u''\big(c(x', y^{P\prime})\big)c_x(x', y^{P\prime}) \,\middle|\, y^P\right]}{u''(c) + \beta(1 + r)^2\mathbb{E}\left[u''\big(c(x', y^{P\prime})\big)c_x(x', y^{P\prime}) \,\middle|\, y^P\right]}. \tag{B.15}$$

This is a recursive equation for the MPC function: the current MPC depends on MPCs at

[24] The consumption policy varies across households with different values of $\beta$; we do not index it to simplify notation.

next-period states, which may be constrained or unconstrained.[25]

Differentiating equation (B.14) with respect to $x$, using the product and chain rules, gives

$$\begin{aligned} &u'''(c)c_x^2 + u''(c)c_{xx} \\ &\quad = \beta(1+r)^2\Bigg\{(1-c_x)\frac{\mathrm{d}}{\mathrm{d}x}\mathbb{E}\left[u''\big(c(x',y^{P\prime})\big)c_x(x',y^{P\prime})\,\middle|\,y^P\right] \\ &\qquad - c_{xx}\mathbb{E}\left[u''\big(c(x',y^{P\prime})\big)c_x(x',y^{P\prime})\,\middle|\,y^P\right]\Bigg\}. \end{aligned} \tag{B.16}$$

On the right-hand side, the first term differentiates the expectation; the second differentiates the factor $1-c_x$. Because $c_x(x',y^{P\prime})$ jumps at the next-period kink $\bar{x}(y^{P\prime})$, the derivatives of the expectation in (B.16)–(B.18) are total derivatives: they include boundary terms from the income draws for which $x'$ crosses that kink as $x$ changes, which the expectation of the derivative would omit. Collecting the terms in $c_{xx}$ yields

$$c_{xx}(x,y^P) = \frac{\beta(1+r)^2(1-c_x)\frac{\mathrm{d}}{\mathrm{d}x}\mathbb{E}\left[u''\big(c(x',y^{P\prime})\big)c_x(x',y^{P\prime})\,\middle|\,y^P\right] - u'''(c)c_x^2}{u''(c) + \beta(1+r)^2\mathbb{E}\left[u''\big(c(x',y^{P\prime})\big)c_x(x',y^{P\prime})\,\middle|\,y^P\right]}. \tag{B.17}$$

Differentiating equation (B.16) once more with respect to $x$ gives

$$\begin{aligned} &u''''(c)c_x^3 + 3u'''(c)c_xc_{xx} + u''(c)c_{xxx} \\ &\quad = \beta(1+r)^2\Bigg\{(1-c_x)\frac{\mathrm{d}^2}{\mathrm{d}x^2}\mathbb{E}\left[u''\big(c(x',y^{P\prime})\big)c_x(x',y^{P\prime})\,\middle|\,y^P\right] \\ &\qquad - 2c_{xx}\frac{\mathrm{d}}{\mathrm{d}x}\mathbb{E}\left[u''\big(c(x',y^{P\prime})\big)c_x(x',y^{P\prime})\,\middle|\,y^P\right] \\ &\qquad - c_{xxx}\mathbb{E}\left[u''\big(c(x',y^{P\prime})\big)c_x(x',y^{P\prime})\,\middle|\,y^P\right]\Bigg\} \end{aligned}$$

[25] One can analogously obtain the MPC at date $t$ out of additional cash-on-hand at date 0, that is, an intertemporal MPC in the spirit of Auclert et al. (2024). Along an income history $y^t = (y_0,\dots,y_t)$, where $y_s$ includes the persistent, transitory and noise components, cash-on-hand evolves as $x_{s+1} = (1+r)\big(x_s - c(x_s,y_s^P)\big) + (1-\tau)y_{s+1} + T$, so an extra unit of cash-on-hand at $t=0$ raises consumption at date $t$ by

$$\frac{\partial c_t}{\partial x_0} = c_x(x_t,y_t^P)\prod_{s=0}^{t-1}(1+r)\big(1-c_x(x_s,y_s^P)\big),$$

where $x_s = x_s(x_0,y^s)$.

and rearranging

$$
\begin{aligned}
c_{xxx}(x, y^P) = \frac{1}{u''(c) + \beta(1+r)^2 \mathbb{E}\left[u''\big(c(x', y^{P\prime})\big) c_x(x', y^{P\prime}) \,\middle|\, y^P\right]} \\
\times \Bigg\{ \beta(1+r)^2 (1 - c_x) \frac{\mathrm{d}^2}{\mathrm{d}x^2} \mathbb{E}\left[u''\big(c(x', y^{P\prime})\big) c_x(x', y^{P\prime}) \,\middle|\, y^P\right] \\
- 2\beta(1+r)^2 c_{xx} \frac{\mathrm{d}}{\mathrm{d}x} \mathbb{E}\left[u''\big(c(x', y^{P\prime})\big) c_x(x', y^{P\prime}) \,\middle|\, y^P\right] \\
- u''''(c) c_x^3 - 3u'''(c) c_x c_{xx} \Bigg\}.
\end{aligned}
\tag{B.18}
$$

Equations (B.17) and (B.18) apply for $x > \bar{x}(y^P)$. On the constrained branch, the derivatives are given by equation (B.13).

**The borrowing-constraint boundary.** At the boundary, saving is zero and the Euler equation gives

$$
u'\big(\bar{x}(y^P)\big) = \beta(1+r)\mathbb{E}\left[u'\left(c\left((1-\tau)y' + T,\ y^{P\prime}\right)\right) \,\middle|\, y^P\right].
$$

The unconstrained solution meets the constrained solution at $x = \bar{x}(y^P)$, where $c = \bar{x}(y^P)$, so consumption is continuous. Since $u'' < 0$ and the expectation in (B.15) is finite and strictly negative, that equation implies $0 < c_x(x, y^P) < 1$ on the unconstrained branch. In the limit $x \downarrow \bar{x}(y^P)$, saving goes to zero and $x'$ tends to $(1-\tau)y' + T$; the expectation in (B.15) remains finite and strictly negative, so $0 < c_x^+(\bar{x}(y^P), y^P) < 1$.

Since $c_x^-(\bar{x}(y^P), y^P) = 1$, the MPC jumps downward at the borrowing constraint.

# C Numerical Algorithm: Partial Equilibrium

In partial equilibrium, the path for prices and transfers is exogenous, so the problem reduces to solving the household problem and aggregating over the induced distribution. We describe first the computation of the steady state and then the transitions.

## C.1 Steady State

We solve for the steady-state household policy functions using the endogenous grid method (EGM) of Carroll (2006). For the asset grid, we use $n_a = 800$ points on $[0, \bar{a}]$, where

$\bar{a} = 4{,}000$. We start from a 200-point double-exponential grid, as in Auclert et al. (2021), which concentrates points near the borrowing limit. We interpolate linearly for assets not on the grid. We discretize the transitory and persistent idiosyncratic income shocks as described in Appendix A. For the *iid* noise shock, we use $y^N \sim \mathcal{N}(-\sigma_N^2/2,\, \sigma_N^2)$ with $\sigma_N = 0.005$.

Expectations in the Euler equation are computed by summing over the discrete transitory and persistent income states with their probabilities and by integrating numerically over the noise shock $y^N$, which arrives every quarter, truncating at eight standard deviations. Because next period's consumption is piecewise linear in cash-on-hand, we split the range of integration at every value of $y^N$ at which next period's cash-on-hand crosses a knot of the consumption function, most importantly the kink at the borrowing constraint, subdivide any piece wider than $2\sigma_N$, and apply 16-point Gauss–Legendre quadrature on each piece.

To obtain the steady state, we use the policy rules to propagate the cross-sectional distribution forward until it converges to the ergodic distribution. Starting from an arbitrary initial distribution over $(a, y^P, \beta)$, we iterate forward using the policy rules and the distribution of shocks until successive iterates differ by less than $10^{-14}$. Savings that fall between asset grid points are split between the two adjacent nodes with weights linear in the distance, as in Young (2010).[26]

## C.2 Nonlinear Transition

The nonlinear response is the aggregate consumption function $\mathcal{C}_t(\Delta)$; at $t = 0$ it is given by equation (2). We compute it by evaluating the steady-state consumption policies at post-transfer cash-on-hand and aggregating over the distribution, which is propagated forward under the same policies. That is,

$$\mathcal{C}_t(\Delta) \equiv \int c\big(x + \Delta\, \mathbf{1}\{t = 0\},\, y^P;\, \beta\big)\, \mathrm{d}\Gamma_t(x, y^P, \beta;\, \Delta),$$

where $\Gamma_t(\cdot\,; \Delta)$ is the distribution of households over cash-on-hand $x$, persistent income $y^P$ and discount factor $\beta$ in period $t$ after a transfer $\Delta$ at $t = 0$; $\Gamma_0$ is the stationary distribution. Note that in partial equilibrium the policy functions are time-invariant. The transfer enters period-0 cash-on-hand directly and, for $t \geq 1$, only through the induced changes in the distribution of future cash-on-hand.

[26] We also solved the normalized sparse linear system $(I - Q)d = 0$, $\mathbf{1}'d = 1$, where $Q$ is the transition matrix over income and assets, but this was approximately 4.6 times slower than forward iteration.

## C.3 Linear Approximation

The linear approximation in equation (4) is the first-order expansion of aggregate consumption around $\Delta = 0$, $\mathcal{C}_t(0) + \mathcal{C}_t'(0)\Delta$. The coefficients $\mathcal{C}_t'(0)$ are the derivatives of the computed aggregate response. For $t = 0$, we compute $\mathcal{C}_0'(0)$ following (C.1).

For $t \geq 1$, we proceed as follows. Let $D_t(s, a_i; \Delta)$ be the mass of households in state $s$ with assets $a_i$ at the beginning of period $t$, and $dD_t(s, a_i)$ its derivative with respect to $\Delta$ at $\Delta = 0$. Since $\mathcal{C}_t(\Delta) = \sum_{s,i} D_t(s, a_i; \Delta)\, \bar{c}_s(a_i)$ for $t \geq 1$,

$$\mathcal{C}_t'(0) = \sum_{s,i} dD_t(s, a_i)\, \bar{c}_s(a_i).$$

From period 1 on, the distribution evolves as follows

$$dD_{t+1}(s', a_k) = \sum_s \Pi(s, s') \sum_i dD_t(s, a_i) \sum_j q_j \int \omega_k\big(A_i + \tilde{y} - c_s(A_i + \tilde{y})\big)\, f_{sj}(\tilde{y})\, d\tilde{y},$$

where $\Pi(s, s')$ is the probability that state $s$ is followed by state $s'$, and $\omega_k(a')$ is the weight that the lottery assigns to grid point $a_k$ when saving is $a'$. For $a' \in [a_k, a_{k+1})$, the lottery assigns weight $(a' - a_k)/(a_{k+1} - a_k)$ to $a_{k+1}$ and the remaining weight to $a_k$.

It remains to compute $dD_1$. For $k = 0, \ldots, K-1$, a household with cash-on-hand in $[x_{sk}, x_{s,k+1})$ saves an amount in $[a_k, a_{k+1})$. A marginal transfer raises its saving at rate $1 - \mathfrak{m}_{sk}$, shifting lottery weight from $a_k$ to $a_{k+1}$ at rate $(1 - \mathfrak{m}_{sk})/(a_{k+1} - a_k)$. Summing over households gives, for interior grid points,

$$dD_1(s', a_k) = \sum_s \Pi(s, s') \left[ \frac{(1 - \mathfrak{m}_{s,k-1})\, M_{s,k-1}}{a_k - a_{k-1}} - \frac{(1 - \mathfrak{m}_{sk})\, M_{sk}}{a_{k+1} - a_k} \right],$$

where $M_{sk} = \sum_{i,j} w_{sij} \Pr\left(x_{sk} \leq A_i + \tilde{y} < x_{s,k+1}\right)$ is the mass of households in state $s$ with cash-on-hand between the two knots. We evaluate these probabilities using the lognormal cumulative distribution function associated with $f_{sj}$. At the lower endpoint, the first term is omitted. Saving above $a_K$ is assigned to $a_K$, so the second term is omitted at the upper endpoint.

As a check, we compute the coefficients using central finite differences:

$$\mathcal{C}_t'(0) \approx \frac{\mathcal{C}_t(+\varepsilon) - \mathcal{C}_t(-\varepsilon)}{2\varepsilon},$$

with $\varepsilon = 10^{-6}$ of quarterly GDP. The two sets of coefficients differ by at most $3 \times 10^{-10}$.

## C.4 Higher-Order Local Approximations

The higher-order approximations in Figure 4 are Taylor expansions of the aggregate consumption function around $\Delta = 0$,

$$\mathcal{C}_0(\Delta) \approx \mathcal{C}_0(0) + \sum_{n=1}^{N} \frac{\mathcal{C}_0^{(n)}(0)}{n!}\,\Delta^n, \qquad N = 2, \ldots, 4.$$

This section describes the baseline computation of the derivatives $\mathcal{C}_0^{(n)}(0)$ from the numerical solution of the steady state. Section C.4.1 describes two alternative computations.

We index households by $(s, i, j)$. The state $s$ collects its discount factor and persistent income $y^P$; $a_i$, a point of the asset grid, is its assets at the beginning of the period; and $j$ indexes the transitory component of its earnings, which has probability $q_j$. $D_s(a_i)$ is the stationary mass of households in state $s$ with assets $a_i$. A household of type $(s, i, j)$ has resources $A_i \equiv (1 + r)a_i + T$ before earnings and after-tax earnings $\tilde{y} = \tilde{y}_{sj} e^{y^N}$, where $\tilde{y}_{sj}$ is implied by the persistent and transitory components and $y^N \sim \mathcal{N}(-\sigma_N^2/2, \sigma_N^2)$ is the noise shock. After a transfer $\Delta$, its cash-on-hand is $x = A_i + \Delta + \tilde{y}$, and it consumes $c_s(x)$.

Aggregate consumption is then

$$\mathcal{C}_0(\Delta) = \sum_{s,i} D_s(a_i)\,\bar{c}_s(a_i; \Delta), \qquad \bar{c}_s(a_i; \Delta) = \sum_j q_j \int c_s(A_i + \Delta + \tilde{y})\, f_{sj}(\tilde{y})\, d\tilde{y},$$

where $\bar{c}_s(a_i; \Delta)$ is the expected consumption of a household that starts the period in state $s$ with assets $a_i$, before its earnings are drawn, and

$$f_{sj}(\tilde{y}) = \frac{1}{\sigma_N\, \tilde{y}}\, \phi\!\left(\frac{\log(\tilde{y}/\tilde{y}_{sj}) + \sigma_N^2/2}{\sigma_N}\right),$$

where $\phi$ is the standard normal density, so that $f_{sj}$ is the lognormal density of after-tax earnings given $\tilde{y}_{sj}$. This is equation (2), with the sum over $s$ in place of the integral over $F$, and the sums over $i$ and $j$, together with $f_{sj}$, in place of the integral over $G$. The transfer is paid only at $t = 0$. In every later period a household with assets $a_i$ has cash-on-hand $A_i + \tilde{y}$, and its expected consumption is $\bar{c}_s(a_i) \equiv \bar{c}_s(a_i; 0)$; the transfer then affects consumption only through the distribution of households over states and assets (Section C.3).

The consumption policy is piecewise linear in cash-on-hand. Its knots are $x_{sk} = c_{sk} + a_k$, $k = 0, \ldots, K$, where $c_{sk}$ is the consumption, from the Euler equation, of a household in state $s$ that ends the period with assets $a_k$, a point of the asset grid. Its derivative, the MPC

$\mathfrak{m}_s(x)$ of equation (1), is constant between knots:

$$c_s(x) = x \;\text{ for } x < x_{s0} = \bar{x}(y^P), \qquad \mathfrak{m}_s(x) = \mathfrak{m}_{sk} \;\text{ on } [x_{sk}, x_{s,k+1}),$$

with $x_{s,K+1} \equiv \infty$, $\mathfrak{m}_{sk} = (c_{s,k+1} - c_{sk})/(x_{s,k+1} - x_{sk})$ for $k < K$, and $\mathfrak{m}_{sK} = \mathfrak{m}_{s,K-1}$. The jump in the MPC at knot $k$ is

$$J_{sk} = \mathfrak{m}_{sk} - \mathfrak{m}_{s,k-1}, \qquad \mathfrak{m}_{s,-1} = 1.$$

At the borrowing kink, $J_{s0} = \mathfrak{m}_{s0} - 1 < 0$; at the interior knots, $J_{sk}$ is the discrete curvature of the consumption function.

Because the MPC is a step function, it is the sum of its jumps, $\mathfrak{m}_s(x) = 1 + \sum_k J_{sk}\,\mathbf{1}\{x \geq x_{sk}\}$. Differentiating under the integral therefore gives the population-average MPC as

$$\mathcal{C}_0'(\Delta) = \sum_{s,i,j} w_{sij} \int \mathfrak{m}_s(A_i + \Delta + \tilde{y})\, f_{sj}(\tilde{y})\, d\tilde{y} = \sum_{s,i,j} w_{sij} \Big[1 + \sum_k J_{sk} \Pr\big(\tilde{y} \geq x_{sk} - A_i - \Delta\big)\Big], \tag{C.1}$$

where $w_{sij} \equiv D_s(a_i)\, q_j$ is the mass of households of type $(s, i, j)$.

The derivative of each probability is the density of earnings at the knot, so that

$$\mathcal{C}_0''(\Delta) = \sum_{s,i,j} w_{sij} \sum_k J_{sk}\, f_{sj}\big(x_{sk} - A_i - \Delta\big).$$

More generally,

$$\mathcal{C}_0^{(n)}(\Delta) = (-1)^n \sum_{s,i,j} w_{sij} \sum_k J_{sk}\, f_{sj}^{(n-2)}\big(x_{sk} - A_i - \Delta\big). \tag{C.2}$$

For Figure 4, we use an asset grid with 10,000 points instead of 800 for computing the fourth order approximation; see Section C.5 for robustness to the grid size.

### C.4.1 Alternative Computations of Higher-Order Derivatives

We also compute the second and third-order derivatives in two alternative ways. First, we evaluate $\mathcal{C}_0''(0)$ and $\mathcal{C}_0'''(0)$ from equations (5) and (6), which depend on the derivatives $c_x$, $c_{xx}$ and $c_{xxx}$ of the unconstrained policy. We compute these derivatives by solving the recursions (B.15), (B.17) and (B.18) as fixed points on the policy grid, interpolating linearly at next period's cash-on-hand. With $\mathfrak{m}^{(\ell)}$ the guess for $c_x$ in iteration $\ell$, at the unconstrained

points of the cash-on-hand grid, the fixed-point update is

$$B^{(\ell)}(x, y^P) = \beta(1+r)^2\, \mathbb{E}\big[u''\big(c(x', y^{P\prime})\big)\mathfrak{m}^{(\ell)}(x', y^{P\prime}) \,\big|\, y^P\big]\,,$$
$$\mathcal{T}[\mathfrak{m}^{(\ell)}](x, y^P) = \frac{B^{(\ell)}(x, y^P)}{u''\big(c(x, y^P)\big) + B^{(\ell)}(x, y^P)},$$

where $x'$ is next period's cash-on-hand, given the saving policy and the income draw. We initialize $\mathfrak{m}^{(0)}$ with the slopes of the computed consumption policy, clipped to $[10^{-8}, 1]$. In each expectation, the guess is interpolated linearly at next period's cash-on-hand, set to one below the borrowing kink (at the kink, we use the unconstrained, right-hand value), and held constant above the last grid point. We sum over the discrete income states and integrate over the noise shock with the same quadrature as for the Euler equation.

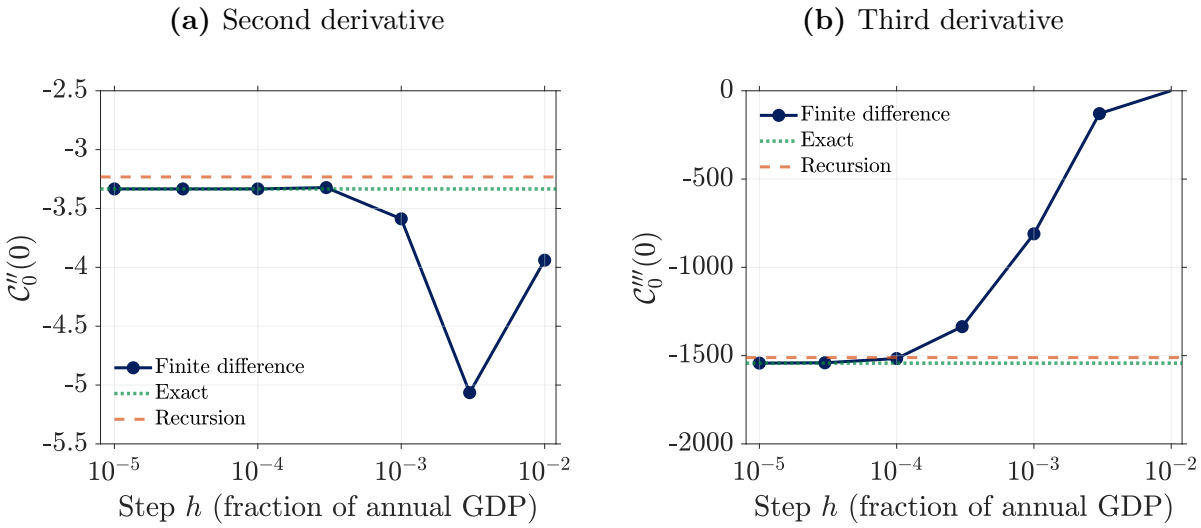
Given $c_x$, the recursions for $c_{xx}$ and $c_{xxx}$ are linear. We solve them in turn on the same grid, with $c_{xx} = c_{xxx} = 0$ below the borrowing kink, and evaluate equations (5) and (6) with the density of cash-on-hand and its derivative at the kink.

Second, we compute the second and third derivatives with central finite differences of the nonlinear response, with step $h$:

$$\widehat{\mathcal{C}_0''}(h) = \frac{1}{3}\left[4\frac{\mathcal{C}_0(h) - 2\mathcal{C}_0(0) + \mathcal{C}_0(-h)}{h^2} - \frac{\mathcal{C}_0(2h) - 2\mathcal{C}_0(0) + \mathcal{C}_0(-2h)}{(2h)^2}\right],$$
$$\widehat{\mathcal{C}_0'''}(h) = \frac{\mathcal{C}_0(2h) - 2\mathcal{C}_0(h) + 2\mathcal{C}_0(-h) - \mathcal{C}_0(-2h)}{2h^3}.$$

Figure C.1 compares these finite differences, for steps $h$ between $10^{-5}$ and $10^{-2}$ of annual GDP, with the exact derivatives from equation (C.2) and with the values from the recursion. As the figure shows, for $h \leq 10^{-4}$, the derivatives computed with finite differences approach the exact derivatives. In addition, the derivatives computed with the recursion are within 3% of the exact derivatives.

**Figure C.1:** Finite-difference, exact and recursion values of the second and third derivatives




*Note:* Finite-difference, exact and recursion values of $\mathcal{C}_0''(0)$ (panel a) and $\mathcal{C}_0'''(0)$ (panel b), as a function of the step $h$ ($\sigma_N = 0.005$). Finite differences are central, with Richardson extrapolation for $\mathcal{C}_0''(0)$. Exact: equation (C.2). Recursion: equations (5) and (6), with the policy derivatives from the recursions of Appendix B.4.

## C.5 Robustness to the Noise Shock and the Asset Grid

We analyze the robustness of the numerical results to the standard deviation of the noise shock (baseline $\sigma_N = 0.005$) and the number of asset-grid points (baseline $n_a = 800$).

Table C.1 reports the constrained share, the ratio of the linear to the nonlinear impact response (L/N), and the transfer above which each approximation of orders two through four becomes less accurate than the linear one. As the table shows, a lower volatility of the noise shock has modest effects on L/N, while a larger one can reduce significantly the degree of nonlinearities. Meanwhile, higher-order approximations are also relatively sensitive to the volatility of the noise shock. On the other hand, grid size has little effect on L/N and the second- and third-order thresholds, while the fourth-order approximation is more sensitive. However, above a grid size of 1,600, the fourth order approximation becomes stable. The results presented for the fourth-order approximation use 10,000 points. Table C.2 reports the associated Taylor coefficients. In addition, Table C.3 shows that for a volatility of the noise shock that is 1/5th of the baseline, the third and fourth order coefficients remain relatively unstable for grid sizes below 9,000.

**Table C.1:** Nonlinearity and accuracy of the local approximations, by noise shock and asset grid

| | Constrained share (%) | L/N for transfer $\Delta$ | | | Less accurate than linear above (% of GDP) | | |
|---|---|---|---|---|---|---|---|
| | | 0.25% | 1% | 10% | 2nd order | 3rd order | 4th order |
| *A. Noise shock, $\sigma_N$ ($n_a = 800$)* | | | | | | | |
| 0.001 | 20.90 | 1.13 | 1.44 | 2.12 | 3.87 | 0.02 | 0.02 |
| 0.0025 | 20.80 | 1.13 | 1.44 | 2.12 | 7.09 | 0.85 | 0.08 |
| 0.005 | 20.50 | 1.13 | 1.44 | 2.11 | 4.41 | 0.52 | 0.32 |
| 0.05 | 13.50 | 1.07 | 1.26 | 1.85 | 2.46 | 1.41 | 2.73 |
| 0.1 | 10.80 | 1.04 | 1.17 | 1.73 | 3.28 | 3.23 | 2.59 |
| *B. Asset grid, $n_a$ ($\sigma_N = 0.005$)* | | | | | | | |
| 400 | 20.50 | 1.13 | 1.44 | 2.10 | 4.31 | 0.51 | 0.47 |
| 800 | 20.50 | 1.13 | 1.44 | 2.11 | 4.41 | 0.52 | 0.38 |
| 1,600 | 20.50 | 1.13 | 1.44 | 2.11 | 4.57 | 0.55 | 0.35 |
| 9,000 | 20.50 | 1.13 | 1.44 | 2.11 | 4.56 | 0.56 | 0.33 |
| 10,000 | 20.50 | 1.13 | 1.44 | 2.11 | 4.56 | 0.56 | 0.32 |

*Note:* Constrained: steady-state share of households at the borrowing constraint. L/N: ratio of the linear to the nonlinear impact response of aggregate consumption to a one-time transfer $\Delta$, in % of annual GDP. Last three columns: transfer, in % of annual GDP, above which the Taylor approximation of the given order is further from the nonlinear impact response than the linear approximation. The baseline results for the fourth-order approximation use 10,000 points.

**Table C.2:** Taylor coefficients of the impact consumption response, by noise shock and asset grid

| | $b_1$ | $b_2$ | $b_3$ | $b_4$ |
|---|---|---|---|---|
| *A. Noise shock, $\sigma_N$ ($n_a = 800$)* | | | | |
| 0.001 | 0.331 | −0.076 | −1.514 | 246.111 |
| 0.0025 | 0.331 | −0.046 | −0.212 | 11.526 |
| 0.005 | 0.329 | −0.067 | −0.411 | −0.571 |
| 0.01 | 0.324 | −0.089 | −0.184 | −0.285 |
| 0.05 | 0.291 | −0.073 | −0.020 | 0.016 |
| 0.1 | 0.273 | −0.047 | 0.000 | 0.007 |
| *B. Asset grid, $n_a$ ($\sigma_N = 0.005$)* | | | | |
| 400 | 0.329 | −0.068 | −0.421 | −0.134 |
| 800 | 0.329 | −0.067 | −0.411 | −0.571 |
| 1,600 | 0.329 | −0.065 | −0.387 | −1.151 |
| 3,200 | 0.329 | −0.065 | −0.377 | −1.287 |
| 9,000 | 0.329 | −0.065 | −0.375 | −1.212 |
| 10,000 | 0.329 | −0.065 | −0.375 | −1.211 |

*Note:* The Taylor expansion is such that $\Delta C_0(s) \simeq \sum_{n=1}^{4} b_n s^n, b_n = \frac{25(0.04)^n}{n!} \mathcal{C}_0^{(n)}(0)$. Baseline: $\sigma_N = 0.005$, $n_a = 800$.

**Table C.3:** Taylor coefficients with low noise volatility

| $n_a$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ |
|---|---|---|---|---|
| 400 | 0.330 | −0.070 | −2.016 | 295.007 |
| 800 | 0.331 | −0.076 | −1.514 | 246.111 |
| 1,600 | 0.331 | −0.075 | −1.219 | 238.632 |
| 3,200 | 0.331 | −0.067 | −1.273 | 198.370 |
| 9,000 | 0.331 | −0.063 | −0.987 | 154.994 |
| 10,000 | 0.331 | −0.063 | −0.963 | 151.673 |

*Notes:* The noise standard deviation is $\sigma_N = 0.001$ in every row instead of the baseline value of $\sigma_N = 0.005$. The Taylor expansion is such that $\Delta C_0(s) \simeq \sum_{n=1}^{4} b_n s^n, b_n = \frac{25(0.04)^n}{n!} \mathcal{C}_0^{(n)}(0)$.

**Figure C.2:** Local approximations to the impact consumption response, by noise shock and asset grid

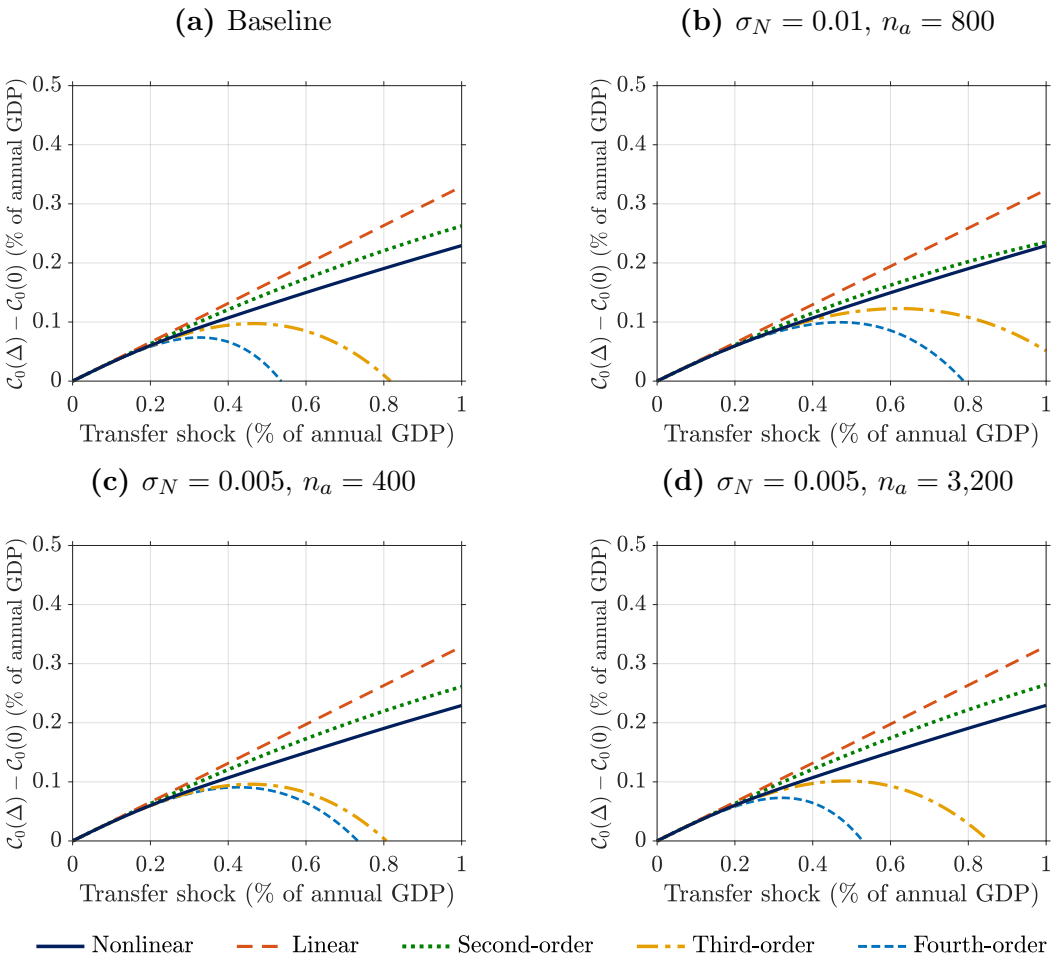
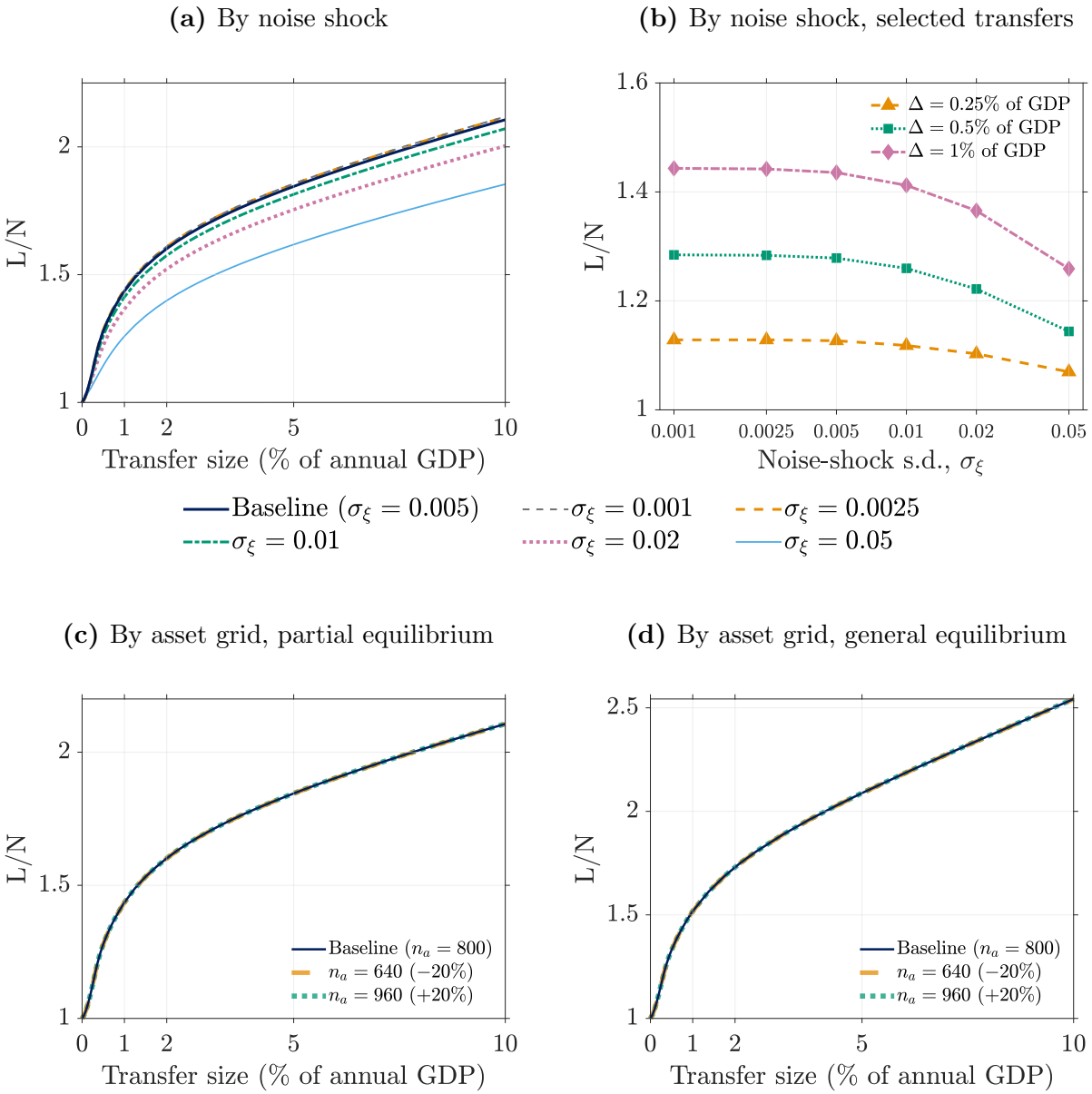


**Figure C.3:** Ratio of the linear to the nonlinear response, by noise shock and asset grid



# D Numerical Algorithm: General Equilibrium

The steady state can be obtained using the same algorithm as in partial equilibrium, with the real interest rate now clearing the asset market, and a real wage of one, as implied by the linear technology. We solve for the linear impulse response following Auclert et al. (2021) and using their Python toolkit. We compute the household Jacobians with their fake-news algorithm, using central finite differences with step $10^{-6}$.

For the nonlinear transition, we proceed as follows:

1. *Guess a sequence* for output, $\mathbf{Y} = (Y_0, \ldots, Y_{\mathbb{T}-1})$. We set $\mathbb{T} = 200$ quarters.

2. *Recover the remaining aggregates.* Production implies $N_t = Y_t$ and goods-market clearing implies $C_t = Y_t - G$, so the wage Phillips curve involves only output and inflation. Iterating it backward from $\pi_{\mathbb{T}} = \bar{\pi}$,

$$\pi_t = \kappa \left[ v'(Y_t) - \frac{1-\tau}{Y_t - G} \right] + \bar{\beta}\, \pi_{t+1},$$

   gives inflation. The monetary rule gives $i_t = \bar{i} + \phi_\pi(\pi_t - \bar{\pi})$, and the Fisher equation gives $r_t = (1 + i_t)/(1 + \pi_{t+1}) - 1$. Finally, the fiscal rule and the government budget constraint give the transfer and debt, forward from $B_{-1} = \bar{B}$ and $r_{-1} = \bar{r}$, the steady-state rate on the real debt issued before the shock:

$$T_t = T - \phi_B \bar{r}\,(B_{t-1} - \bar{B}) + \Delta\, \mathbf{1}\{t = 0\}, \qquad B_t = (1 + r_{t-1})B_{t-1} + G + T_t - \tau Y_t,$$

   where $\mathbf{1}\{t = 0\}$ equals one at $t = 0$ and zero otherwise.

3. *Backward EGM from the terminal steady state.* Starting from the steady-state policy functions at $t = \mathbb{T}$, iterate the Euler equation backward with EGM, given the paths of after-tax labor income, interest rates and transfers implied by $\mathbf{Y}$, to obtain the policy functions for $t = \mathbb{T} - 1, \ldots, 0$.

4. *Propagate the distribution forward* from the steady-state distribution, $\Gamma_0 = \bar{\Gamma}$, using these policies. This delivers aggregate consumption $C_t(\mathbf{Y}; \Delta)$, asset demand $A_t(\mathbf{Y}; \Delta)$ and next period's distribution $\Gamma_{t+1}$.

5. *Compute the residual* of asset-market clearing for $t = 0, \ldots, \mathbb{T} - 1$:

$$F_t(\mathbf{Y}; \Delta) = A_t(\mathbf{Y}; \Delta) - B_t(\mathbf{Y}; \Delta).$$

When the asset market clears, the household and government budget constraints imply that the goods market clears as well, so $C_t(\mathbf{Y};\Delta) = Y_t - G$, as used in step 2.

6. *Quasi-Newton update.*

$$\mathbf{Y}^{(k+1)} = \mathbf{Y}^{(k)} - \alpha_k \, \mathbf{J}^{-1}\mathbf{F}\big(\mathbf{Y}^{(k)};\Delta\big), \qquad \mathbf{J} \equiv \left.\frac{\partial \mathbf{F}}{\partial \mathbf{Y}}\right|_{(\bar{\mathbf{Y}};0)},$$

where $\mathbf{J}$ is the $\mathbb{T} \times \mathbb{T}$ steady-state equilibrium Jacobian. It is computed once, before iterating: the household Jacobians are obtained with the fake-news algorithm of Auclert et al. (2021), using central finite differences with step $10^{-6}$. The step size $\alpha_k$ starts at 1 and is halved until the residual falls.[27]

7. *Iterate until convergence*, that is, until $\max_t |F_t| < 10^{-8}$.

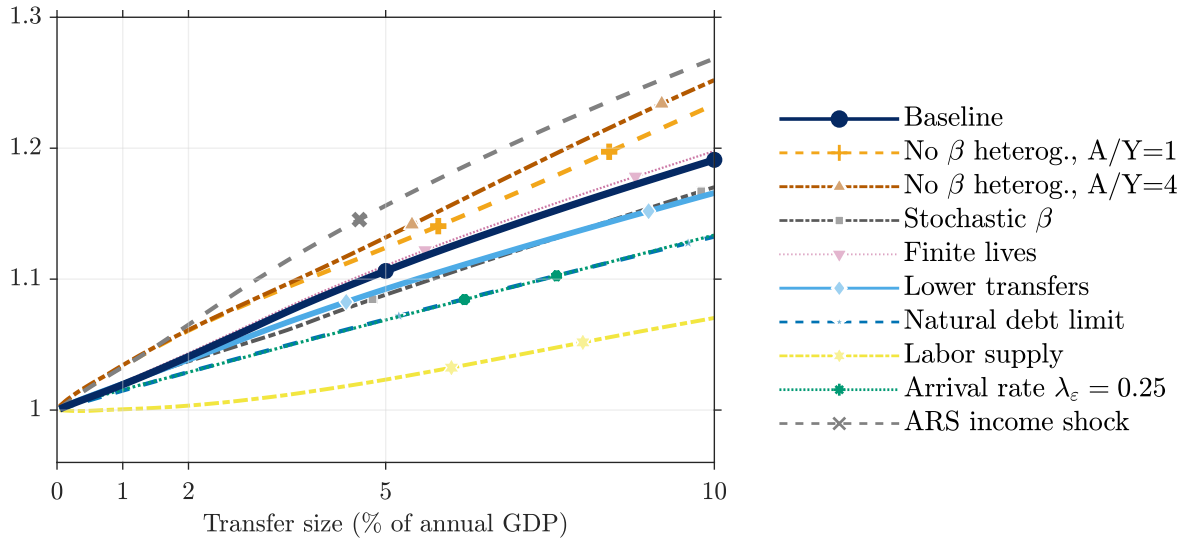
We use as a first guess the linear transition. Along a grid of transfer sizes $\Delta_1 < \Delta_2 < \cdots$, we start each size from the previous size's nonlinear transition scaled by $\Delta_i/\Delta_{i-1}$.

Given the steady state, computing the household Jacobians takes about 20 seconds and the linear solution less than a second. Each iteration of the nonlinear solution takes about 15 seconds, and starting from the linear solution or from the steady state takes 5 iterations to converge for a transfer of 1% of annual GDP. These times are for a laptop with an Intel Core Ultra 7 268V processor (8 cores) and 32 GB of memory, using 8 parallel processes.

[27] We keep $\mathbf{J}$ fixed at its steady-state value. Convergence can be accelerated with a Broyden update.

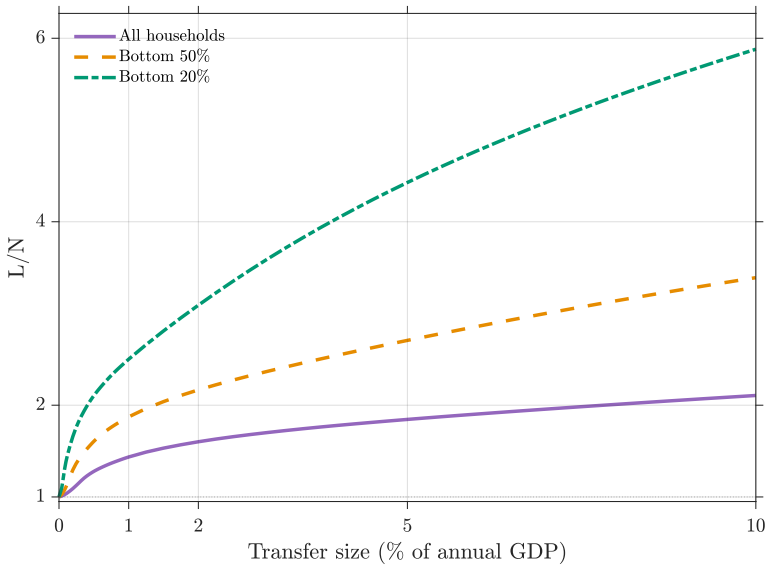
# E Partial Equilibrium: Additional Results

**Figure E.1:** Ratio of Linear to Nonlinear First-Year Aggregate Consumption Responses



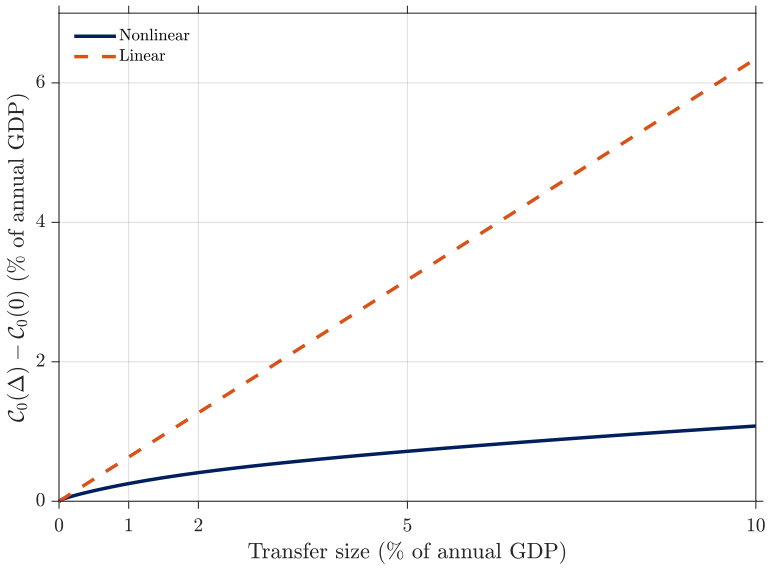

*Note:* The figure presents the ratio of the linear to the nonlinear consumption response over the first four quarters, for different model specifications, in the partial-equilibrium model.

**Figure E.2:** Targeted versus Untargeted Transfers in Partial Equilibrium



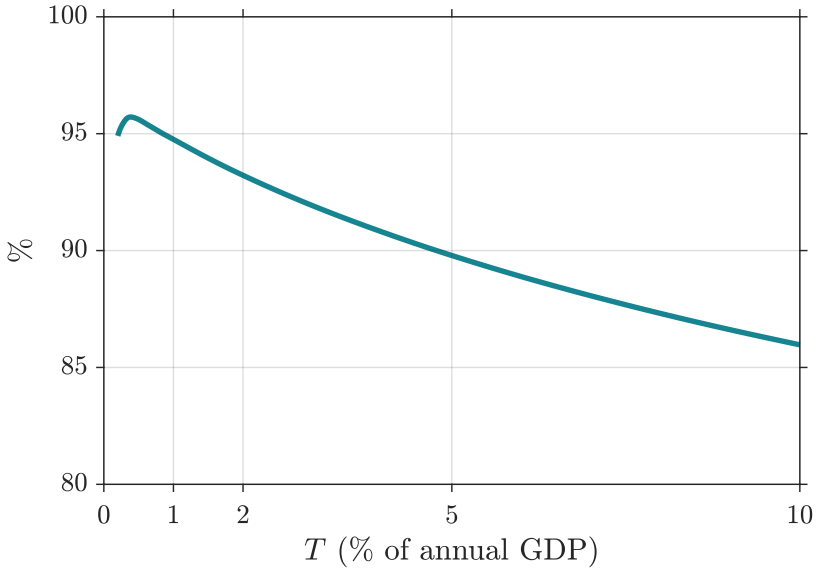

*Note:* Ratio of the linear to the nonlinear partial-equilibrium impact consumption response to a one-time transfer, as a function of transfer size, when the period-0 transfer is directed to all households, the bottom 50%, or the bottom 20% of the income distribution.

**Figure E.3:** Targeted Transfers in Partial Equilibrium




*Note:* Consumption response in the baseline partial-equilibrium model where transfers in period 0 are directed toward households in the bottom 20% of the income distribution. Partial-equilibrium counterpart of Figure G.1.

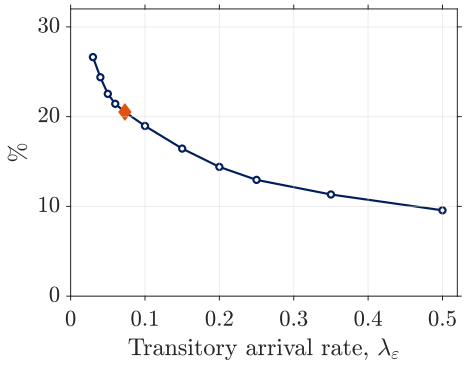
**Figure E.4:** Share of the linear–nonlinear gap explained by movement in the constrained share



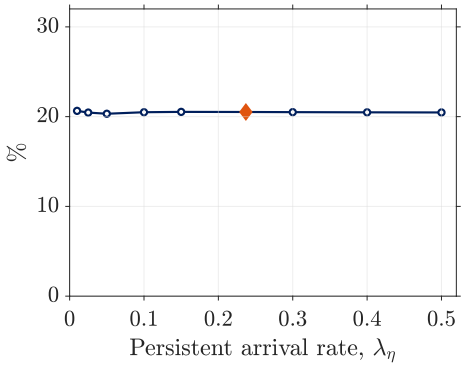

*Note:* Share of the gap between the linear and nonlinear impact consumption responses accounted for by households moving off the constraint, the first term of (7), as a function of transfer size.

**Figure E.5:** Arrival rates, the constrained share, and nonlinearity

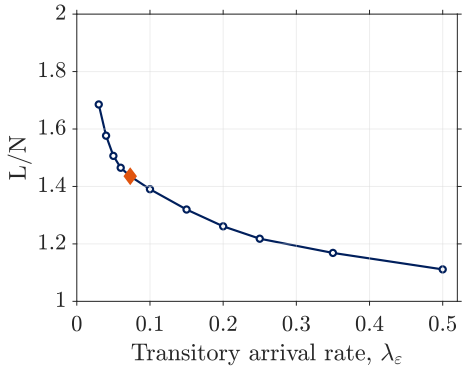
**(a)** Constrained share: transitory



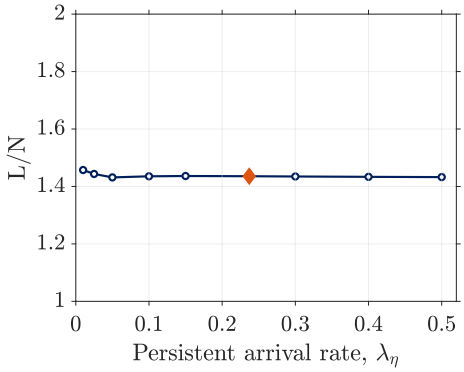

**(b)** Constrained share: persistent




**(c)** Linear/nonlinear, PE: transitory



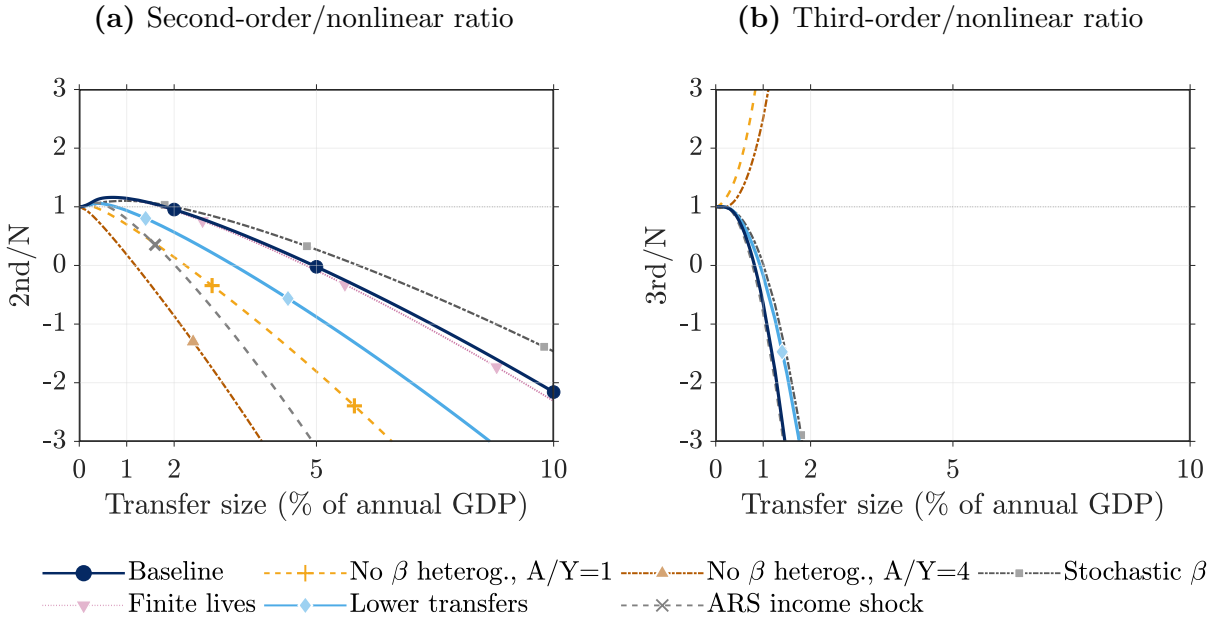

**(d)** Linear/nonlinear, PE: persistent




*Note:* Panels (a) and (b) report the steady-state share of constrained households; panels (c) and (d) report the ratio of the linear to the nonlinear impact consumption response to a one-time transfer of 1% of annual GDP. Each curve varies one arrival rate, holding the other at its estimated value ($\lambda_\eta = 0.237$ or $\lambda_\varepsilon = 0.073$), the persistence at 0.987, and total persistent and transitory risk ($\lambda_\eta\sigma_\eta^2$ and $\lambda_\varepsilon\sigma_\varepsilon^2$) at their estimated values. The rest of the parameters are recalibrated. The diamond is the estimated calibration.

## E.1 Additional Results on Higher-Order Approximations

**Figure E.6:** Higher-order local approximations across robustness calibrations




*Note:* Each line reports the indicated local approximation divided by the nonlinear impact consumption response.

# F Deterministic Analytical Example

This appendix provides a simple deterministic example where increasing the order of a Taylor approximation around zero need not recover the consumption response for a fixed transfer.

Consider a version of our baseline model without uncertainty from date 1 on. Households have a common discount factor $\beta \in (0,1)$ and face a no-borrowing constraint, and the interest rate satisfies $\beta(1+r)=1$. At date 0 they differ in cash-on-hand $x \in (0,\infty)$ before the transfer, distributed according to $G$, which has a density $g$. Assume that $\mathbb{E}[x] < \infty$. From date 1 on, every household earns the same income $y > 0$ with certainty. At date 0 every household receives an unexpected one-time transfer $T_0 = \Delta \geq 0$, and $T_t = 0$ afterwards. Each household's consumption is constant for $t \geq 1$, while households with low cash-on-hand are constrained at $t = 0$. The date-0 consumption policy, holding the common future income $y$ fixed, is

$$c(x) = \begin{cases} x, & x < y, \\ (1-\beta)\, x + \beta y, & x \geq y. \end{cases}$$

Unlike in the baseline model, $c$ is linear above the kink with constant MPC $1-\beta$.[28]

Aggregate consumption after the transfer is $\mathcal{C}_0(\Delta) = \int c(x+\Delta)\,\mathrm{d}G(x)$. Let $\Lambda(\Delta) \equiv G(y-\Delta)$ be the share of households that remain constrained, and let $\bar{\Lambda} \equiv \Lambda(0)$. Since the aggregate MPC is $\mathcal{C}_0'(\Delta) = (1-\beta) + \beta\Lambda(\Delta)$, integration gives

$$\mathcal{C}_0(\Delta) - \mathcal{C}_0(0) = \mathcal{C}_0'(0)\,\Delta + \beta \int_0^{\Delta} \left[\Lambda(u) - \bar{\Lambda}\right] du, \tag{F.1}$$

where $\mathcal{C}_0'(0) = (1-\beta) + \beta\bar{\Lambda}$. The first term of (F.1) is the linear response, which holds the constrained share fixed at $\bar{\Lambda}$; all nonlinearity comes from the path of the constrained share.

Assume that $g$ is infinitely differentiable just below $y$, with finite left derivatives of every order at $y$. Since we consider positive transfers, derivatives of $\mathcal{C}_0$ and $\Lambda$ at zero are taken from the right, while derivatives of $g$ at $y$ are taken from the left. The higher-order derivatives are

$$\mathcal{C}_0^{(k)}(0) = \beta\,\Lambda^{(k-1)}(0) = (-1)^{k-1}\beta\,g^{(k-2)}(y), \qquad k \geq 2.$$

The order-$n$ Taylor approximation around zero is

$$\mathcal{C}_0^{[n]}(\Delta) = \mathcal{C}_0(0) + \sum_{k=1}^{n} \frac{\mathcal{C}_0^{(k)}(0)}{k!}\Delta^k.$$

**Proposition 1.** *Let $x_{\min} \equiv \inf\{x : G(x) > 0\}$ denote the lower endpoint of the cash-on-hand distribution. Suppose $x_{\min} < y$, and define $\bar{\Delta} \equiv y - x_{\min}$. Then:*

*(i) For $\Delta \geq \bar{\Delta}$,*

$$\mathcal{C}_0(\Delta) - \mathcal{C}_0(0) = (1-\beta)\Delta + \beta \int_0^{\bar{\Delta}} \Lambda(u)\,du.$$

*(ii) $\mathcal{C}_0$ is not analytic at $\bar{\Delta}$.*

*(iii) The Taylor series of $\mathcal{C}_0$ around zero does not converge to $\mathcal{C}_0$ throughout any open interval of transfers above $\bar{\Delta}$.*

*Proof of Proposition 1. (i)* For $\Delta \geq \bar{\Delta}$, no household is constrained and $\Lambda(\Delta) = 0$. Substituting into (F.1) gives the stated expression.

*(ii)* By the definition of $x_{\min}$, $\Lambda(\Delta) > 0$ for $0 \leq \Delta < \bar{\Delta}$. Hence $\mathcal{C}_0'(\Delta) > 1-\beta$ below $\bar{\Delta}$, while part (i) gives $\mathcal{C}_0'(\Delta) = 1-\beta$ above it. Suppose that $\mathcal{C}_0$ were analytic at $\bar{\Delta}$. On a

[28] A household that is never constrained consumes the annuity value of its wealth, $c = (1-\beta)x + \beta y$, and saves $\beta(x-y)$, which is feasible if and only if $x \geq y$. A household with $x < y$ consumes all its cash-on-hand at date 0 and $y$ for all $t \geq 1$.

right neighborhood of $\bar{\Delta}$ it coincides with the line in part (i). The identity theorem would then imply that it coincides with the same line on a left neighborhood of $\bar{\Delta}$, contradicting $\mathcal{C}_0'(\Delta) > 1 - \beta$ there.

*(iii)* Define

$$A \equiv \beta \int_0^{\bar{\Delta}} \Lambda(u)\, du > 0.$$

Suppose that the Taylor series around zero converges to $\mathcal{C}_0$ throughout an open interval above $\bar{\Delta}$, and choose two points $a < b$ in that interval. Convergence at $b$ implies that the series defines an analytic function $S$ on $(-b, b)$. By part (i) and the assumed convergence,

$$S(\Delta) = \mathcal{C}_0(0) + (1 - \beta)\Delta + A, \qquad \Delta \in (a, b).$$

The identity theorem implies that this equality holds throughout $(-b, b)$. At zero it gives $S(0) = \mathcal{C}_0(0) + A$. But the constant term of the Taylor series is $\mathcal{C}_0(0)$, so $S(0) = \mathcal{C}_0(0)$, contradicting $A > 0$. □

To illustrate the results, consider a uniform distribution (Figure F.1, panel (a)): $G$ is uniform on $[x_{\min}, x_{\max}]$, with $x_{\min} < y < x_{\max}$. Then the constrained share falls linearly, $\Lambda(\Delta) = \bar{\Lambda} \max\{1 - \Delta/\bar{\Delta}, 0\}$, and (F.1) gives

$$\mathcal{C}_0(\Delta) - \mathcal{C}_0(0) = \begin{cases} \mathcal{C}_0'(0)\, \Delta - \frac{\beta\bar{\Lambda}}{2\bar{\Delta}}\, \Delta^2, & 0 \le \Delta \le \bar{\Delta}, \\ (1 - \beta)\Delta + \frac{\beta\bar{\Lambda}\bar{\Delta}}{2}, & \Delta \ge \bar{\Delta}. \end{cases}$$

The second-order Taylor expansion is exact up to $\bar{\Delta}$, and all higher-order terms are zero. Beyond $\bar{\Delta}$ it understates the response by $\frac{\beta\bar{\Lambda}}{2\bar{\Delta}}(\Delta - \bar{\Delta})^2$.

Now let the density of $G$ rise linearly from zero at $x_{\min}$ up to the kink, as for a triangular distribution with mode at or above $y$ (panel (b)). Then the constrained share falls quadratically, $\Lambda(\Delta) = \bar{\Lambda} \max\{1 - \Delta/\bar{\Delta}, 0\}^2$, and

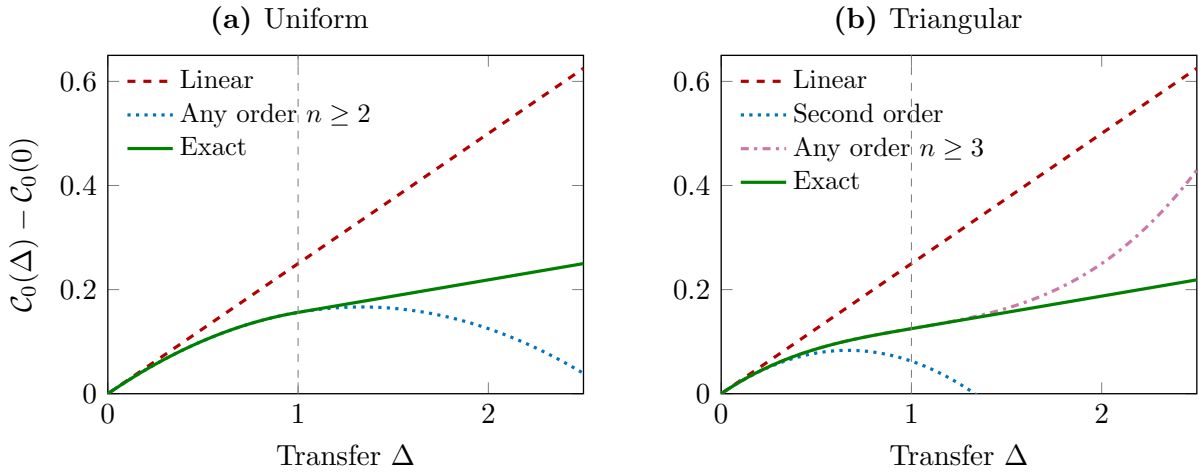
$$\mathcal{C}_0(\Delta) - \mathcal{C}_0(0) = \begin{cases} \mathcal{C}_0'(0)\, \Delta - \frac{\beta\bar{\Lambda}}{\bar{\Delta}}\, \Delta^2 + \frac{\beta\bar{\Lambda}}{3\bar{\Delta}^2}\, \Delta^3, & 0 \le \Delta \le \bar{\Delta}, \\ (1 - \beta)\Delta + \frac{\beta\bar{\Lambda}\bar{\Delta}}{3}, & \Delta \ge \bar{\Delta}. \end{cases}$$

The third-order Taylor expansion is exact up to $\bar{\Delta}$, and all higher-order terms are zero. Beyond $\bar{\Delta}$ it overstates the response by $\frac{\beta\bar{\Lambda}}{3\bar{\Delta}^2}(\Delta - \bar{\Delta})^3$.

In the uniform and triangular examples, the Taylor series around $\Delta = 0$ terminates after finitely many terms and therefore has an infinite radius of convergence. For transfers above

$\bar{\Delta}$, however, its sum differs from the true consumption response. More generally, part (iii) of Proposition 1 rules out that the Taylor series of $\mathcal{C}_0$ around zero converges to $\mathcal{C}_0$ throughout any open interval of transfers above $\bar{\Delta}$, whether the radius of convergence is finite or infinite.

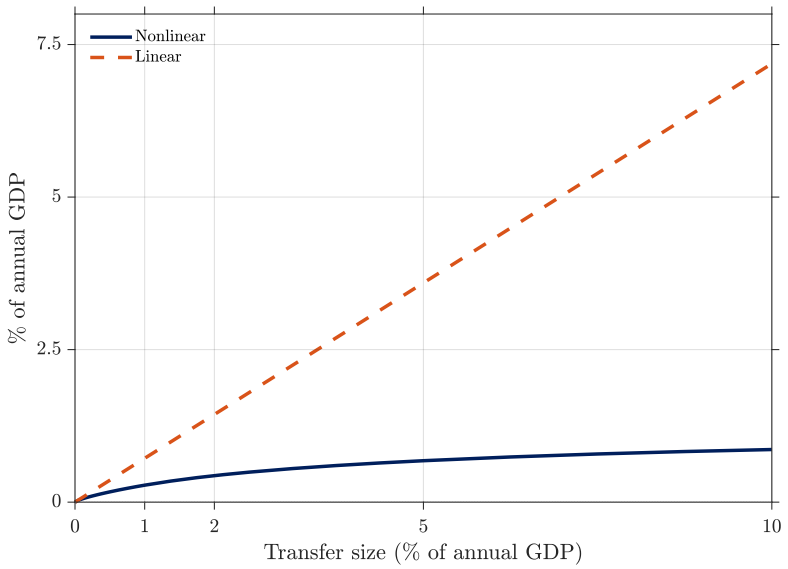
**Figure F.1:** Exact response and local approximations




*Note:* $\beta = 15/16$, $y = 1$, and $G$ is uniform on $[0, 5]$ in panel (a) and triangular on $[0, 5]$ with mode 1 in panel (b), so that $\bar{\Lambda} = 0.2$ in both panels. The gray dashed line represents the value of $\Delta$ at which the borrowing constraint stops binding for all households, $\bar{\Delta} = 1$.

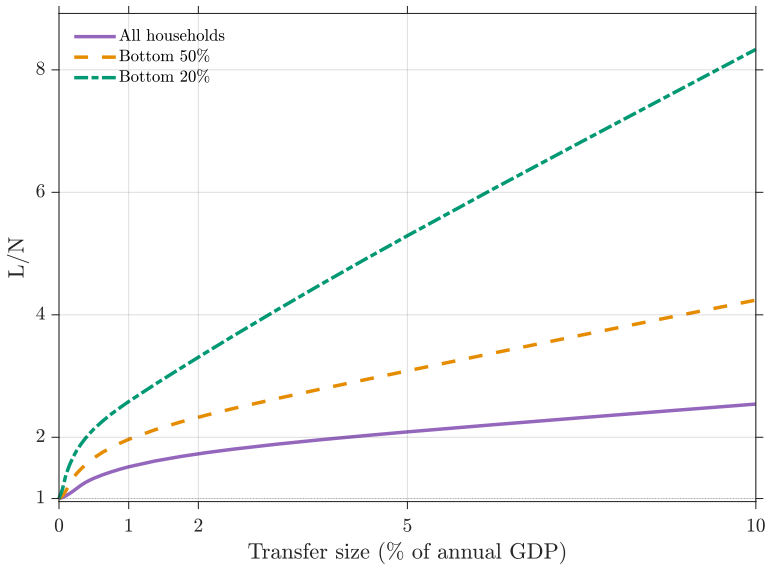
# G General Equilibrium: Additional Results

**Figure G.1:** Targeted Transfers in General Equilibrium



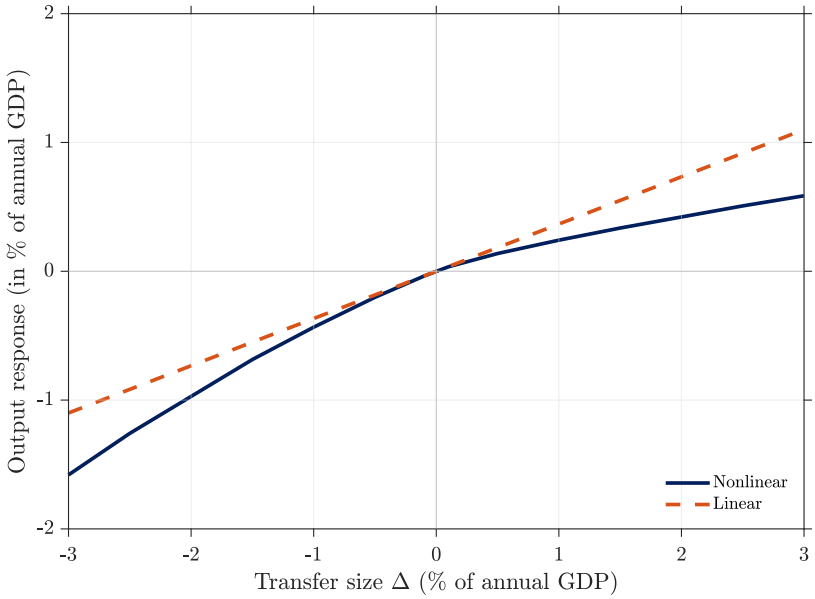

*Note:* Output response in the baseline HANK model where transfers in period 0 are directed toward households in the bottom 20% of the income distribution.

**Figure G.2:** Targeted versus Untargeted Transfers in General Equilibrium



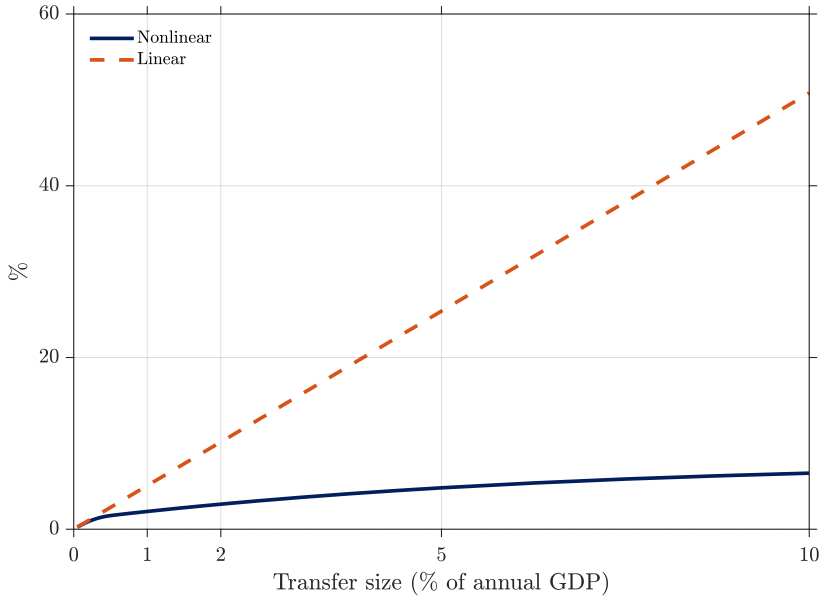

*Note:* The figure presents the ratio of the linear to the nonlinear general-equilibrium impact output response to a one-time transfer, as a function of transfer size, when the period-0 transfer is directed to all households, the bottom 50%, or the bottom 20% of the income distribution.

**Figure G.3:** Fiscal Expansions and Contractions



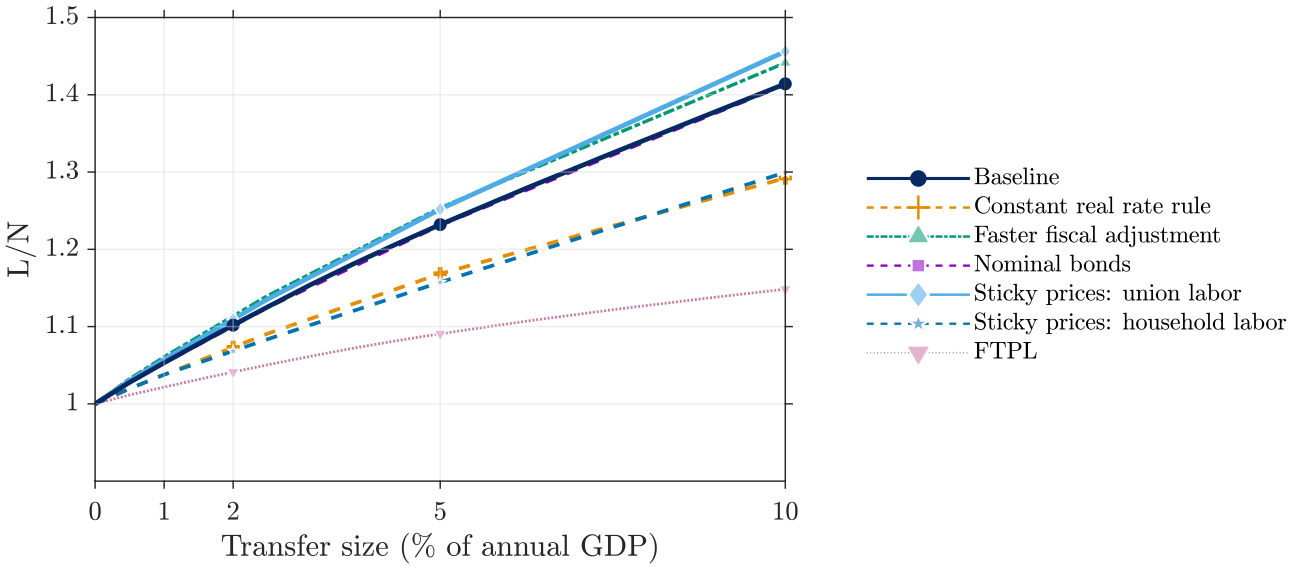

*Note:* First-quarter output response in the baseline HANK model to one-time lump-sum transfers of both signs, expansions ($\Delta > 0$) and contractions ($\Delta < 0$), in percent of annual GDP.

**Figure G.4:** Real Interest Rate Response with Flexible Wages and Household Labor Supply



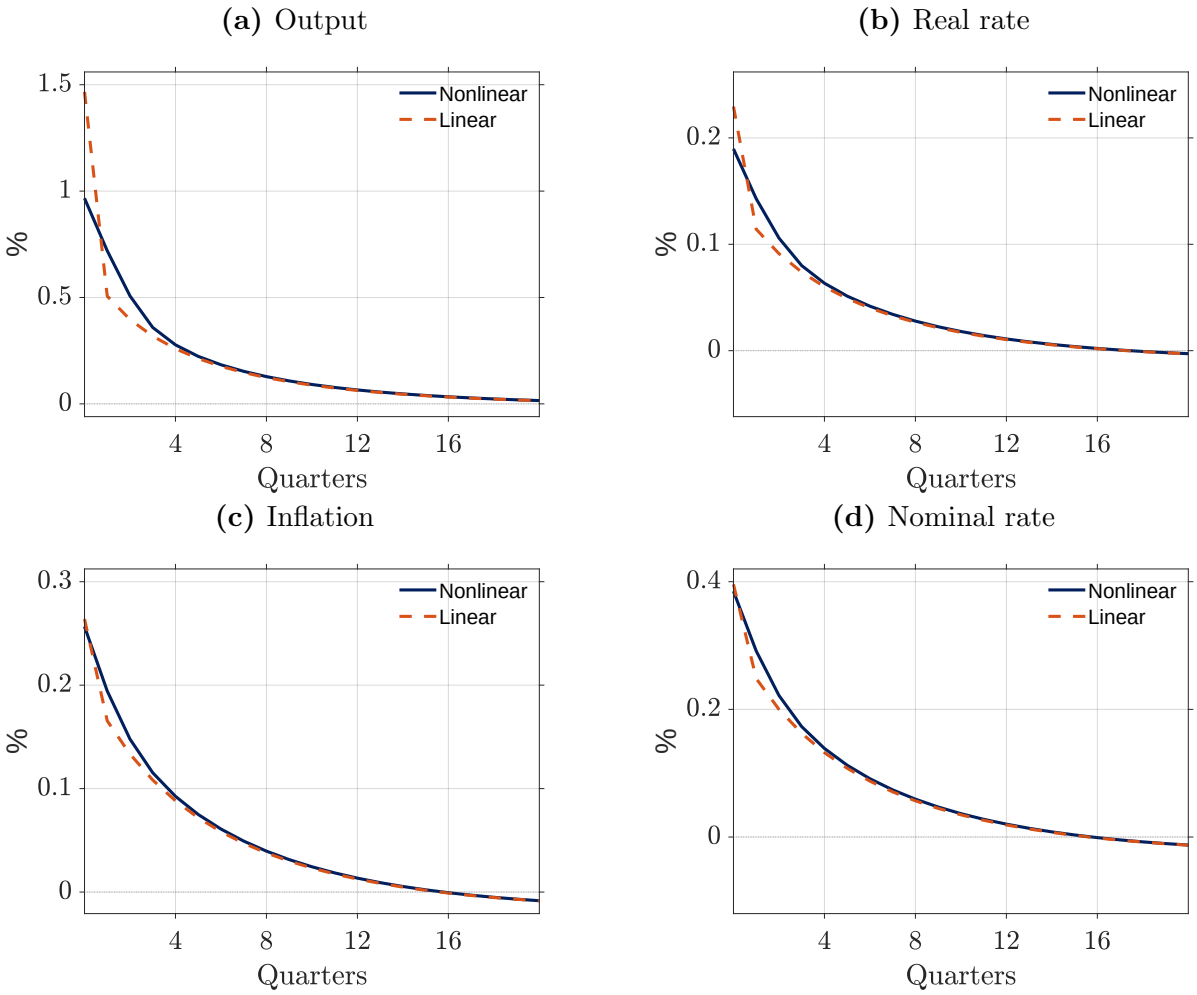

*Note:* First-quarter real interest-rate response, in the linear and nonlinear solutions, in an economy with flexible prices and wages in which households choose labor supply, as a function of the transfer size in percent of annual GDP. Rate deviations are annualized and expressed in percentage points.

**Figure G.5:** Sensitivity in HANK: Annual Responses



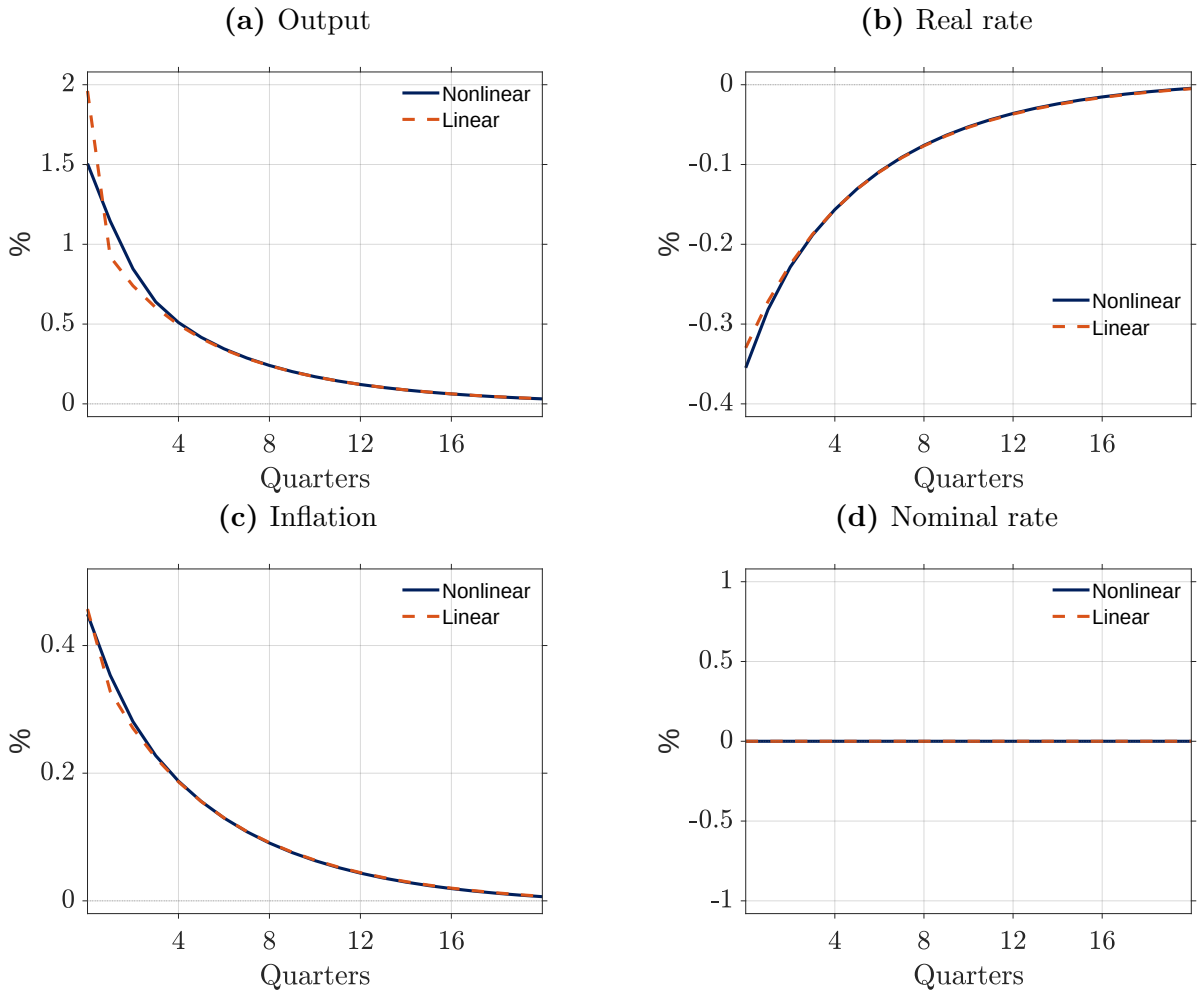

*Note:* The figure presents the ratio of the linear to the nonlinear output response over the first four quarters, for different model specifications, in the general-equilibrium model.

**Figure G.6:** Impulse Responses in the Baseline HANK Model




*Note:* Output, real rate, inflation, and nominal rate for a transfer equal to 1% of annual GDP. All variables are expressed as deviations from steady state. Output is expressed as a percent deviation from steady-state quarterly output. The real rate, inflation, and nominal rate are expressed as annualized percentage-point deviations.

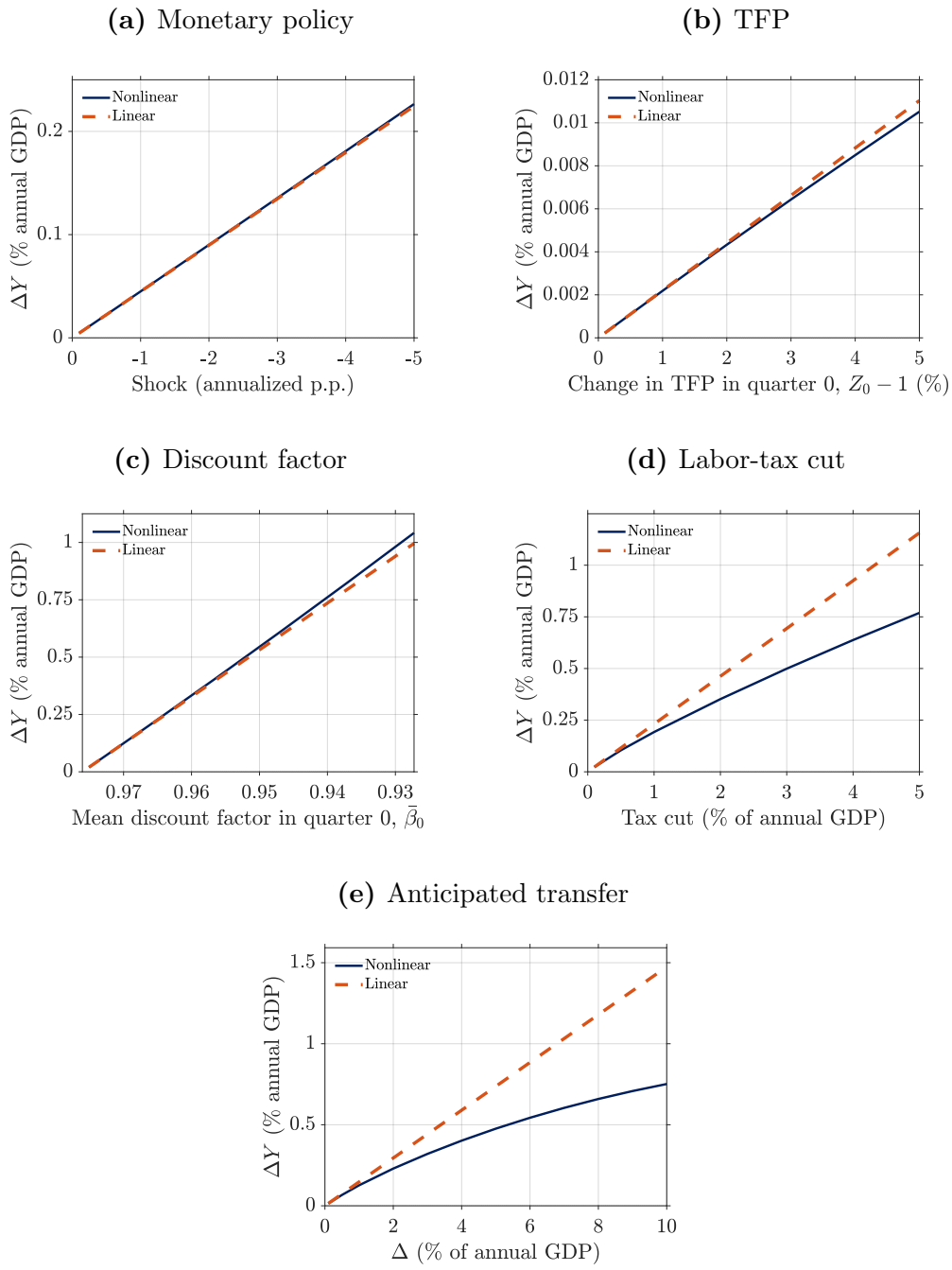
**Figure G.7:** Impulse Responses: FTPL (Unfunded Fiscal Stimulus)




*Note:* Output, real rate, inflation, and nominal rate for a transfer equal to 1% of annual GDP. In this version of the model, the government issues nominal debt and $\phi_B = \phi_\pi = 0$ (Section 3.4), so the nominal rate is pegged at $\bar{i}$. All variables are expressed as deviations from steady state. Output is expressed as a percent deviation from steady-state quarterly output. The real rate, inflation, and nominal rate are expressed as annualized percentage-point deviations.

# H Other Shocks: Additional Results

**Figure H.1:** Nonlinearities in the Output Response across Shocks




*Note:* First-quarter output response against the size of each shock, measured as follows. Each shock lasts one quarter. (a) Monetary shock, $i_t = \bar{i} + \phi_\pi(\pi_t - \bar{\pi}) - \varepsilon_i$, shown as $-\varepsilon_i$ in annualized percentage points. (b) Productivity, $Z_0 - 1$, in percent. (c) Discount-factor shock $\widetilde{\beta}_{i0} = \beta_i(1 - \varepsilon_\beta)$, shown as the mean $\bar{\beta}_0 = \bar{\beta}(1 - \varepsilon_\beta)$, with $\bar{\beta} = 0.976$. (d) Labor-tax cut $\varepsilon_\tau$, its ex-ante revenue loss in percent of annual GDP ($\tau_0 = \tau - 4\varepsilon_\tau$, $\tau = 27.4\%$). (e) Transfer $\Delta$ announced at $t = 0$ and paid at $t = 1$.