Foreign Reserve Management

Manuel Amador[1] Javier Bianchi[2] Luigi Bocola[3]
Fabrizio Perri[4]

[1]Minneapolis Fed, University of Minnesota

[2]Minneapolis Fed

[3]Stanford University

[4]Minneapolis Fed

December 11, 2019

Foreign reserves and exchange rates

- A Central Bank (CB) sets an exchange rate and interest policy that makes domestic assets attractive.
  - $\Rightarrow$ Capital flows in.
- The CB has a problem if:
  - Domestic interest rates cannot fall to restore equilibrium.
- One option:
  - Accumulate foreign assets and reverse the inflow.
- And this can work, in a world with limited arbitrage.
  - Focus of our previous work (ABBP 19)

Foreign reserve management

- CB needs to decide how to invest the accumulate assets.
- Standard answer (Backus Kehoe, 89)
  - with perfect international arbitrage: it does not matter
- Our take: with imperfect international arbitrage, it does.

Foreign reserve management

With imperfect international arbitrage

- CB buys/sells foreign reserves and affects prices
- But doing so involves costs

  ... arbitrage losses to foreigners

Results

- Portfolio of foreign reserves determines losses.
- Optimal portfolio depends on openness to capital flows.

Framework

- Two-period model (similar to *Backus-Kehoe, 89*)
  - Small open economy (government + households)
  - International Financial Market
  - International Arbitrageurs
- Time $t \in \{1, 2\}$
- Uncertainty realized at $t = 2$, $s \in S \equiv \{s_1, ..., s_N\}$
- Probability $\pi(s) \in (0, 1]$
- One good – no production
- Law of one price – foreign price normalized to 1

Asset markets: complete but segmented

International financial market

- Full set of Arrow-Debreu (real) securities:
  - Security s: 1 unit of consumption good only in state s
  - Price $q(s)$ in terms of goods at $t = 1$

Domestic financial market

- Full set of Arrow-Debreu (nominal) securities:
  - Security s: 1 unit of domestic currency only in state s
  - Price $p(s)$ in terms of domestic currency at $t = 1$

Asset markets: complete but segmented

Foreign intermediaries

- Trade securities with SOE & IFM .. but have limited capital

Model: Small open economy

- Endowment: $(y_1, \{y_2(s)\})$, transfers: $\{T_2(s)\}$)

$$\max_{c_1,\{c_2,a,f\}} \left\{ u(c_1) + \beta \sum_{s\in S} \pi(s) u(c_2(s)) \right\}$$

$$y_1 = c_1 + \sum_{s\in S} \left[q(s)f(s) + p(s)\frac{a(s)}{e_1} \right]$$

$$y_2(s) + T_2(s) + f(s) + \frac{a(s)}{e_2(s)} = c_2(s) \quad \forall s \in S$$

$$f(s) \geq 0 \quad \forall s \in S$$

e_1, $e_2(s)$: exchange rates at $t = 1$ and $t = 2$

$f(s), a(s)$: holdings of foreign and domestic security s

m: money holdings, $\bar{x}$: satiation point of h

Model: Foreign intermediaries

- Endowed with capital $\overline{w}$

$$\max_{d_1, \{d_2, a^\star, f^\star\}} d_1^\star + \sum_{s \in S} \pi(s) \Lambda(s) d_2^\star(s)$$

subject to:

$$\overline{w} = d_1^\star + \sum_{s \in S} p(s) \frac{a^\star(s)}{e_1} + \sum_{s \in S} q(s) f^\star(s)$$

$$d_2^\star(s) = \frac{a^\star(s)}{e_2(s)} + f^\star(s)$$

$$a^\star(s) \geq 0, f^\star(s) \geq 0$$

Consider $\Lambda(s) = \frac{q(s)}{\pi(s)}$ (same SDF as IFM)

Model: Central Bank

- Amounts invested at home and abroad, $A(s)$ and $F(s)$; and transfers $\{T_2(s)\}$.
- Budget constraint:

$$\sum_s p(s)\frac{A(s)}{e_1} + \sum_s q(s)F(s) = 0$$

$$T_2(s) = \frac{A(s)}{e_2(s)} + F(s) \quad \forall s \in S$$

Model: Central Bank

Monetary policy objective: $\{i, e_1, e_2(s)\}$; where

$$1 + i = \frac{1}{\sum_{s \in S} p(s)} \tag{NIRC}$$

- Nominal interest rate link to the prices individual securities
- Note that $i = 0$ (ZLB)

Model: Central Bank

Monetary policy objective: $\{i, e_1, e_2(s)\}$; where

$$1 + i = \frac{1}{\sum_{s \in S} p(s)} \tag{NIRC}$$

- Nominal interest rate link to the prices individual securities
- Note that $i = 0$ (ZLB)

Can the CB achieve the objective? Are there costs? Are the costs affected by the CB balance sheet?

Equilibrium definition

Equilibrium given policy objective

HH's consumption, $(c_1, \{c_2(s)\})$, and asset positions, $\{a(s), f(s)\}$; foreign intermediaries dividend policy, $(d_1^\star, \{d_2^\star(s)\})$, and asset positions $(\{a^\star(s), f^\star(s)\})$; government transfers $\{T_2(s)\}$, asset $\{A(s), F(s)\}$; such that

1. HH and intermediaries maximize taking prices as given,
2. the government budget constraint holds, and
3. the domestic financial markets clear:

$$a(s) + a^\star(s) + A(s) = 0 \quad \forall s \in S$$

Government objective

- Government desires to implement $(i, e_1, \{e_2(s)\})$ – this is given.

- Chooses policy $\{A(s), F(s)\}$ and $\{T_2(s)\}$ as to implement the equilibrium that maximizes household welfare.
  - *optimal equilibrium / optimal equilibrium allocation.*

Government objective

- Government desires to implement $(i, e_1, \{e_2(s)\})$ – this is given.

- Chooses policy $\{A(s), F(s)\}$ and $\{T_2(s)\}$ as to implement the equilibrium that maximizes household welfare.
  - *optimal equilibrium / optimal equilibrium allocation.*

- For the rest of the talk: no income in the second period:

$$y_1(s) = 0 \text{ for all } s$$

Preliminary: Trade deficit

- Trade deficits and net foreign assets:

$$\underbrace{c_1 - y_1}_{\text{trade deficit}} = \underbrace{\frac{\sum_s p(s) a^\star(s)}{e_1}}_{\text{foreign liabilities}} - \underbrace{\sum_s q(s) \left[f(s) + F(s) \right]}_{\text{foreign assets}}$$

$$c_2(s) - y_2(s) = f(s) + F(s) - \frac{a^\star(s)}{e_2(s)} \quad \forall s \in S$$

Preliminary: First best (real) allocation

First best (real) allocation, $(c_1^{fb}, \{c_2^{fb}(s)\})$:

$$\max_{(c_1, \{c_2(s)\})} \left\{ u(c_1) + \beta \sum_{s \in S} \pi(s) u(c_2(s)) \right\}$$

$$\text{s.t.: } y_1 - c_1 + \sum_{s \in S} q(s) \left(y_2(s) - c_2(s) \right) = 0$$

The capital of the intermediaries is irrelevant.

There is always a monetary policy objective s.t. FB is eqm.

Characterizing monetary equilibria: Arbitrage returns

Characterizing monetary equilibria: Arbitrage returns

- **Arbitrage return** for security s:

$$\kappa(s) \equiv \frac{\frac{e_1}{e_2 p(s)}}{\frac{1}{q(s)}} - 1 \tag{1}$$

Characterizing monetary equilibria: Arbitrage returns

- **Arbitrage return** for security s:

$$\kappa(s) \equiv \frac{\frac{e_1}{e_2 p(s)}}{\frac{1}{q(s)}} - 1 \tag{1}$$

- Using the HH's FOC

$$u'(c_0) = \beta\pi(s)\frac{e_1}{e_2(s)p(s)}u'(c_2(s))$$

Characterizing monetary equilibria: Arbitrage returns

- **Arbitrage return** for security s:

$$\kappa(s) \equiv \frac{\frac{e_1}{e_2 p(s)}}{\frac{1}{q(s)}} - 1 \tag{1}$$

- Using the HH's FOC

$$u'(c_0) = \beta\pi(s)\frac{e_1}{e_2(s)p(s)}u'(c_2(s))$$

we get

$$\kappa(s) = \frac{q(s)u'(c_1)}{\beta\pi(s)u'(c_2(s))} - 1$$

Characterizing monetary equilibria: Arbitrage returns

- $\kappa(s) = 0$: security s: same real return in all markets
- $\kappa(s) > 0$: security s: higher return at home than abroad
- $\kappa(s) < 0$: security s: higher return at abroad than home

(One direction arbitrages). In any equilibrium,

$$0 \leq \kappa(s) \quad \forall s \in S$$

and $f(s) = 0$ if strict.

$\Rightarrow$ return on domestic securities weakly higher than foreign.

Resource losses

Using the HH and CB budget constraints, plus market clearing,

$$y_1 - c_1 + \sum_{s \in S} q(s)(y_2(s) - c_2(s))$$

L: Potential "arbitrage losses"

Resource losses

Using the HH and CB budget constraints, plus market clearing,

$$y_1 - c_1 + \sum_{s \in S} q(s)(y_2(s) - c_2(s)) - \underbrace{\left(\sum_{s \in S} \kappa(s) \frac{p(s) a^\star(s)}{e_1} \right)}_{L} = 0$$

L: Potential "arbitrage losses"

Intermediaries profits

In any equilibrium, $\{a^\star(s)\}$ solves

$$L = \max_{\{a^\star(s)\}} \left\{ \sum_{s \in S} \kappa(s) \frac{p(s)a^\star(s)}{e_1} \right\} \text{ subject to}$$

$$\sum_{s \in S} \frac{p(s)a^\star(s)}{e_1} \leq \bar{w}$$

$$\frac{a^\star(s)}{e_2(s)} \geq 0 \text{ for all } s \in S$$

The present value of intermediaries dividends is $\Pi = \overline{w} + L$.

Note: intermediaries invest in the highest κ security

Intermediaries profits

$$L = \overline{\kappa} + \overline{w}$$

where

$$\overline{\kappa} = \max_{s \in S} \kappa(s)$$

Arbitraging the bonds

Consider (risk-adjusted) return differential on the bonds:

$$\Delta(i) = \mathbb{E}\left[\Lambda(s)\left(\frac{e_1}{e_2(s)}(1+i) - (1+i^\star)\right)\right]$$

- $\kappa(s) \geq 0$ for all $s \in S$ implies $\Delta(i) \geq 0$.
- $\Delta(i) > 0$ then $\kappa(s) \geq \Delta(i)$ for some $s \in S$
- $L \geq \Delta(i)\overline{w}$

Implementability

$(c_1, \{c_2(s)\})$ is part of an equilibrium if and only if

$$\kappa(s) \geq 0; \text{ for all } s \in S$$
$$\sum_{s \in S} \frac{q(s)}{1+\kappa(s)} \frac{e_1}{e_2(s)} = (1+i)^{-1}$$
$$y_1 - c_1 + \sum_{s \in S} q(s) \left[y_2(s) - c_2(s) \right] = \overline{\kappa} \times \overline{w}$$

Equal gaps allocations

- Consider allocations such that all securities are distorted equally: $\kappa(s) = \overline{\kappa}$ for all $s \in S$
- Arbitrage gap is the same for all securities (and thus for any portfolio):

$$\overline{\kappa} = \Delta(i)$$

- Associated $(c_1, \{c_2(s)\})$ is

$$u'(c_1)q(s) = \beta(1+\bar{\kappa})\pi(s)u'(c_2(s)) \ \forall s$$
$$y_1 - c_1 + \sum_{s \in S} q(s)(y_2(s) - c_2(s)) = \bar{\kappa} \times \bar{w}$$

- First best is a special case.

Equal gaps always a choice

For a given monetary policy objective, either (i) the set of implementable allocations is empty or (ii) the equal gap allocation with $\kappa(s) = \Delta(i)$ for all s is implementable.

Equal gaps always a choice

For a given monetary policy objective, either (i) the set of implementable allocations is empty or (ii) the equal gap allocation with $\kappa(s) = \Delta(i)$ for all s is implementable.

- Only objectives with $\Delta(i) \geq 0$ are implementable
- $\Delta(i) = 0 \Rightarrow$ first best allocation is implementable
- $y_1 + \sum_s y_2(s) > \Delta(i)\overline{w}$ and Inada $\Rightarrow$ the set of implementable allocations is non-empty.
- Equal gaps minimizes the losses, L, among all implementable allocations.

$\Delta(i) = 0$

- The first best allocation is the only equilibrium allocation
- $\overline{w}$ is sufficiently large:
  - $\Rightarrow$ $F(s) = 0$ for all $s \in S$ optimal
  - A neighborhood of $F(s) = 0$ is also optimal
- Backus-Kehoe benchmark: perfect capital mobility and irrelevance of CB's balance sheet.

$\Delta(i) > 0$: A relaxed problem

Relax NIRC, and let $\overline{\kappa}_0$ a gap upper bound:

$$\hat{V}(\overline{\kappa}_0) = \max_{(c_1,\{c_2(s)\}} \left\{ u(c_1) + \beta \sum_{s \in S} \pi(s) u(c_2(s)) \right\} \text{ s. t.}$$

$$y_1 - c_1 + \sum_{s \in S} q(s)(y_2(s) - c_2(s)) = \overline{\kappa}_0 \bar{w}$$

$$\sum_{s \in S} \frac{\beta \pi(s) e_1 u'(c_2(s))}{e_2(s) u'(c_1)} \leq \frac{1}{1+i}$$

$$1 \leq \frac{q(s) u'(c_1)}{\beta \pi(s) u'(c_2(s))} \leq 1 + \overline{\kappa}_0 \text{ for all } s \in S$$

Optimal allocation:

$$\overline{\kappa} = \arg \max_{\overline{\kappa}_0 \geq \Delta(i)} \hat{V}(\kappa_0)$$

A potential trade-off

- Higher $\overline{\kappa}_0$ increases losses
- Higher $\overline{\kappa}_0$ relaxes the NIRC constraint

Goals not necessarily aligned $\Rightarrow$ trade-off depends on $\overline{w}$

Main results

$$\sum_{s \in S} \frac{\beta \pi(s) e_1 u'(c_2(s))}{e_2(s) u'(c_1)} \leq \frac{1}{1+i}$$

Main results

$$\sum_{s \in S} \frac{\beta \pi(s) e_1 u'(c_2(s))}{e_2(s) u'(c_1)} \leq \frac{1}{1+i}$$

Result: Suppose that $\pi(s)/q(s)$ is constant and u is DARA. Then $\kappa(s)$ is (weakly) decreasing in $e_2(s)$.

- When $e_2(s)$ is low (appreciation) $\Rightarrow$ increase $c_2(s)$ ($\kappa(s)$).

Main results

$$\sum_{s \in S} \frac{\beta \pi(s) e_1 u'(c_2(s))}{e_2(s) u'(c_1)} \leq \frac{1}{1+i}$$

Result: Suppose that $\pi(s)/q(s)$ is constant and u is DARA. Then $\kappa(s)$ is (weakly) decreasing in $e_2(s)$.

- When $e_2(s)$ is low (appreciation) $\Rightarrow$ increase $c_2(s)$ ($\kappa(s)$).
- Dispersion in $\kappa(s)$ increases losses proportional to $\overline{w}$

Main results

$$\sum_{s \in S} \frac{\beta \pi(s) e_1 u'(c_2(s))}{e_2(s) u'(c_1)} \leq \frac{1}{1+i}$$

Result: Suppose that $\pi(s)/q(s)$ is constant and u is DARA. Then $\kappa(s)$ is (weakly) decreasing in $e_2(s)$.

- When $e_2(s)$ is low (appreciation) $\Rightarrow$ increase $c_2(s)$ ($\kappa(s)$).
- Dispersion in $\kappa(s)$ increases losses proportional to $\overline{w}$

Result: When $\overline{w}$ is large $\Rightarrow$ equal gaps is optimal.

Reserve Management

Suppose that $y_2(s)$ is constant.

- For all $\kappa(s_1) < \overline{\kappa}$:

$$c_2(s_1) = y_2(s_1) + F(s_1)$$

(there are no private flows, CB has to do the trades)

- Let $\overline{S} \subset S$ s.t. $\kappa(s) = \overline{\kappa}$:

$$\sum_{s \in \overline{S}} q(s)\left(c_2(s_1) - y_2(s_1)\right) + (1 + \overline{\kappa})\overline{w} = \sum_{s \in S} q(s)F(s_1)$$

Reserve Management

Suppose that $y_2(s)$ is constant.

- If equal gaps is optimal, then it suffices to invest everything in a risk-free foreign security.

Conclusion

- Optimal porfolio hinges on *degree of openness* ($\overline{w}$)
  - Relatively closed economies:
    - Invest in foreign assets that pay when the currency appreciates
  - Relatively open economies:
    - Invest in safe foreign assets

- CB can affect all domestic security prices by intervening

  ... and not just the nominal risk-free bond.
- More instruments is better!
- But the more open the economy is – the more costly it is to use them.