International Reserve Management under Rollover Crises

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Motivation

To reduce the vulnerability to a debt crisis:

- Should the government reduce the debt or increase reserves?

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- Should the government reduce the debt or increase reserves?

Answer unclear:

- Reserves provide liquidity, but their return is lower than borrowing costs

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    - Sunspot shocks, deterministic income
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- If heavily indebted, optimal to initially reduce debt and keep zero reserves
- Once debt is reduced sufficiently, optimal to increase debt and accumulate reserves
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- Hernandez (2019): numerical simulations w/ fundamental and sunspot shocks

Cole-Kehoe (2001); Corsetti-Dedola (2016); Aguiar-Amador (2020); Bianchi-Mondragon (2022); Bianchi and Sosa-Padilla (2023); Corsetti-Maeng (2023ab)

Model

Environment

- Discrete time, infinite horizon. Constant endowment: $y_t = y$

- Government trades two assets ...
  - short-term risk-free reserves, a
  - long-term defaultable debt, b

    a bond issued in t promises to pay

$$\kappa \left[1, (1-\delta), (1-\delta)^2, ...\right]$$

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- Markov equilibrium w/ Cole-Kehoe (2000) timing:
  - Borrowing at the beginning of the period
  - Settlement (repay/default) at the end

Preferences and resource constraint

- Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \phi d_t]$$

  where $d_t = 0$ (1) denotes repayment (default)

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- If the government repays:

$$c_t = \underbrace{y + a_t - \kappa b_t}_{\text{resources avail.}} - \underbrace{\frac{a_{t+1}}{1+r}}_{\text{reserve purchases}} + \underbrace{q_t \left[b_{t+1} - (1-\delta) b_t\right]}_{\text{debt issuance}}$$

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- If the government defaults:

$$c_t = y + a_t - \frac{a_{t+1}}{1+r} \qquad \text{Gov. saves on bond payments}$$

and faces permanent exclusion and utility loss ϕ

Recursive Government Problem

- State is $s \equiv (a, b, \zeta)$

  ζ denotes an iid sunspot that coordinates the lenders

- The government chooses to repay or default

$$V(a, b, \zeta) = \max\{V_R(a, b, \zeta), V_D(a)\}$$

If indifferent, assume repay

Value of Default

$$V_D(a) = \max_{a' \geq 0} \left\{ u(c) - \phi + \beta V_D(a') \right\}$$

subject to

$$c \leq y + a - \frac{a'}{1+r}$$

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- Given $\beta(1+r) = 1$, we have constant consumption

$$V_D(a) = \frac{u(y + (1-\beta)a) - \phi}{1-\beta}$$

Value of Repayment

Two cases, depending on whether the investors want to rollover the debt

If investors **want** to rollover:

$$V_R^+(a,b) = \max_{a' \geq 0, b'} \left\{ u(c) + \beta \mathbb{E} V(a', b', s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + q(a', b') \left(b' - (1-\delta) b \right)$$

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Bond price depends on the portfolio and reflects default prob:

$$q(a', b') = \frac{1}{1+r} \mathbb{E} \left[\left(1 - d(s') \right) \left(\kappa + (1-\delta) q(a'', b'', s') \right) \right]$$

Value of Repayment

Two cases, depending on whether the investors want to rollover the debt

If investors **don't want** to rollover:

$$V_R^-(a,b) = \max_{a' \geq 0} \left\{ u(c) + \beta \mathbb{E} V(a', (1-\delta)b, s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + \quad q(a', b') \left(b' - (1-\delta)b \right)$$

0

To pay debt, need to use reserves or cut consumption

Multiplicity of Equilibria

- Coordination failure may lead to self-fulfilling crises (Cole-Kehoe)

- If lenders expect...

  ... repayment, then they rollover, and the govt repays

  ... default, then they don't rollover, and the govt defaults

Characterization

Default thresholds

For a given level of reserves, two thresholds

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$$V^D(a) > V_R^+(a, b)$$

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$V_R^-(a, b) \geq V^D(a)$ $\qquad$ $V^D(a) > V_R^+(a, b)$

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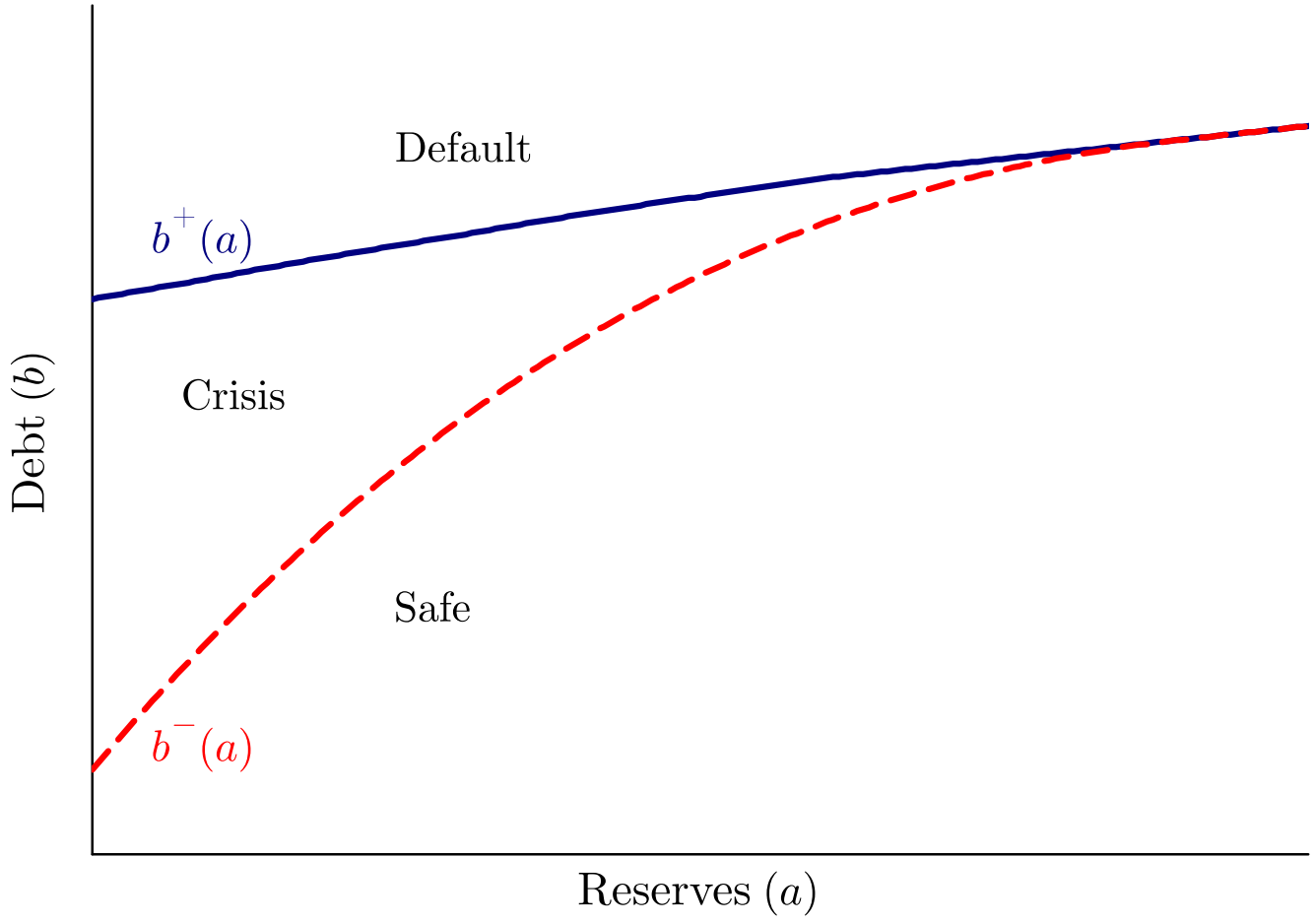
$$V_R^-(a,b) < V^D(a) \leq V_R^+(a,b)$$

Default thresholds

For a given level of reserves, two thresholds

Sunspot: government faces a run w/ prob π when initial portfolio (a, b) is in the crisis zone

The Three Zones



Proposition 2 establishes: $\frac{\partial b^-(a)}{\partial a} \geq \frac{\partial b^+(a)}{\partial a} > 0$

▸ More

Escaping the Crisis Zone

How to Exit the Crisis Zone?

Remaining in the crisis zone is risky:

- in case of a run, the gov't defaults

But exiting is also costly:

- requires cutting consumption and improving NFA

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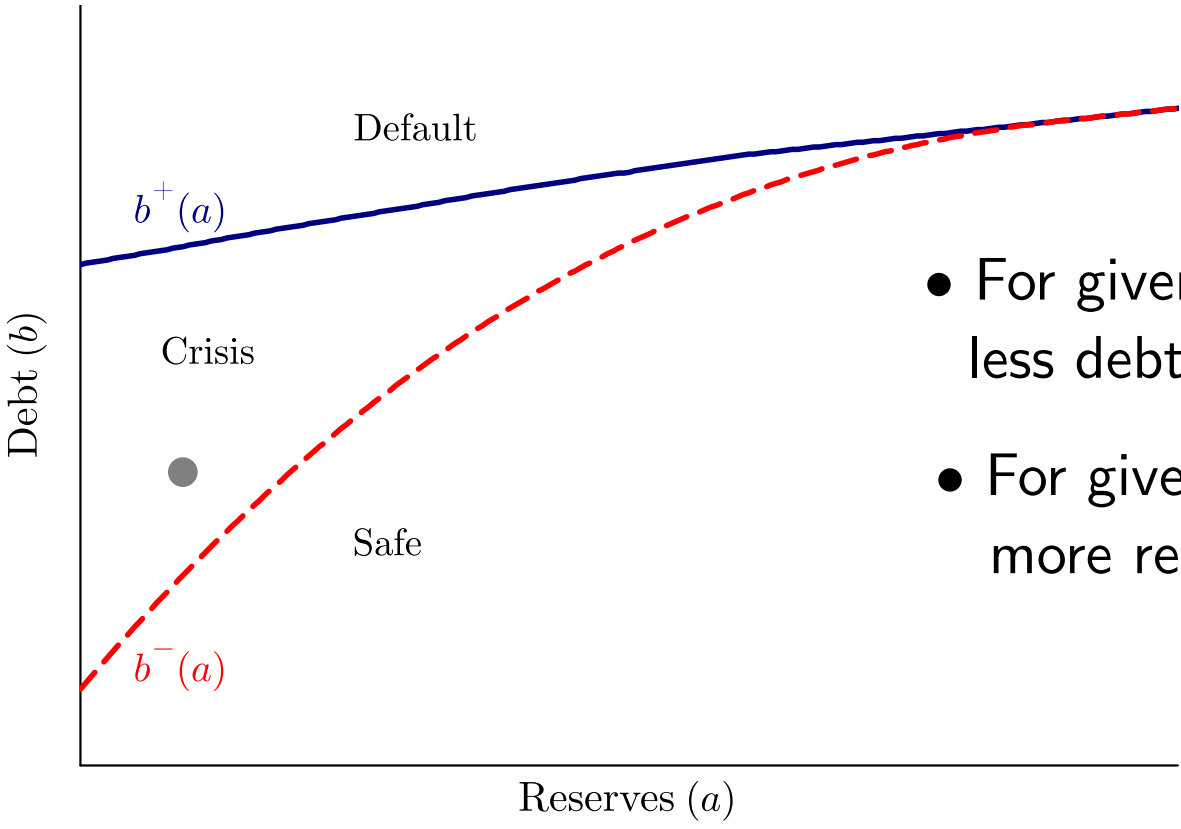
What's the best exit strategy for a country that is in the crisis zone (but didn't face a run today) ?

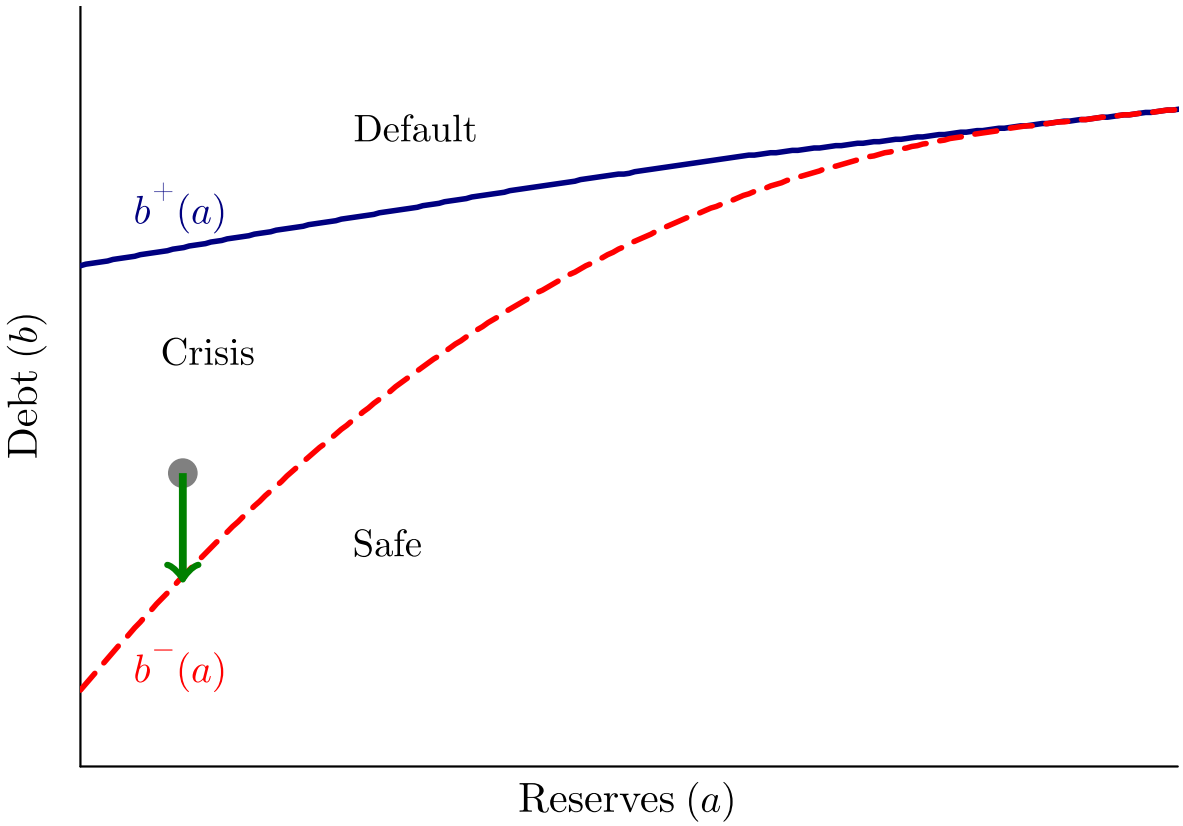
- Accumulate reserves ($a \uparrow$) or reduce debt ($b \downarrow$)?

Possible Exit Paths



- For given reserves level:
  less debt lowers vulnerability
- For given debt level:
  more reserves lower vulnerability

Possible Exit Paths



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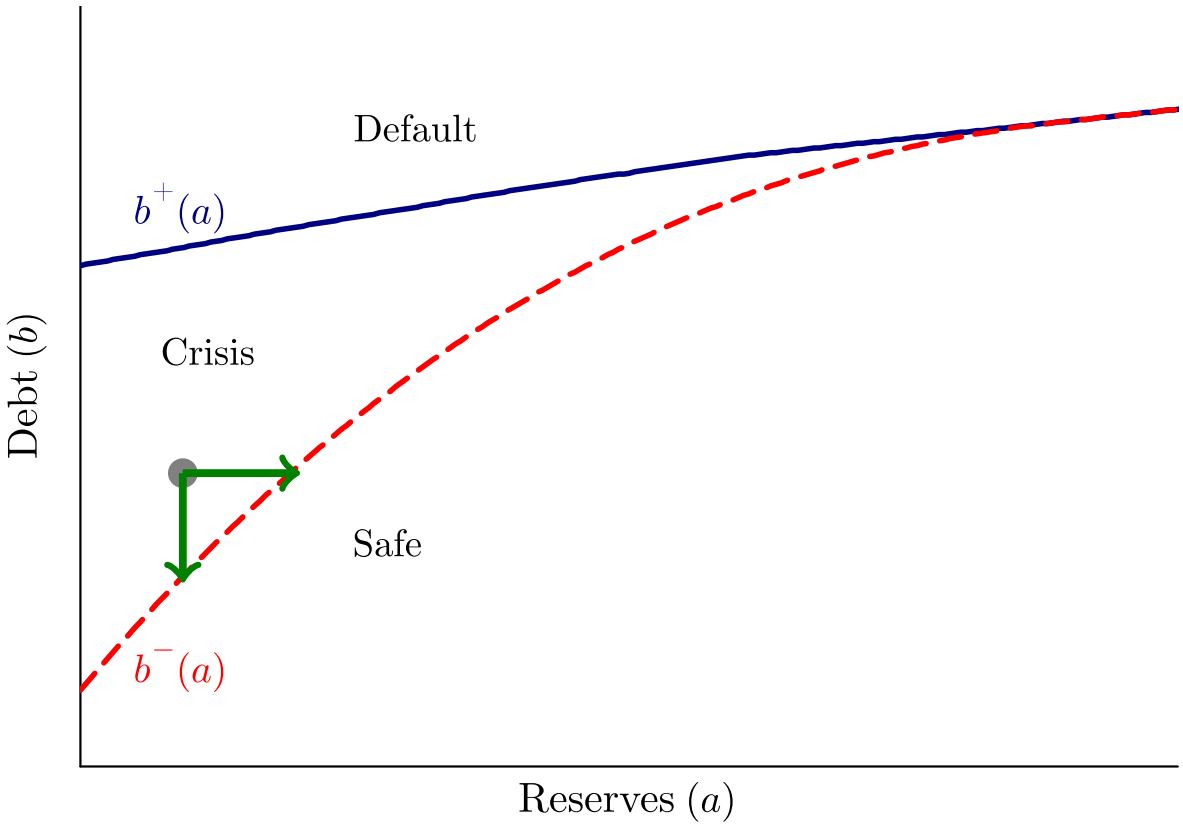

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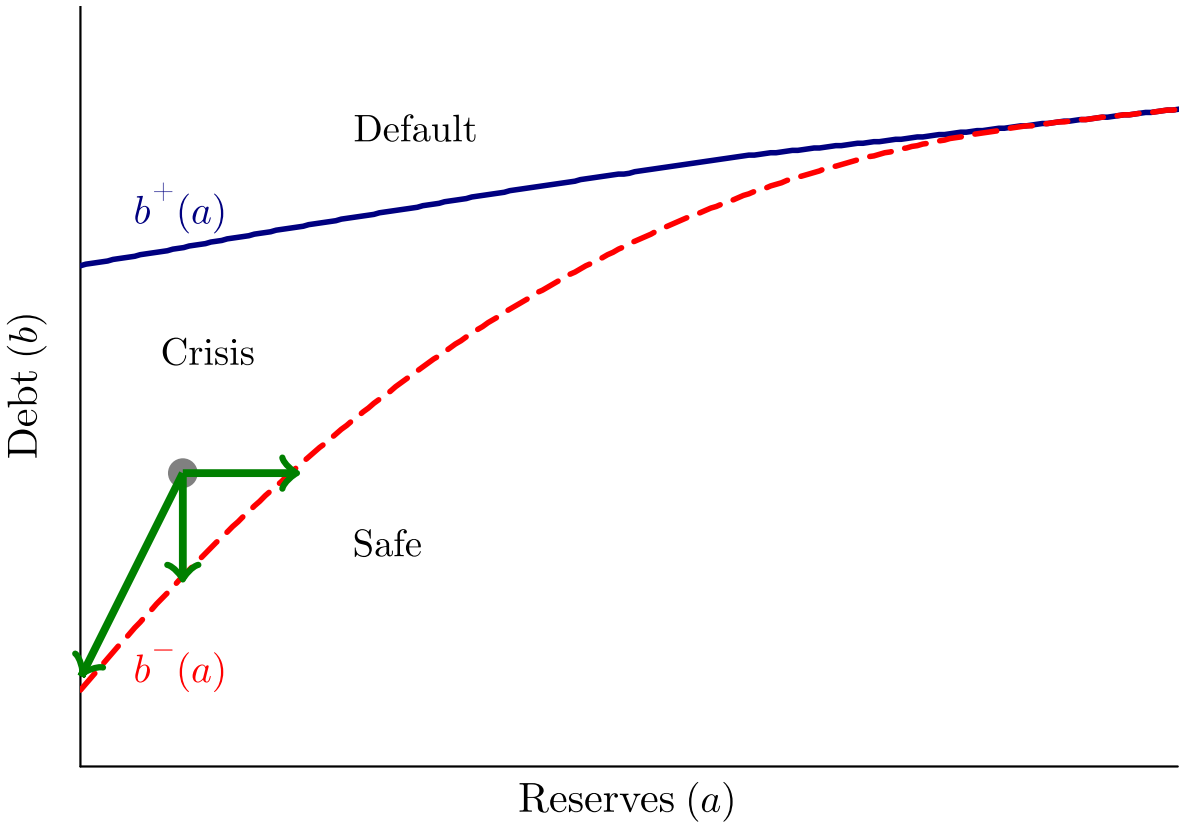

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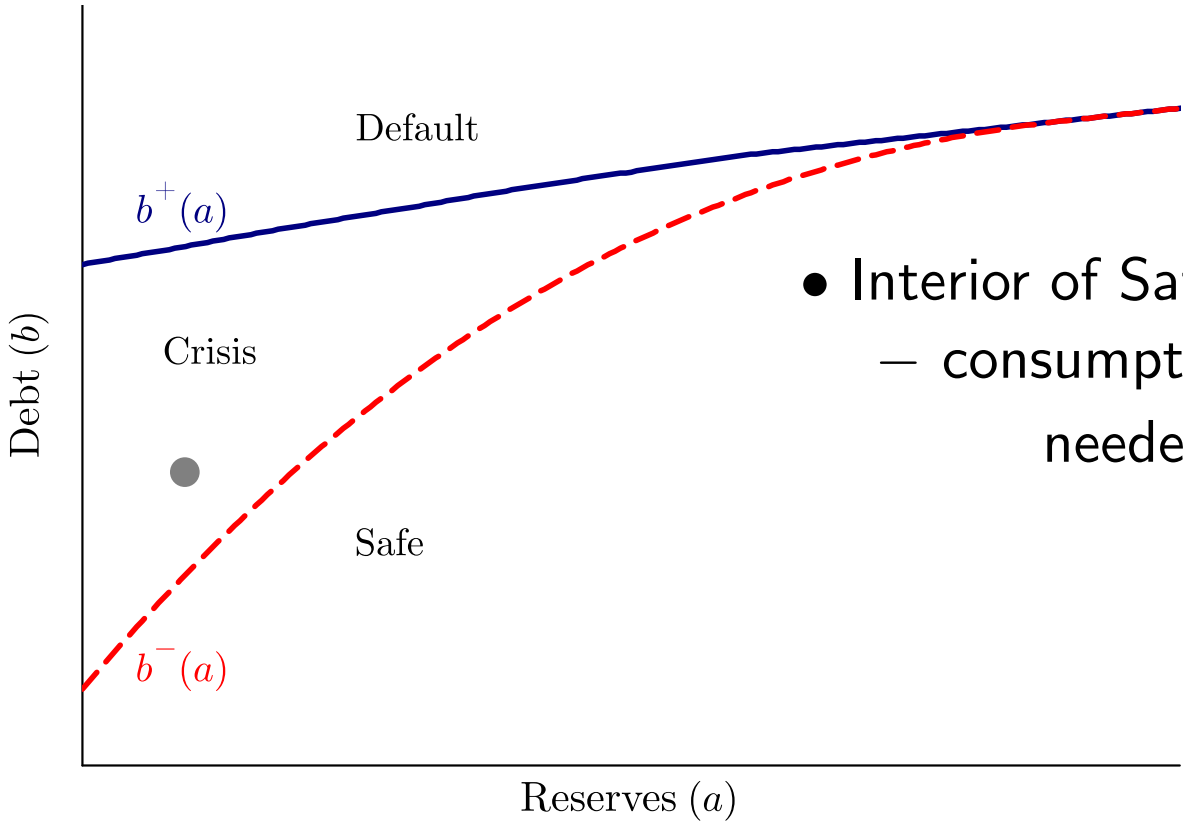

- Interior of Safe zone isn't optimal
  — consumption cut larger than needed to be safe

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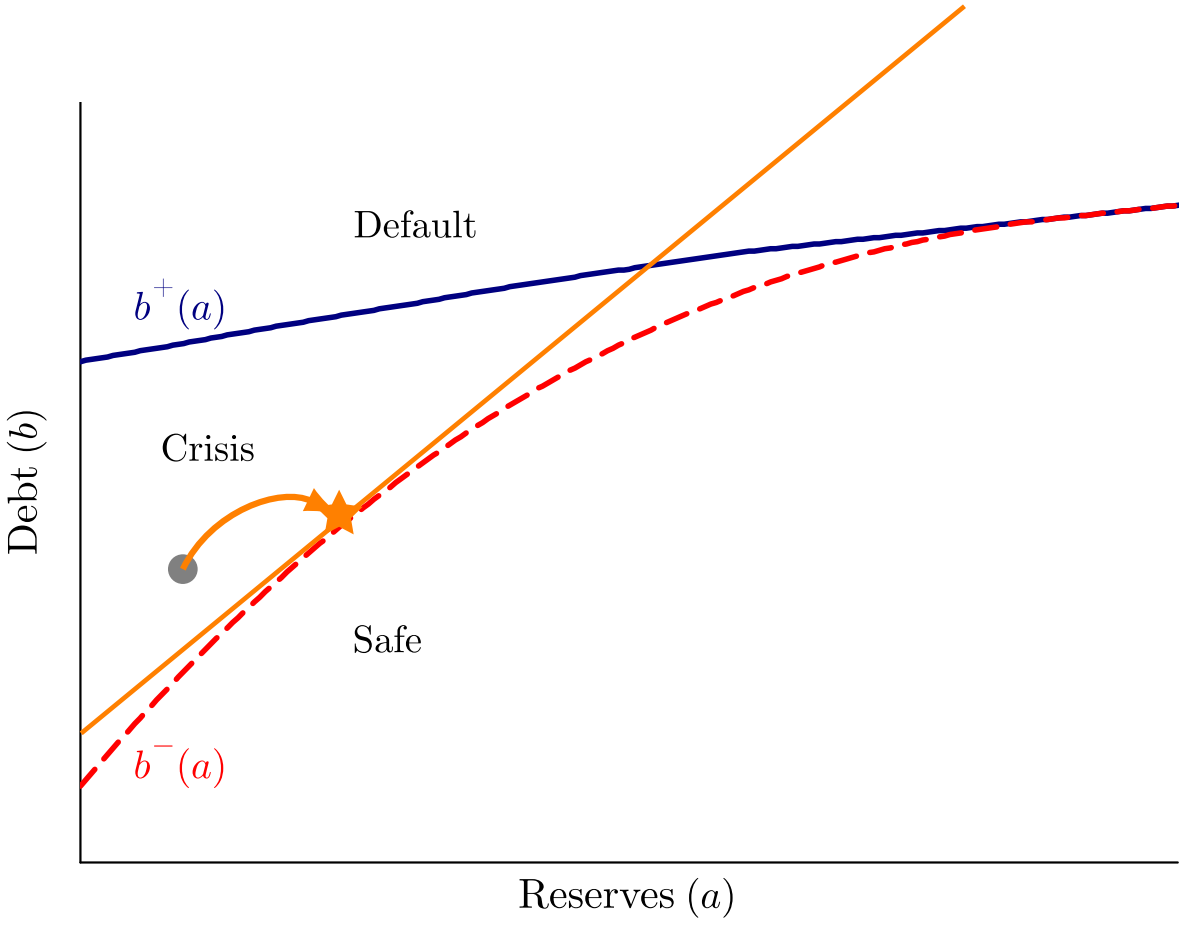

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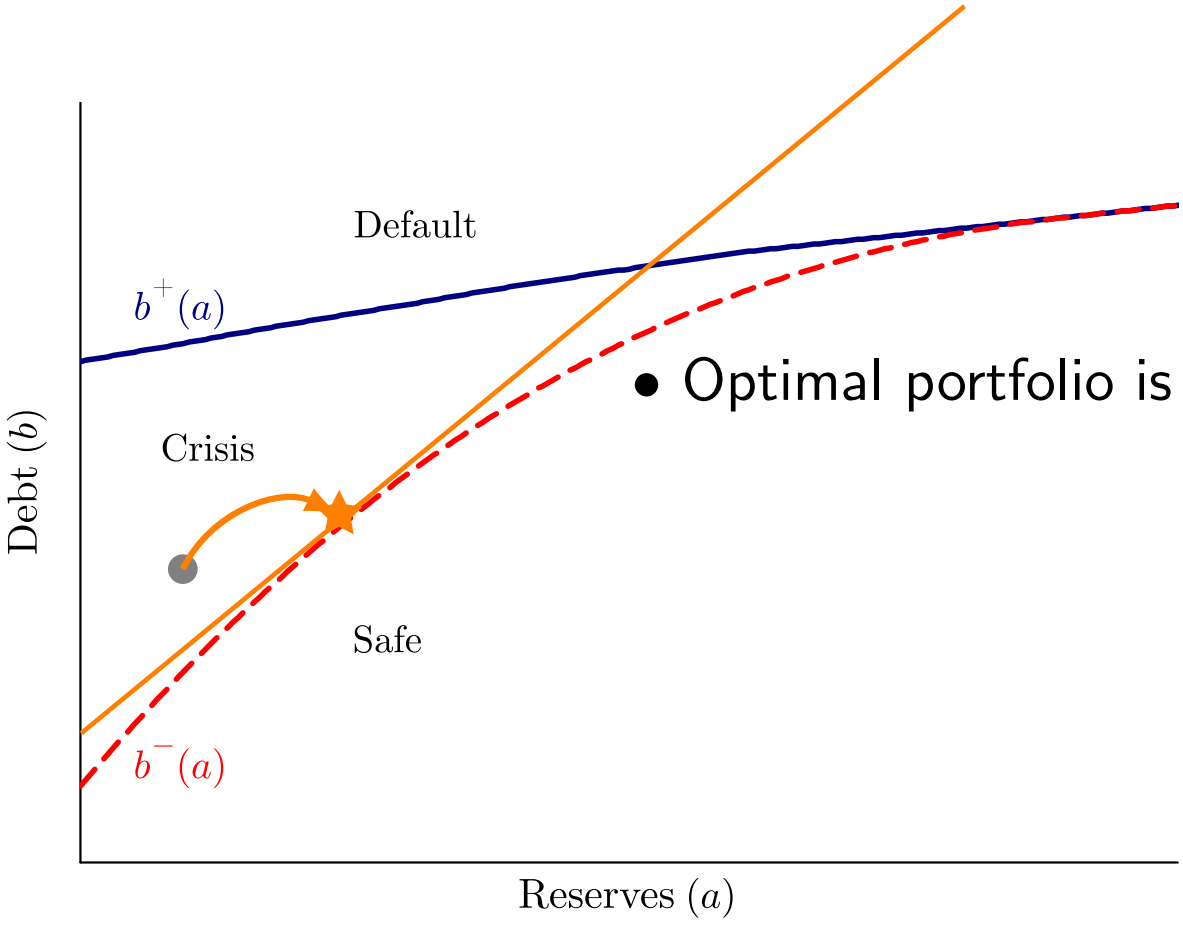
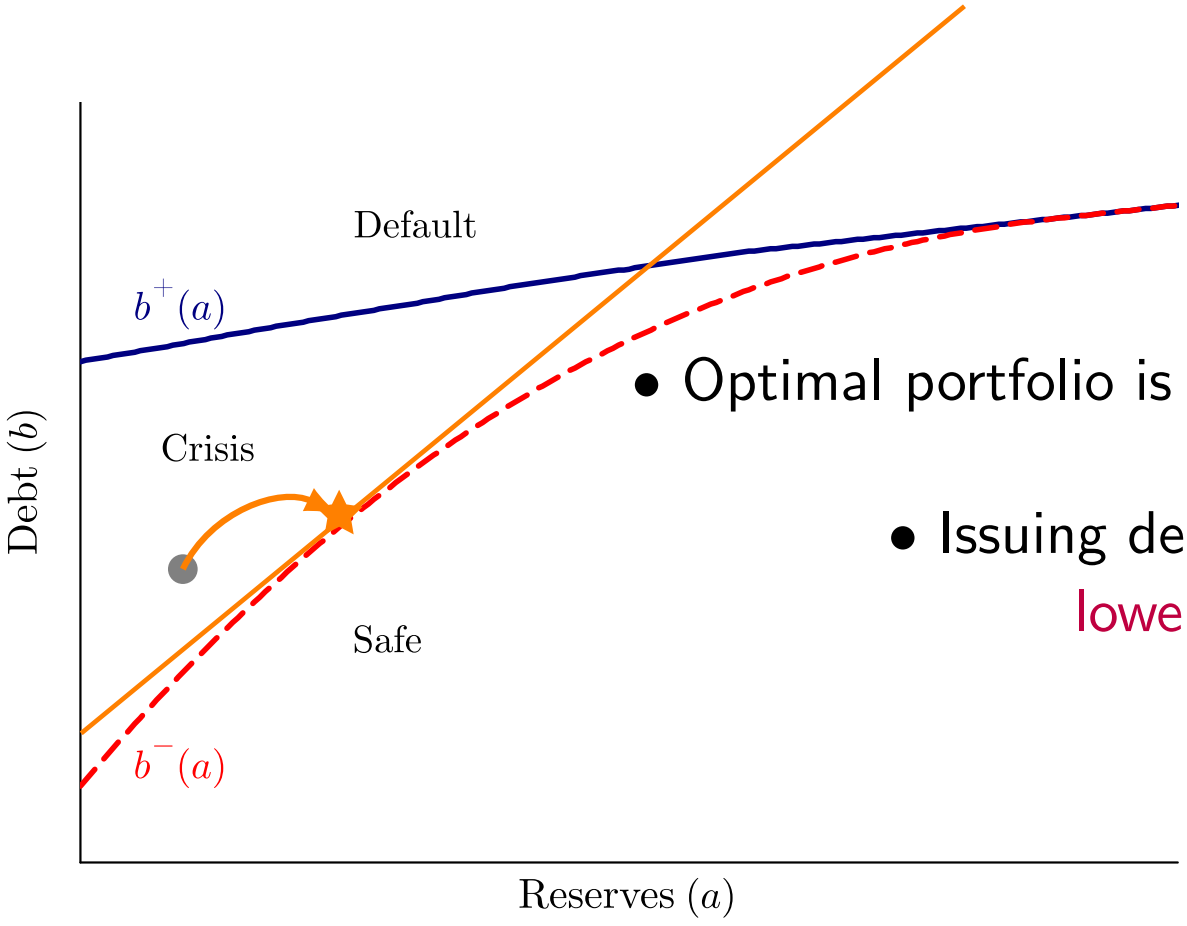


- Optimal portfolio is tangent to threshold b^-

Possible Exit Paths



- Optimal portfolio is tangent to threshold b^-
- Issuing debt to buy reserves
  lowers spreads!

Why do reserves help exit the crisis zone?

Getting to the safe zone requires $V_R^-(a, b) \geq V_D(a)$

- Accumulating reserves helps sustain higher net debt

  ... even though reserves increase default value $V_D(a)$.

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- Only a fraction κ of debt is due every period
- Reserves are liquid and can be used in a run:

$$c = y + \underbrace{a - \kappa b}_{\text{more resources}} - \frac{a'}{1 + r}$$

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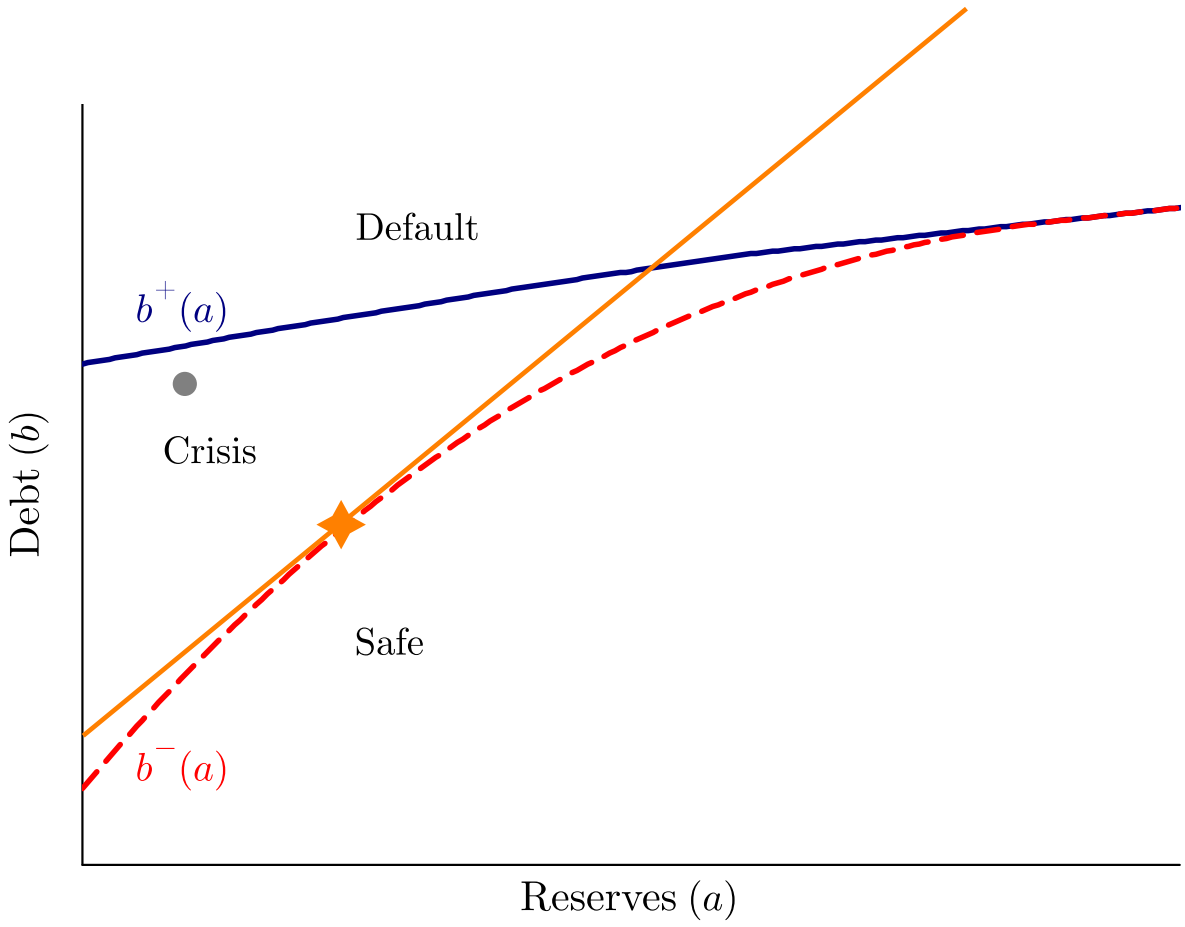
Borrowing to accumulate reserves reduces vulnerability

Deep in the Crisis Zone

Country has **higher** initial debt level: what to do?

Deep in the Crisis Zone



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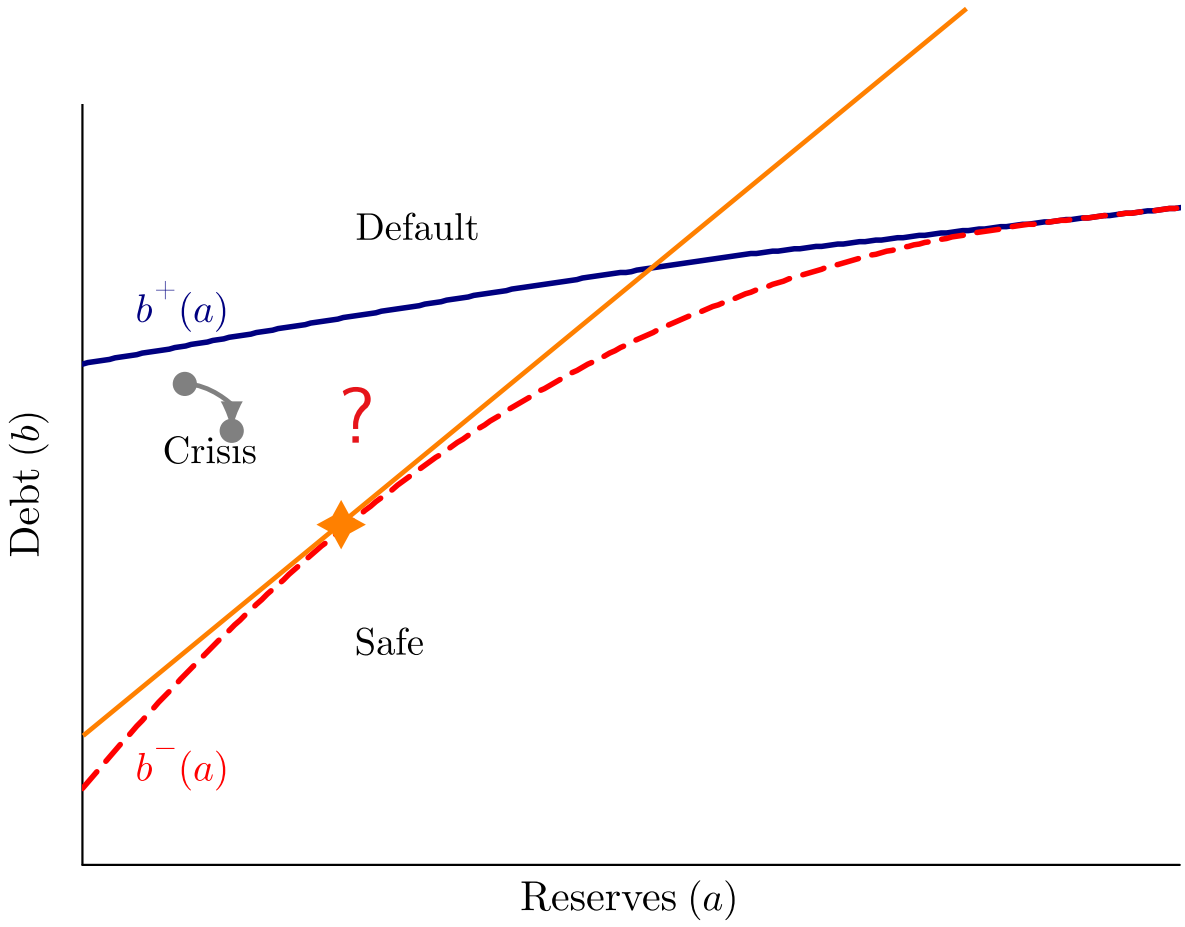

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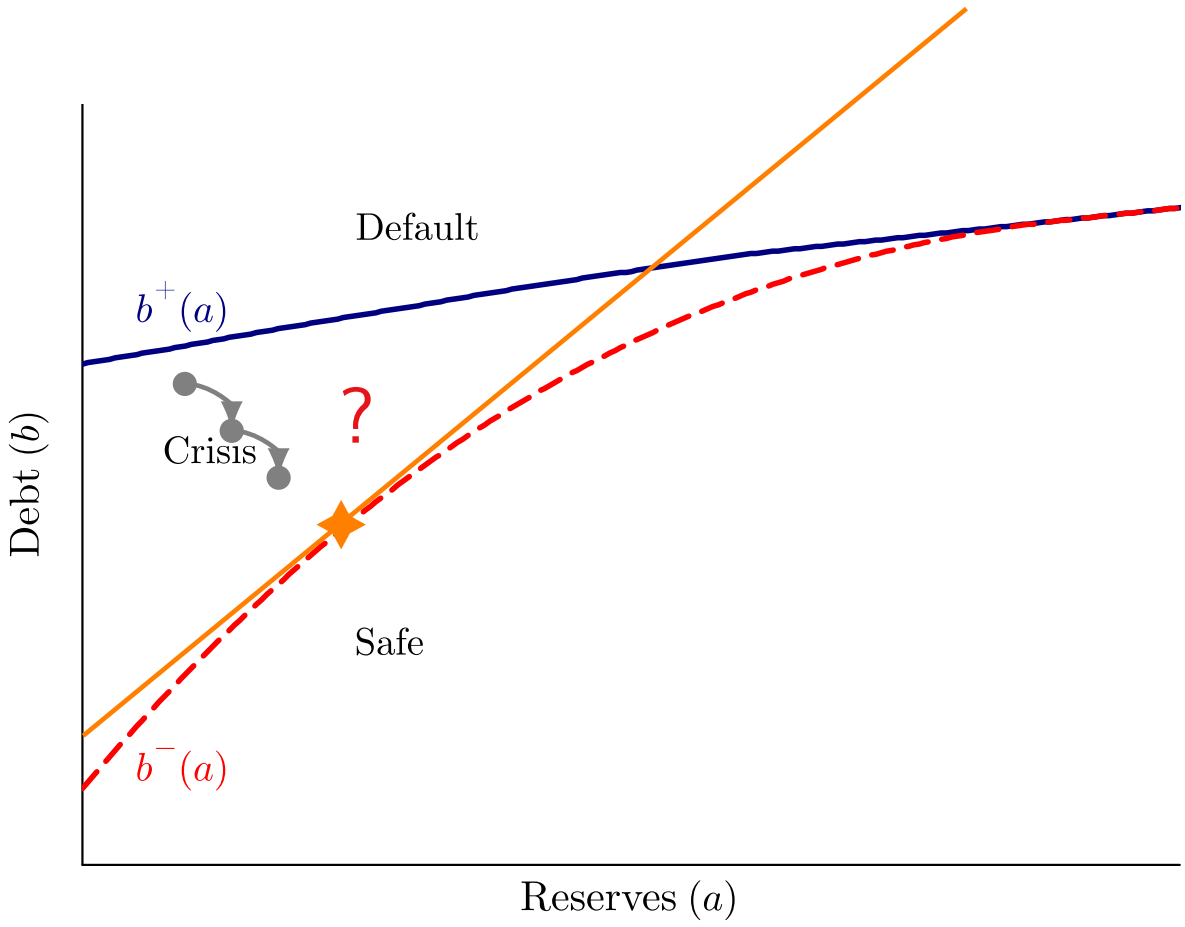

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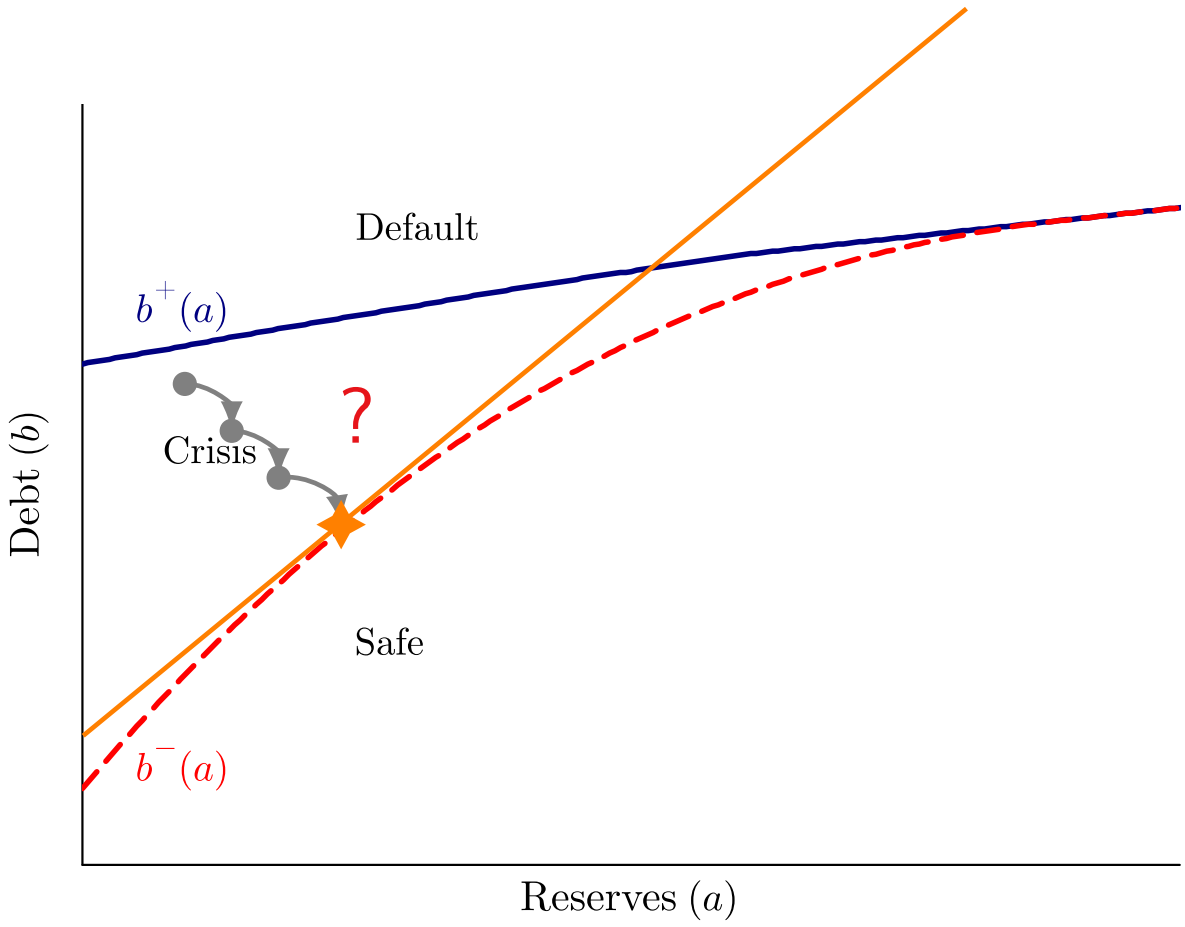

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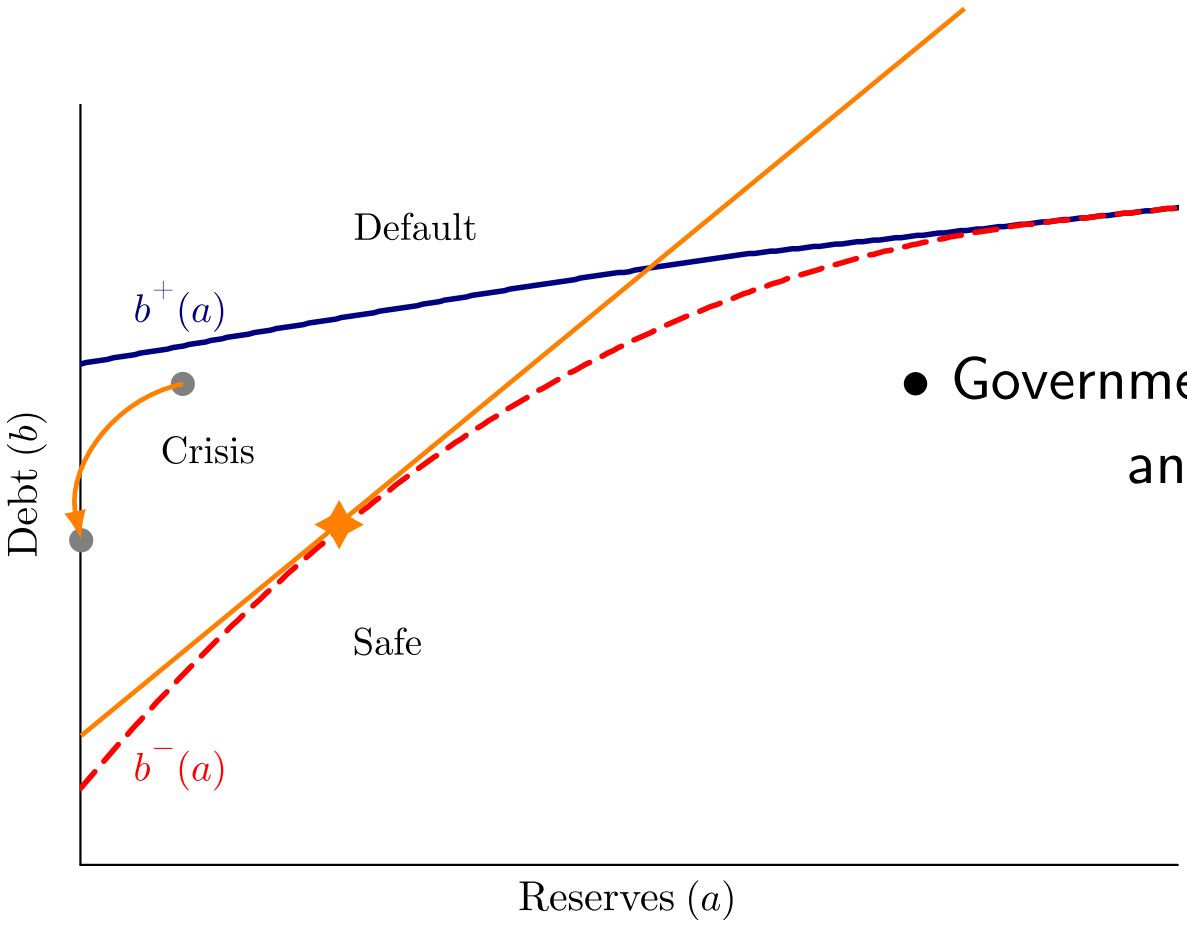
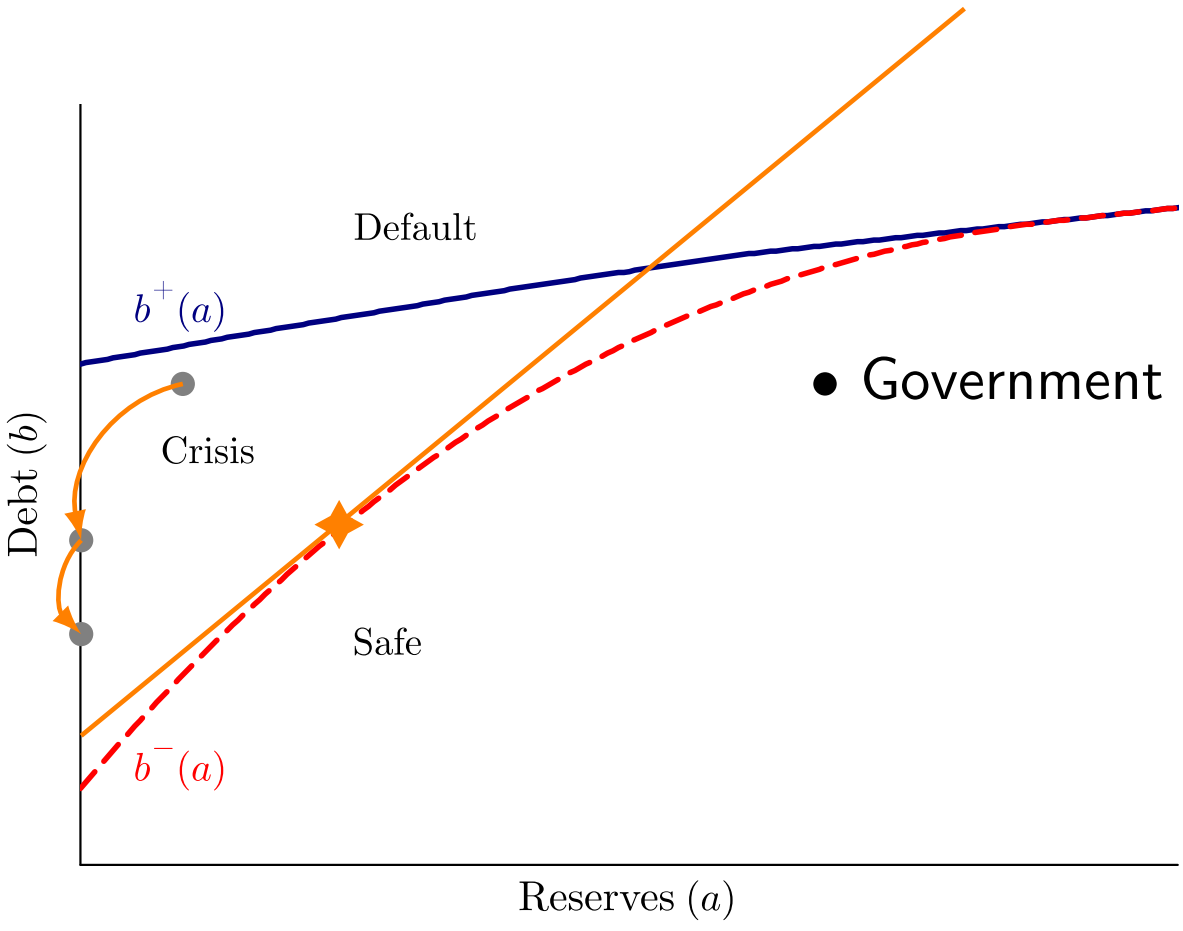


- Government sells reserves and lowers debt

Deep in the Crisis Zone



- Government keeps reducing debt

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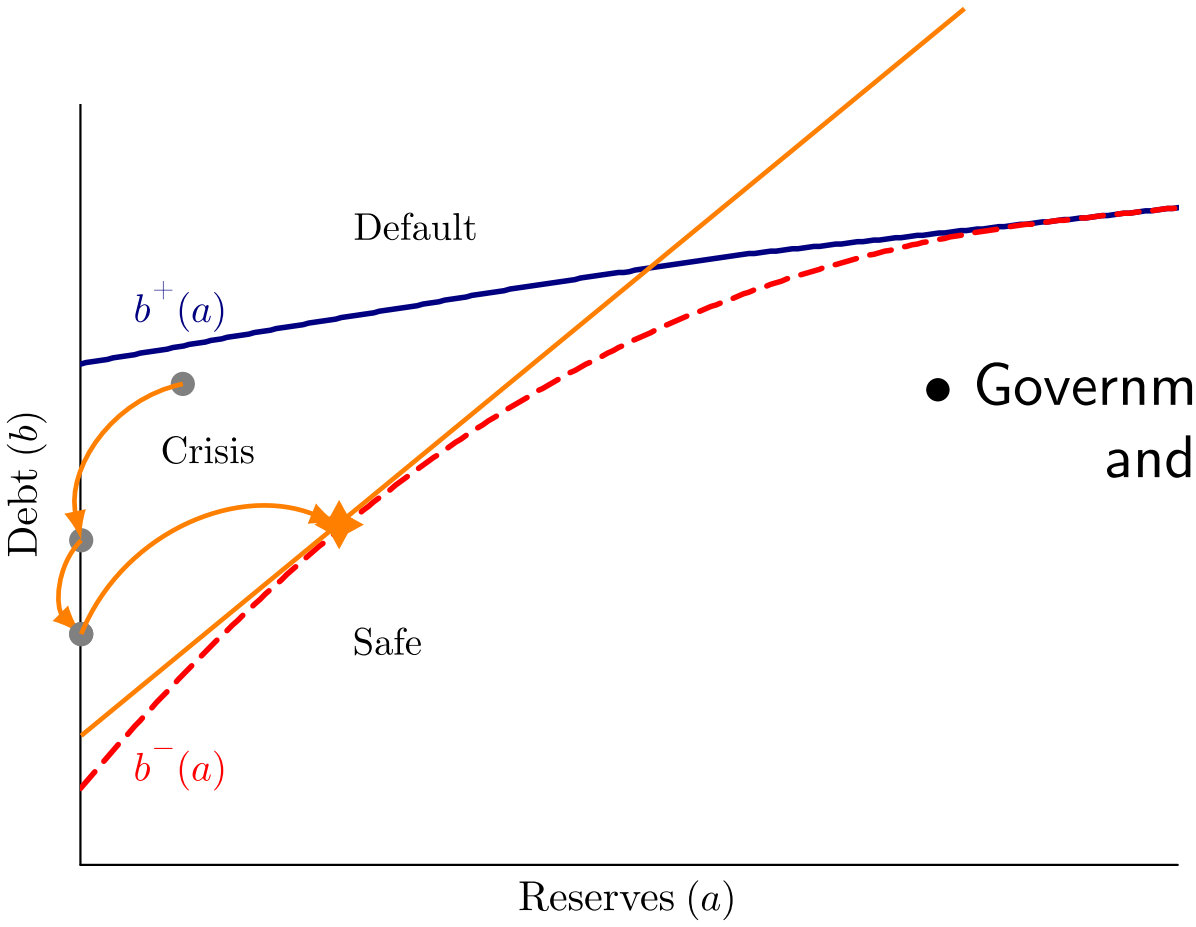


- Government issues debt and buys reserves

Why selling reserves (initially)?

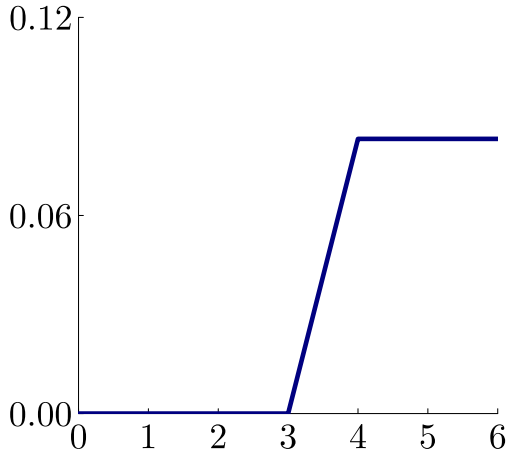
- When the government is 'deep' in the Crisis Zone, on the margin reserves do not change the probability of a run

- Using the reserves to lower debt allows the govt to save on interest payments and helps deleveraging

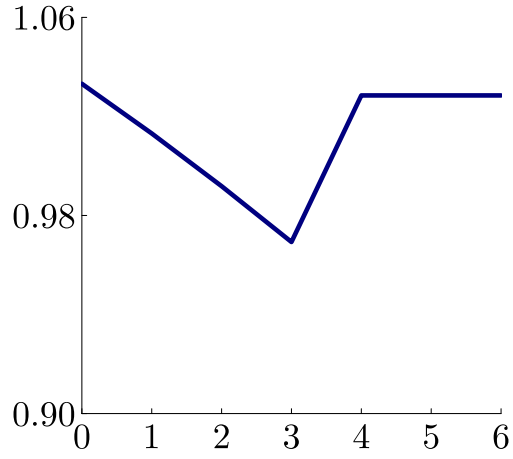
Deleveraging Dynamics

▸ More

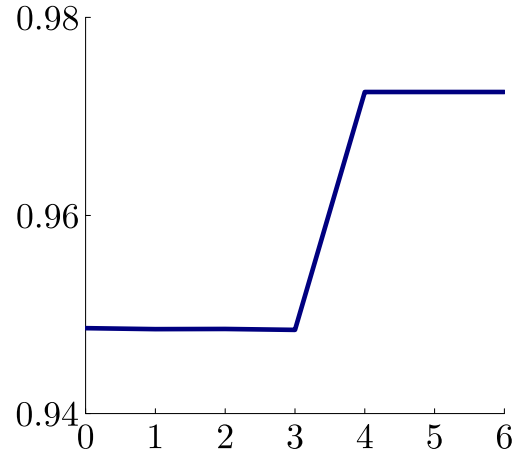
Reserves, a



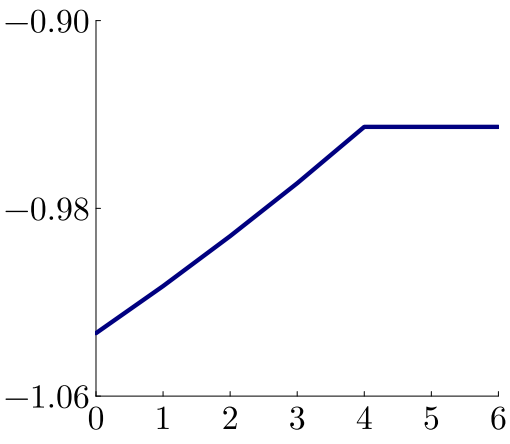
Debt, b



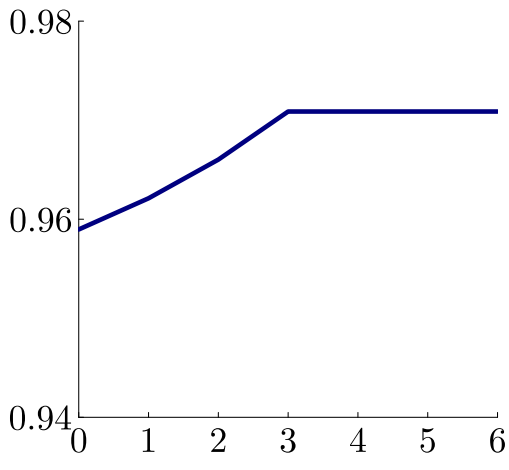
Consumption



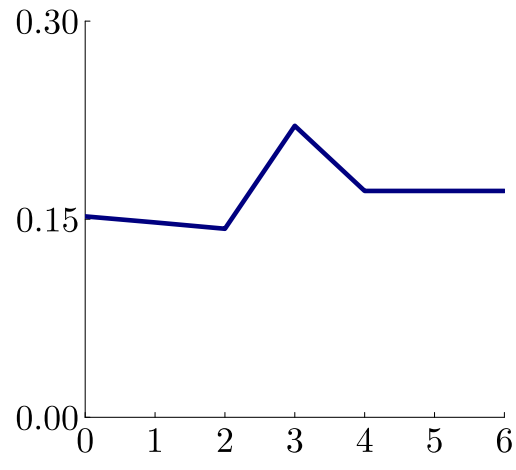
Net Foreign Assets



Debt Price, $q(a', b', s)$



Issuance, $b' - (1 - \delta)b$



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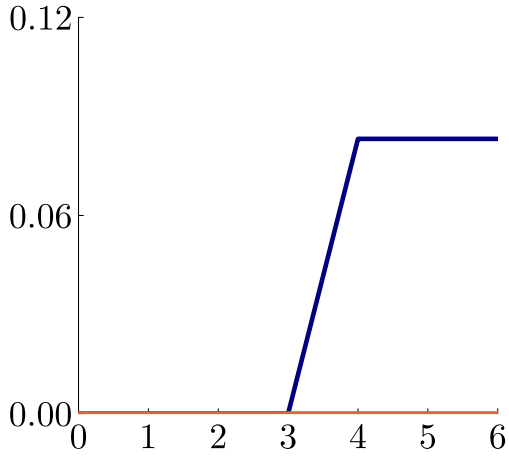

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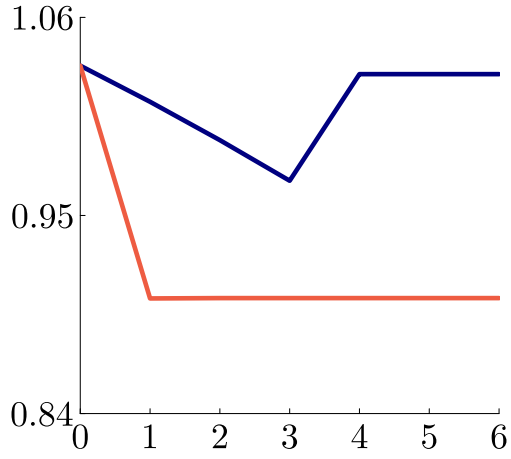

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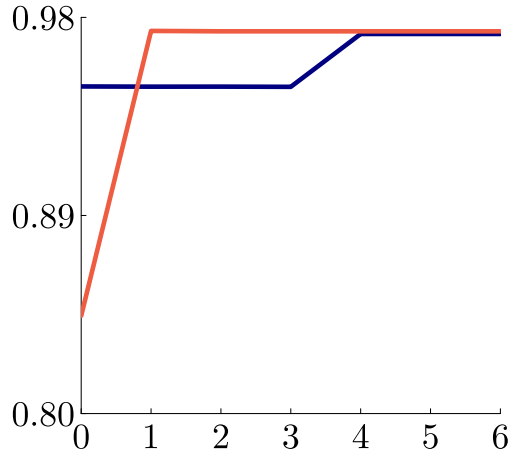

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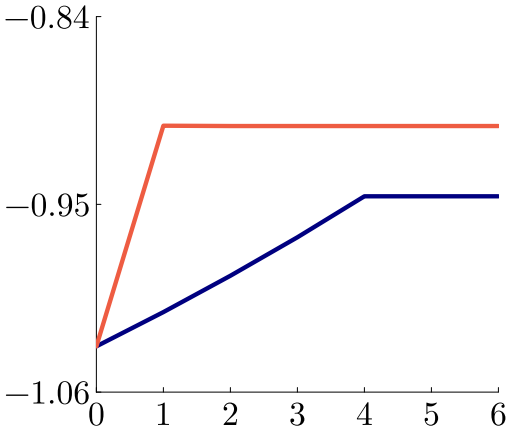

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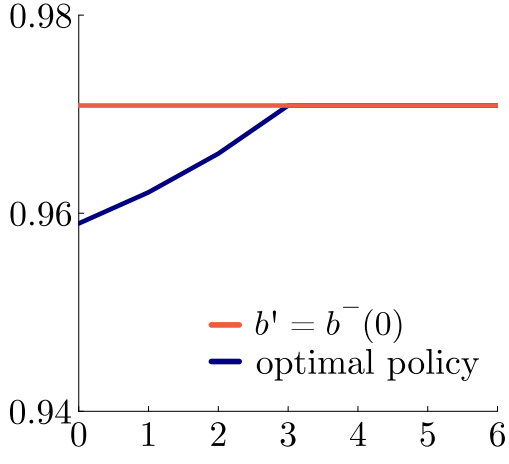

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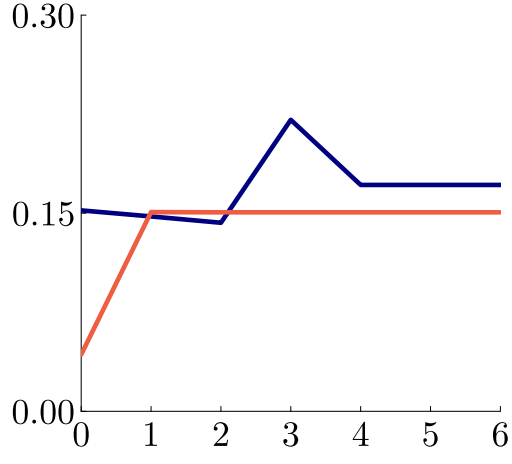
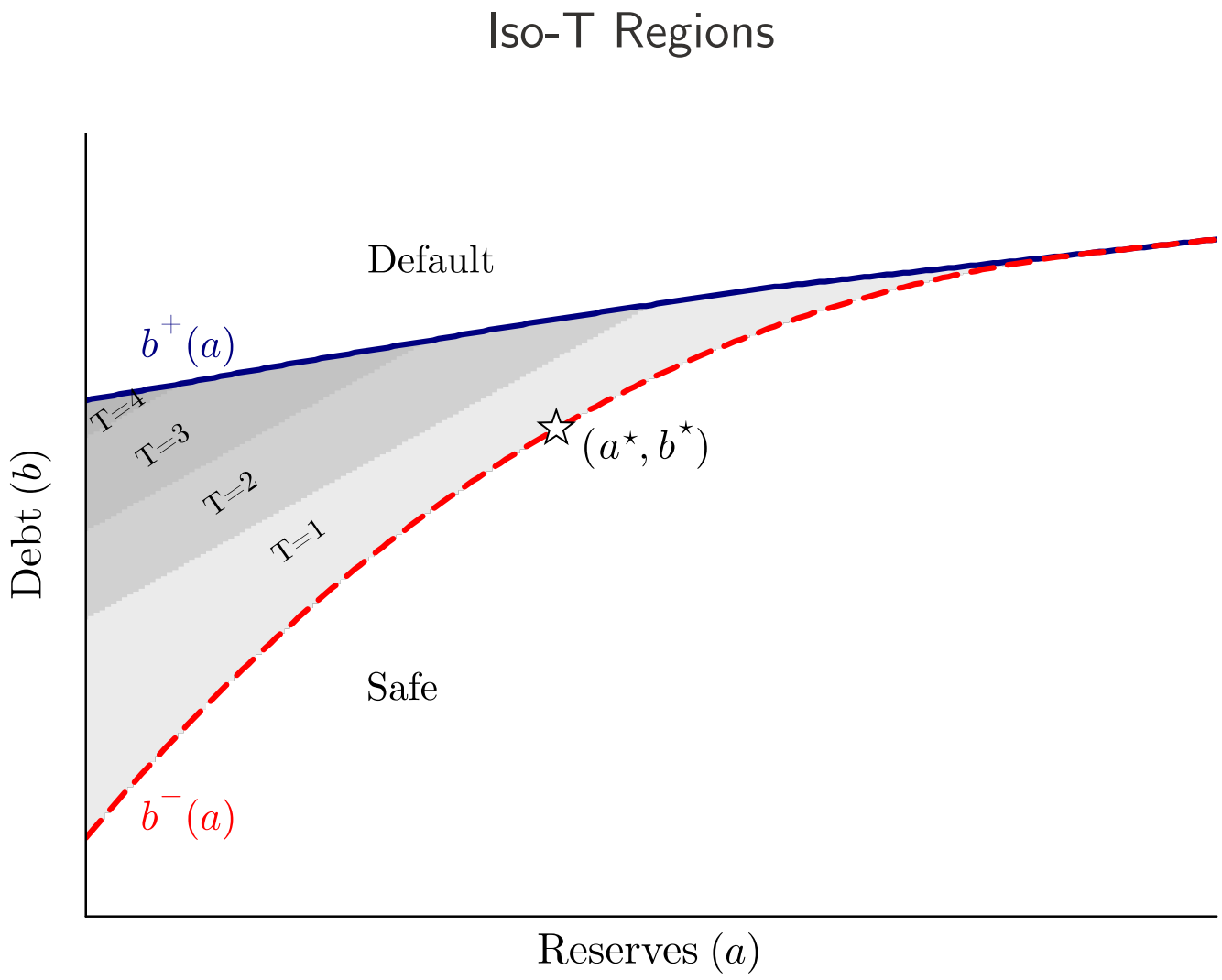


How many periods until exit?



Formalizing the Results: (a^*, b^*) portfolio

$(a^\star, b^\star)$ is a focal point – we call it **Lowest-NFA safe portfolio**

When do we have $a^\star > 0$?

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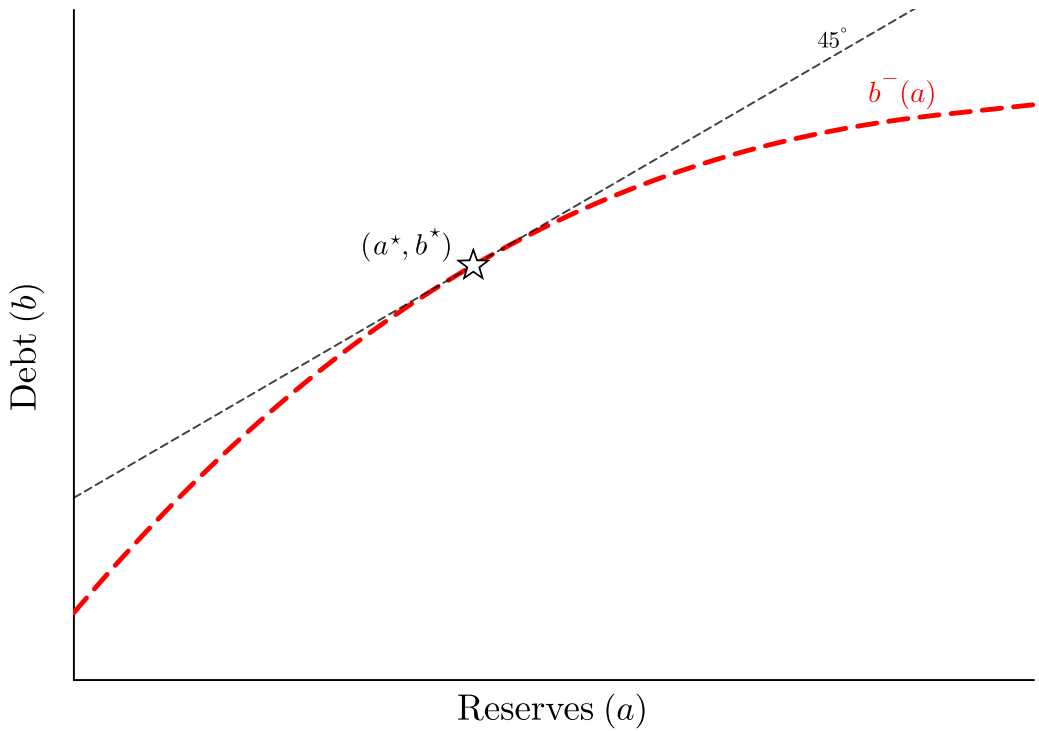

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Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves $b^-(0)$ satisfies

$$\beta(1-\delta)\left[u'\left(y-\kappa b^-(0)\right)-u'\left(y-(1-\beta)(1-\delta)b^-(0)\right)\right] > u'(y)$$

Then, the lowest-NFA safe portfolio has strictly positive reserves, $a^\star > 0$

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When does it fail?

1. low risk-aversion,

Lowest-NFA safe portfolio, $(a^\star, b^\star)$

$(a^\star, b^\star)$ is a focal point. When do we have $a^\star > 0$?

Answer: when $\left.\frac{\partial b^-(a)}{\partial a}\right|_{a=0} > 1$

Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves $b^-(0)$ satisfies

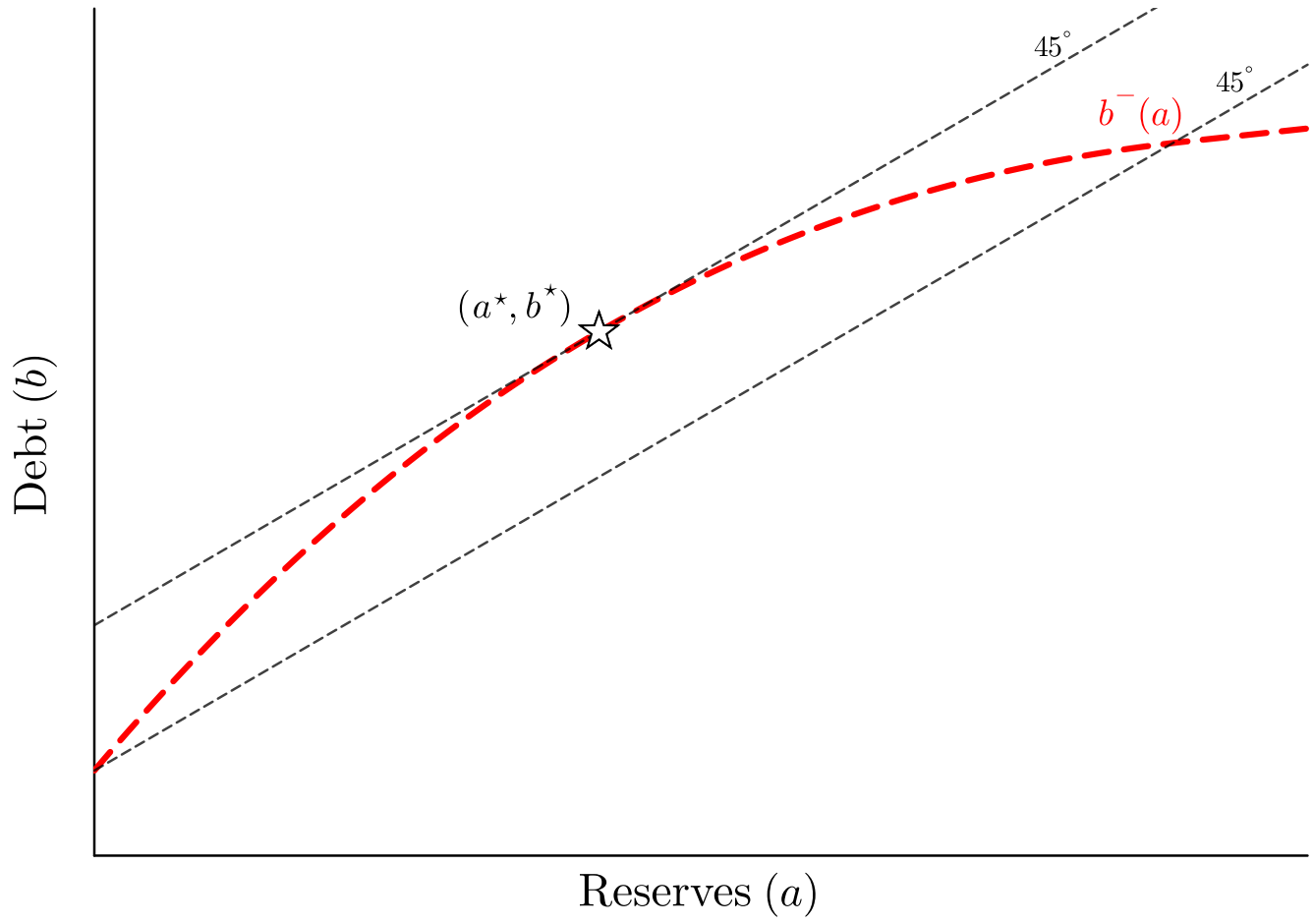
$$\beta(1-\delta)\left[u'\left(y-\kappa b^-(0)\right)-u'\left(y-(1-\beta)(1-\delta)b^-(0)\right)\right] > u'(y)$$

Then, the lowest-NFA safe portfolio has strictly positive reserves, $a^\star > 0$

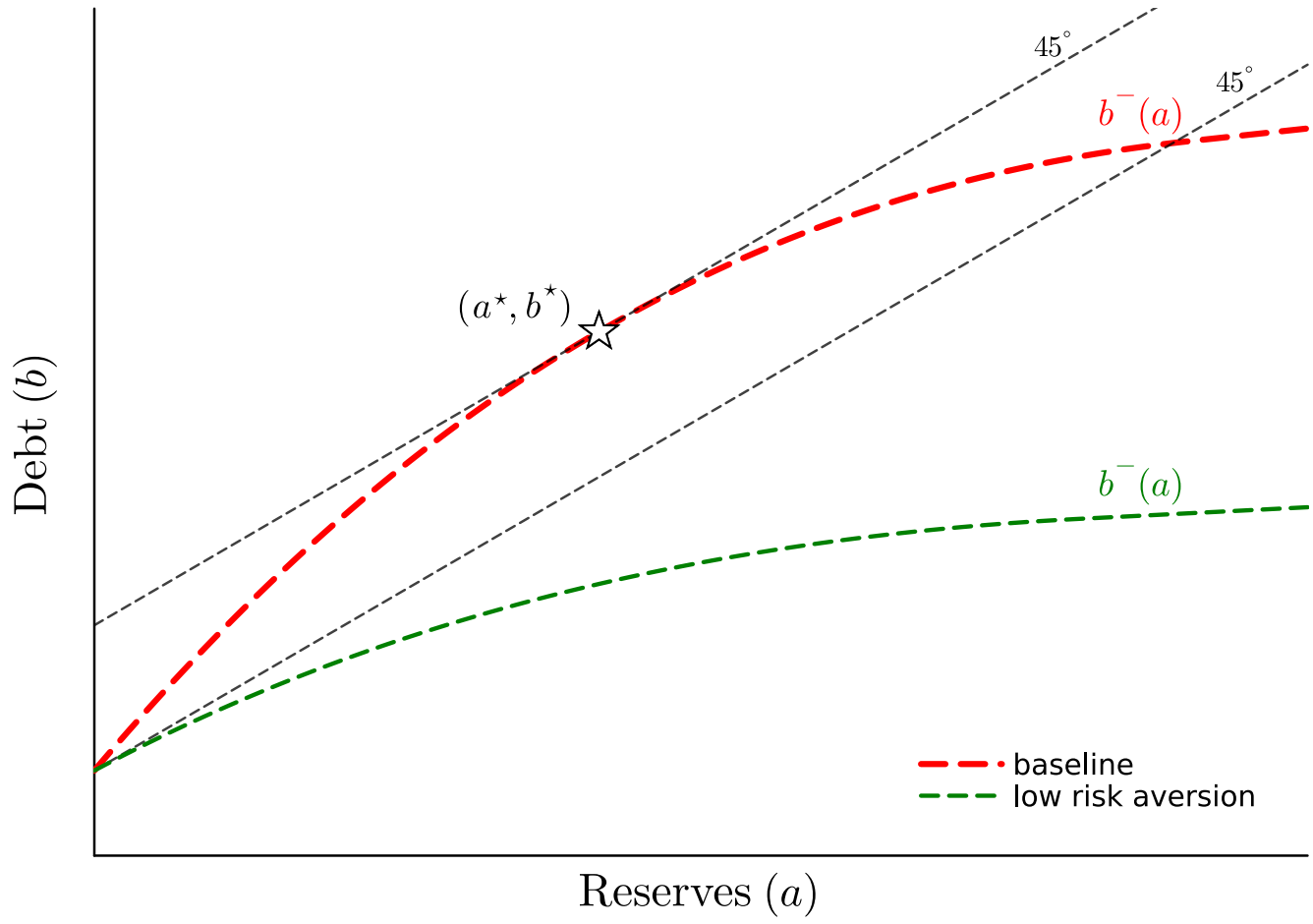
When does it fail?

1. low risk-aversion,
2. one-period debt ($\delta = 1$) [**Prop. 4**]

Lowest-NFA safe portfolio, $(a^\star, b^\star)$



Lowest-NFA safe portfolio, $(a^\star, b^\star)$



Formalizing the Results: Optimal portfolio

To exit crisis zone, first deleverage, then raise debt and reserves

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Proposition 5 (Optimal portfolio)

Consider an initial portfolio $(a, b) \in \mathbf{C}$. The optimal portfolio satisfies:

- If initial portfolio (a, b) is such that $a - b < a^\star - b^\star$ and $(a', b') \in \mathbf{S}$. Then we have $a' = a^\star$, $b' = b^\star$
- If initial portfolio (a, b) is such that $(a', b') \in \mathbf{C}$. Then, the optimal solution features $a' = 0$.

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Remark on maturity:

- With one-period debt, $\delta = 1$: ${V_R}^-$ and ${V_R}^+$ are unaffected by equal increases in debt and reserves $\Rightarrow$ issuing debt to accumulate reserves increases spreads
  - Zero reserves are optimal

▸ Sensitivity

Conclusions

- Simple theory of optimal res. management w/ rollover crises
- Optimal to accumulate reserves to reduce vulnerability
    - However, only after debt has been reduced towards safety

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Conclusions

- Simple theory of optimal res. management w/ rollover crises
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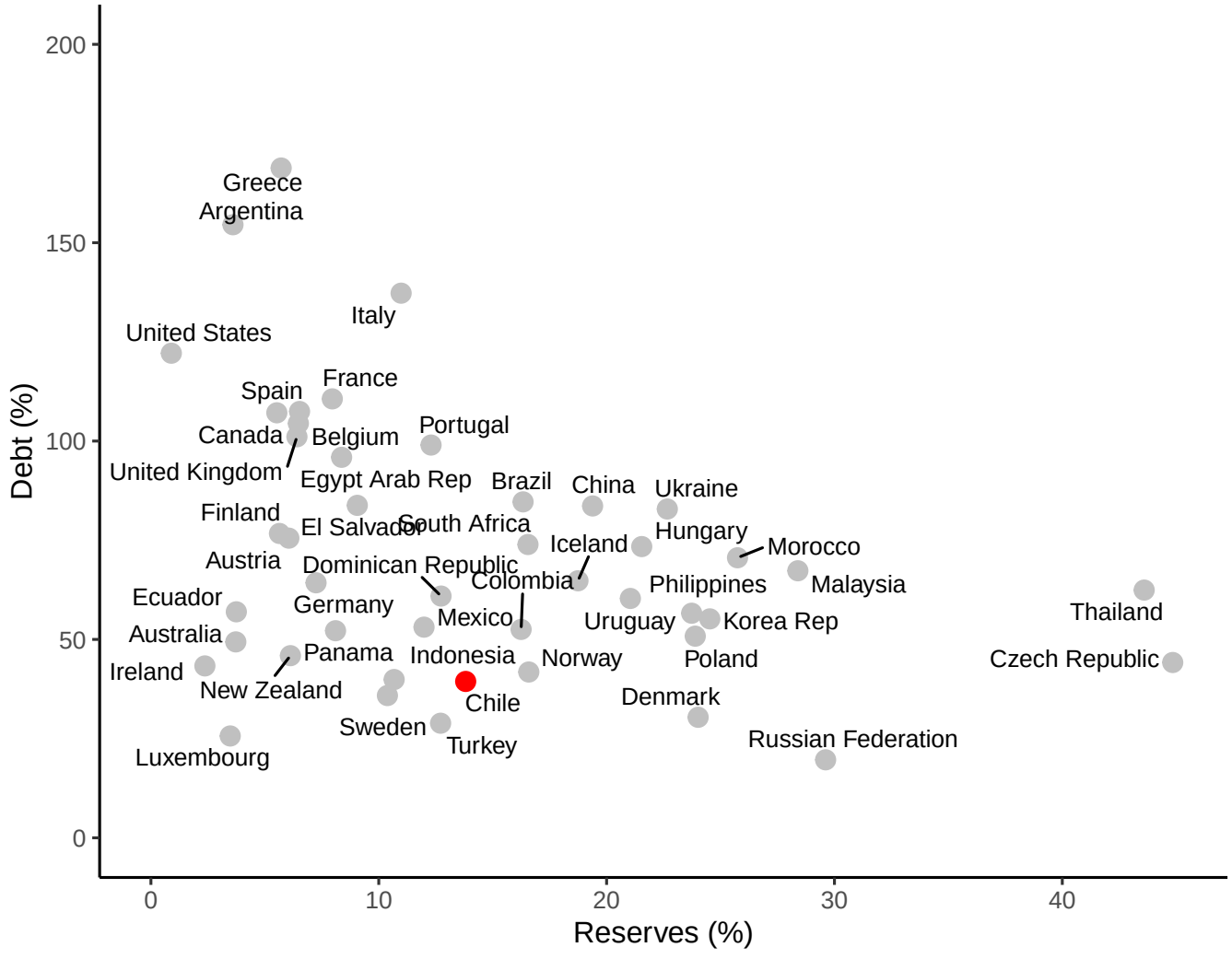
- Reserves as 'buffer': after buildup, no use of reserves
  - Not using them doesn't mean they're unnecessary
- Issuing debt to accumulate reserves can reduce spreads
- Findings speak to policy discussions on appropriate level of FX reserves (e.g. IMF)
  - Following a debt crisis, IMF often prescribes increasing reserves
  - However, we find holding reserves not optimal at beginning of deleveraging process

Scan to find the paper!

THANKS!

Data: Government Debt and International Reserves

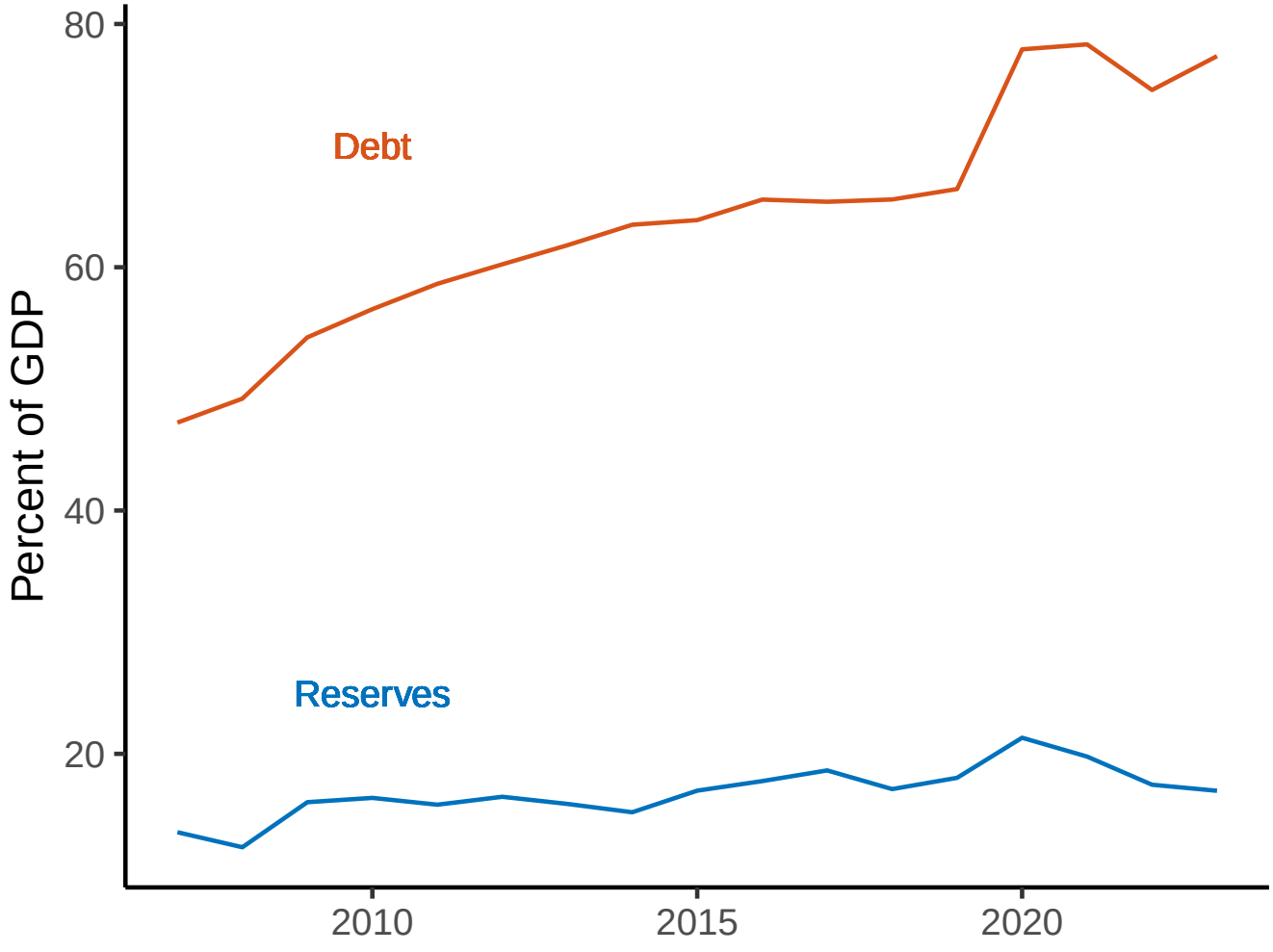
back



Government debt and reserves (as % of GDP), 2023

Evolution of Debt and Reserves

back



Avg. Government debt and reserves (as % of GDP)

Characterization: Value in the Safe zone

back

- If $(a, b) \in \mathbf{S}$: we assume gov. stays in safe zone

$$V^S(a-b) = \frac{u(y + (1-\beta)(a-b))}{1-\beta}$$

- **Note:** relevant state variable is the NFA, $a - b$

For a high enough δ: can establish that gov. finds it optimal to stay in **S**

Characterization: Crisis zone

back

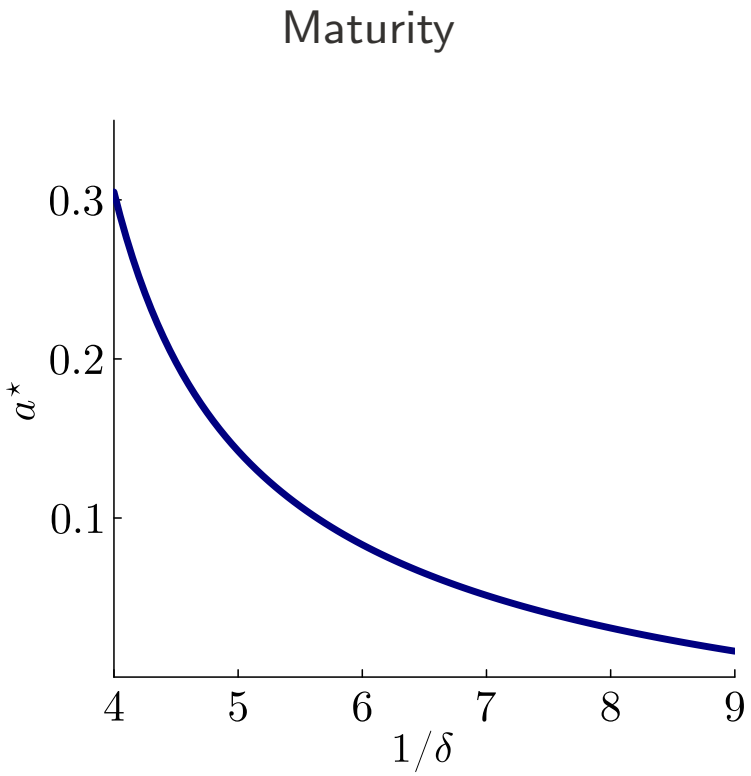
- If $(a, b) \in \mathbf{C}$, govt. seeks to exit in finite time (may default along the way if bad sunspot hits)
  - Staying in the crisis zone implies eventually costly default
  - Speed of exit depends on curvature of $u(\cdot)$ and probability of bad sunspot

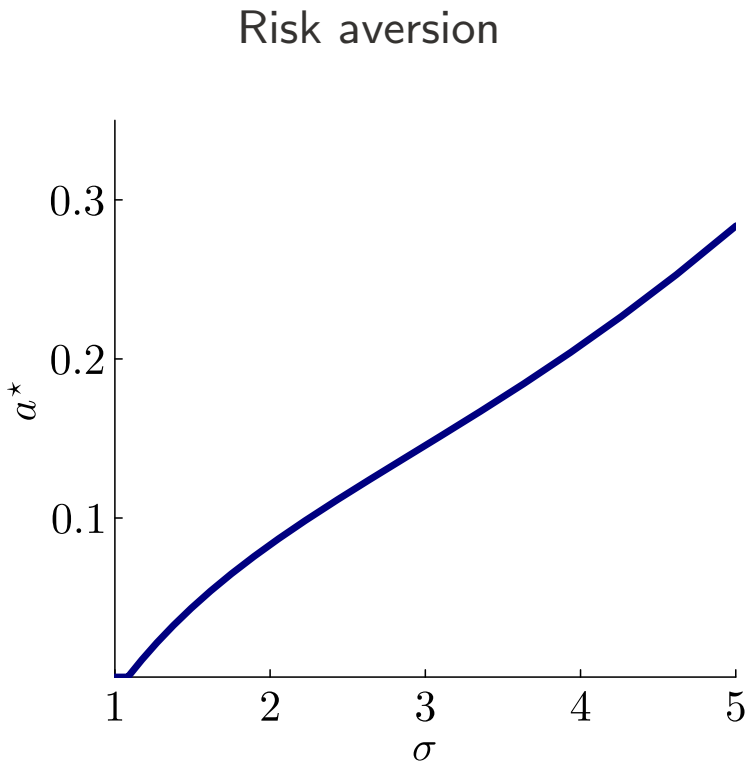
Continuation value:

$$\mathbb{E}V(a', b', \zeta') = \begin{cases} V^S(a' - b') & \text{if } (a', b') \in \mathbf{S} \\ (1-\lambda)V_R^+(a', b') + \lambda V_D(a') & \text{if } (a', b') \in \mathbf{C} \\ V_D(a') & \text{if } (a', b') \in \mathbf{D} \end{cases}$$

Sensitivity: effect of maturity and risk-aversion on $a^\star$

back

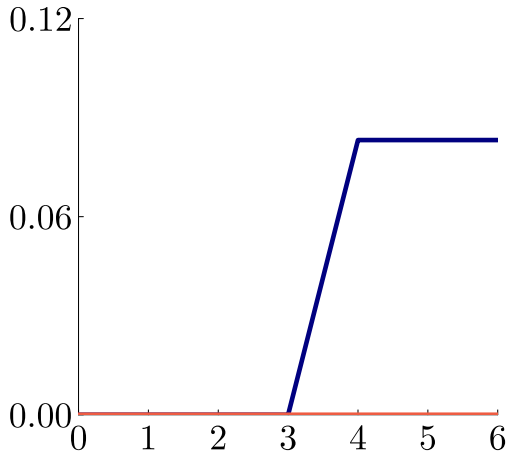




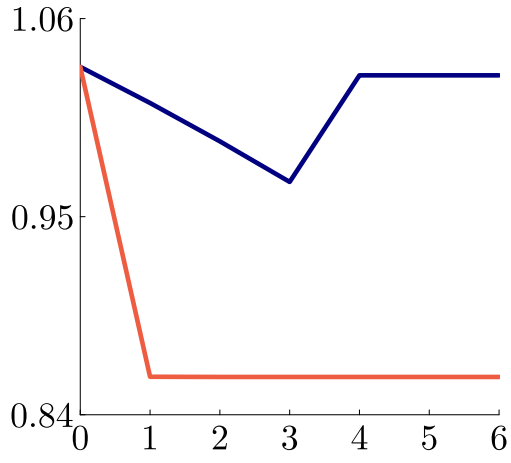
Deleveraging Dynamics: $b' = (1-\delta)b_0$

Back

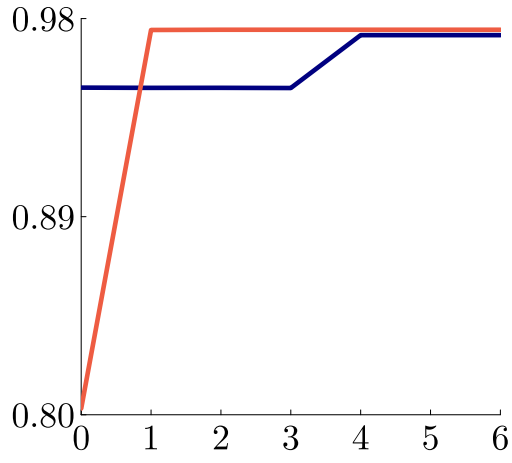
Reserves, a



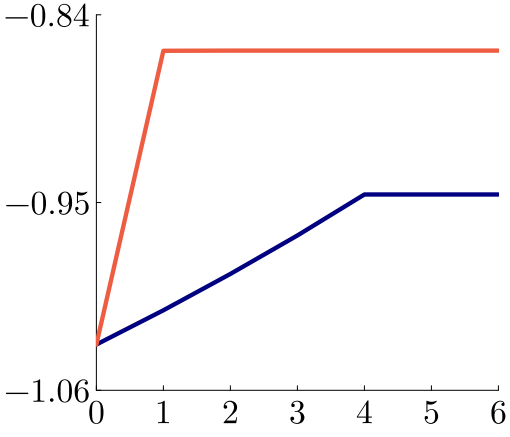
Debt, b



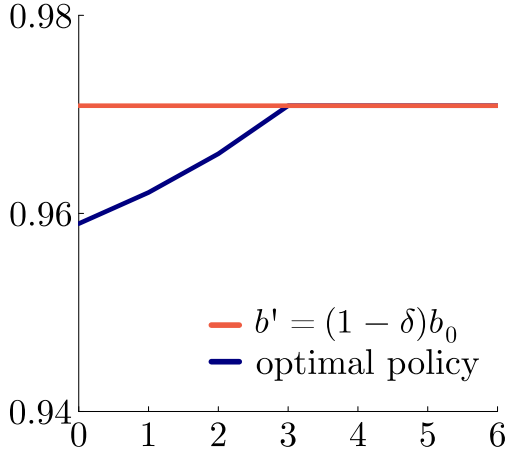
Consumption



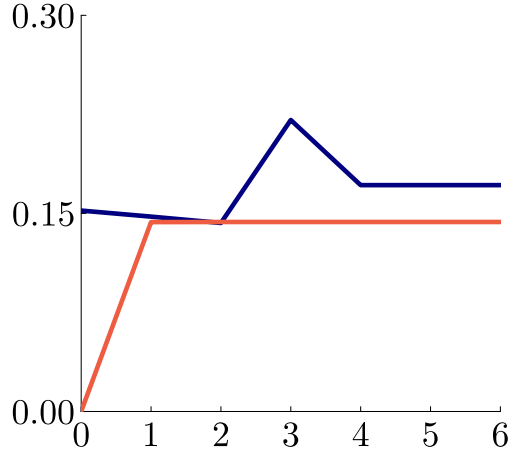
Net Foreign Assets



Debt Price, $q(a', b', s)$



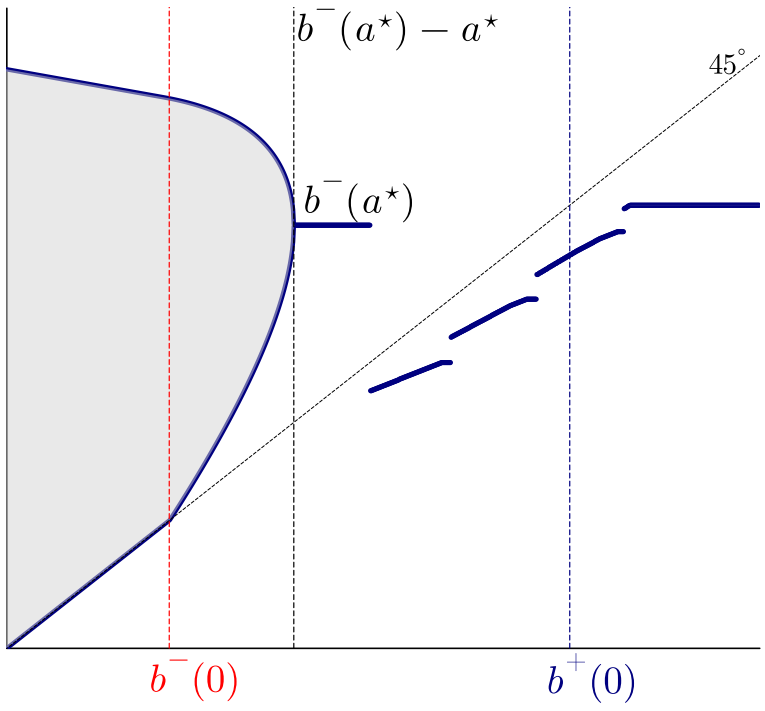
Issuance, $b' - (1-\delta)b$



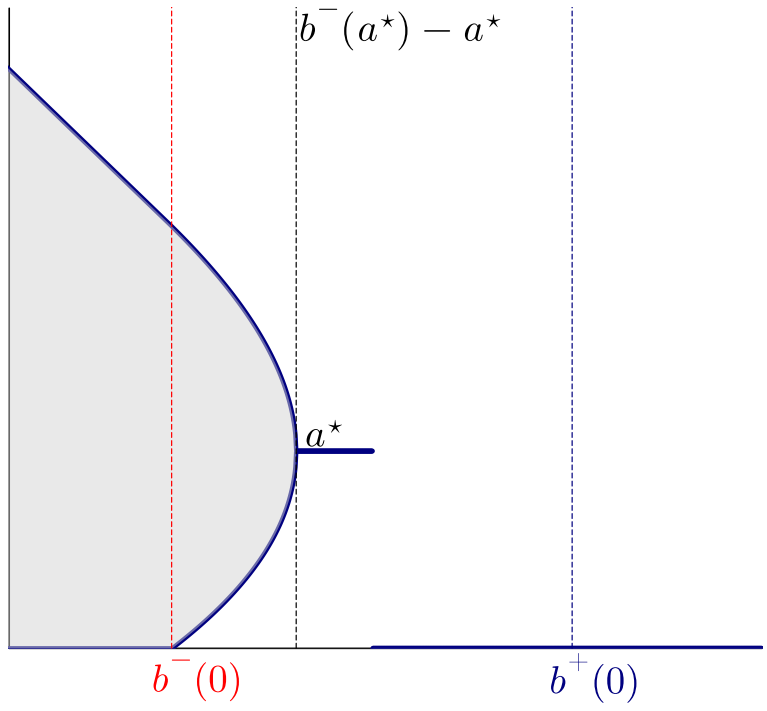
Policies

Back

Debt, b'



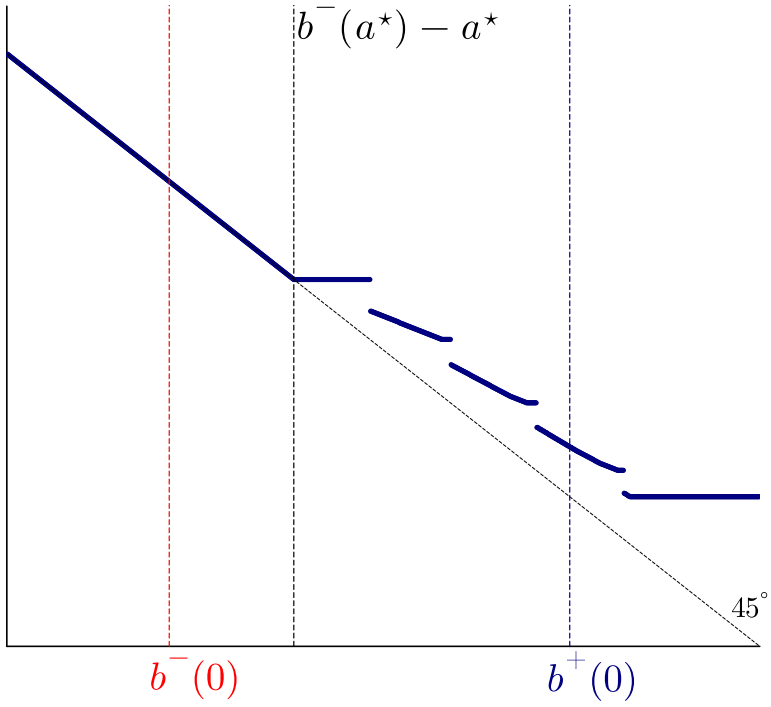
Reserves, a'



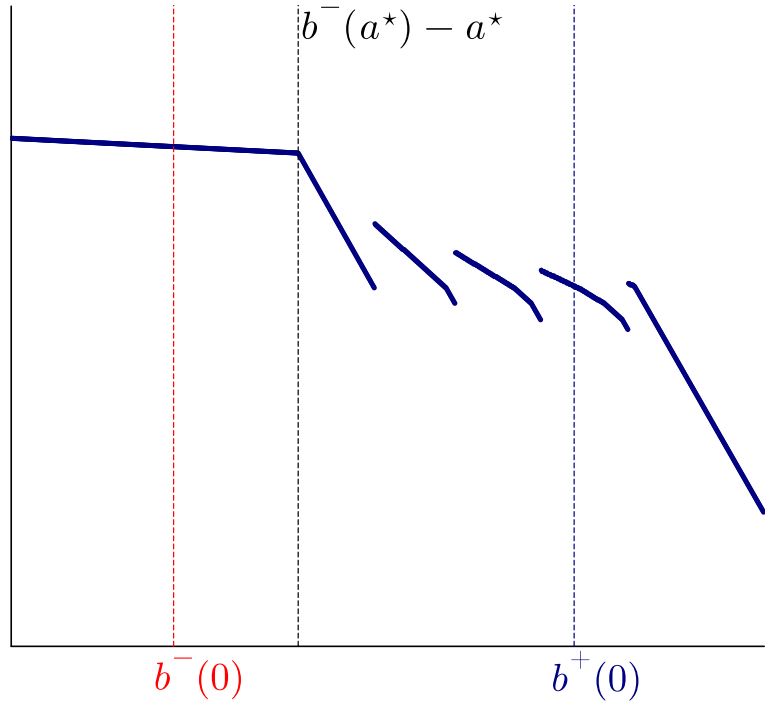
Policies

Back

Net Foreign Assets, $a' - b'$



Consumption



Parametrization

Back

$$u(c) = \frac{(c - \underline{c})^{1-\sigma}}{1-\sigma}$$

Parameter	Value	Description	Source
y	1	Endowment	Normalization
σ	2	Risk-aversion	Standard
r	3%	Risk-free rate	Standard
$1/\delta$	6	Maturity of debt	Italian Debt
$\underline{c}$	0.68	Consumption floor	Bocola-Dovis (2019)
β	0.97	Discount factor	$\beta(1+r)=1$
λ	0.5%	Sunspot probability	Baseline
ϕ	0.33	Default Cost	Debt-to-income =100%
κ	$\frac{\delta+r}{1+r}$	Coupon	Normalization

Experiment – How reserves help exit crisis zone

- Assume gov. starts w/ portfolio (a, b), **but** from t+1 onward, $a' = 0$

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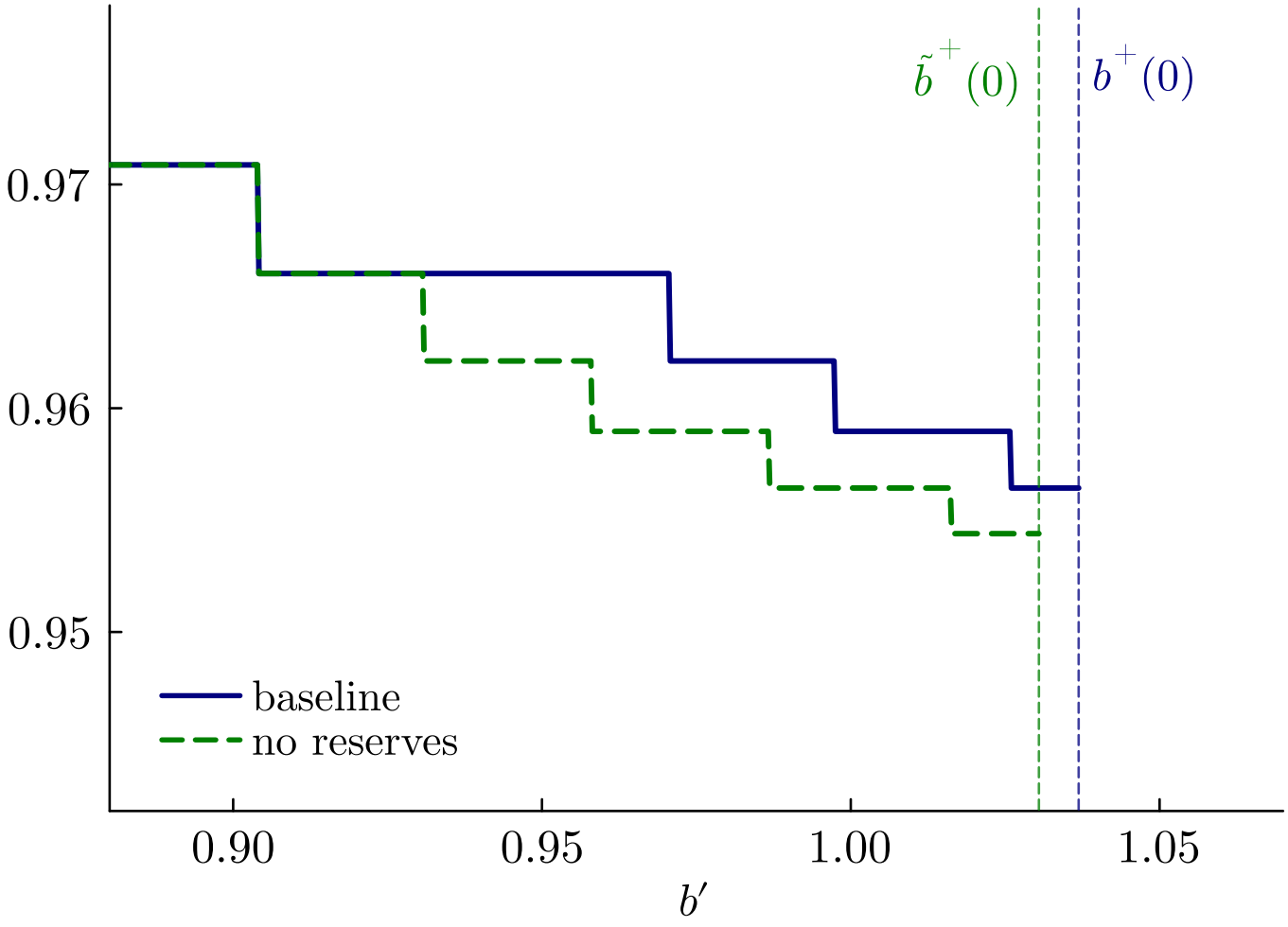
Without reserves: $\downarrow b^{+}$. More costly to deleverage $\Rightarrow$ lower debt-carrying capacity

▸ Default zone expands

▸ back

Price Schedule, $q(0, b')$

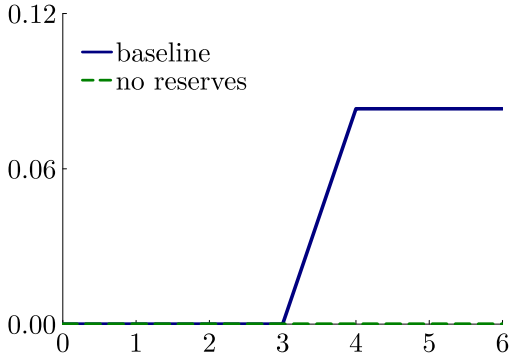
back



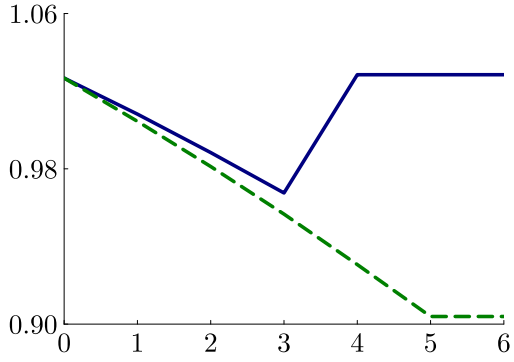
Lower consumption without reserves

back

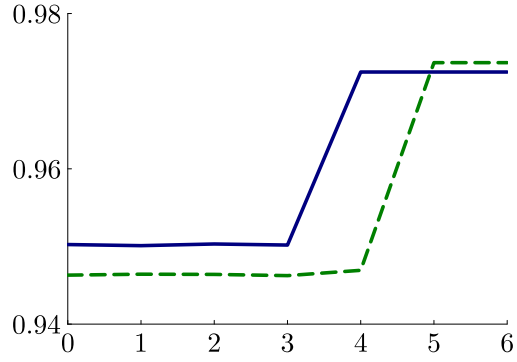
Reserves, a



Debt, b



Consumption



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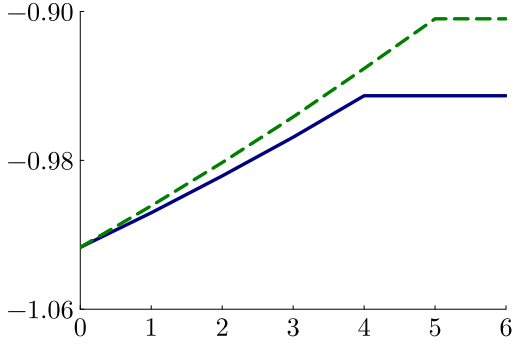

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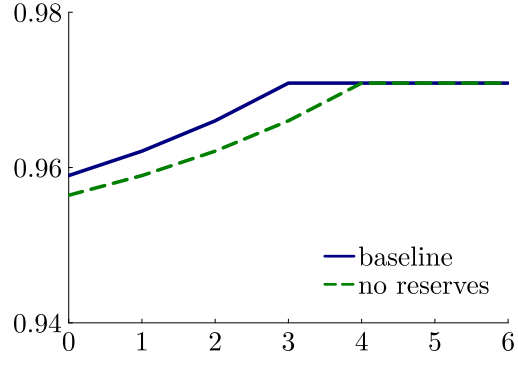
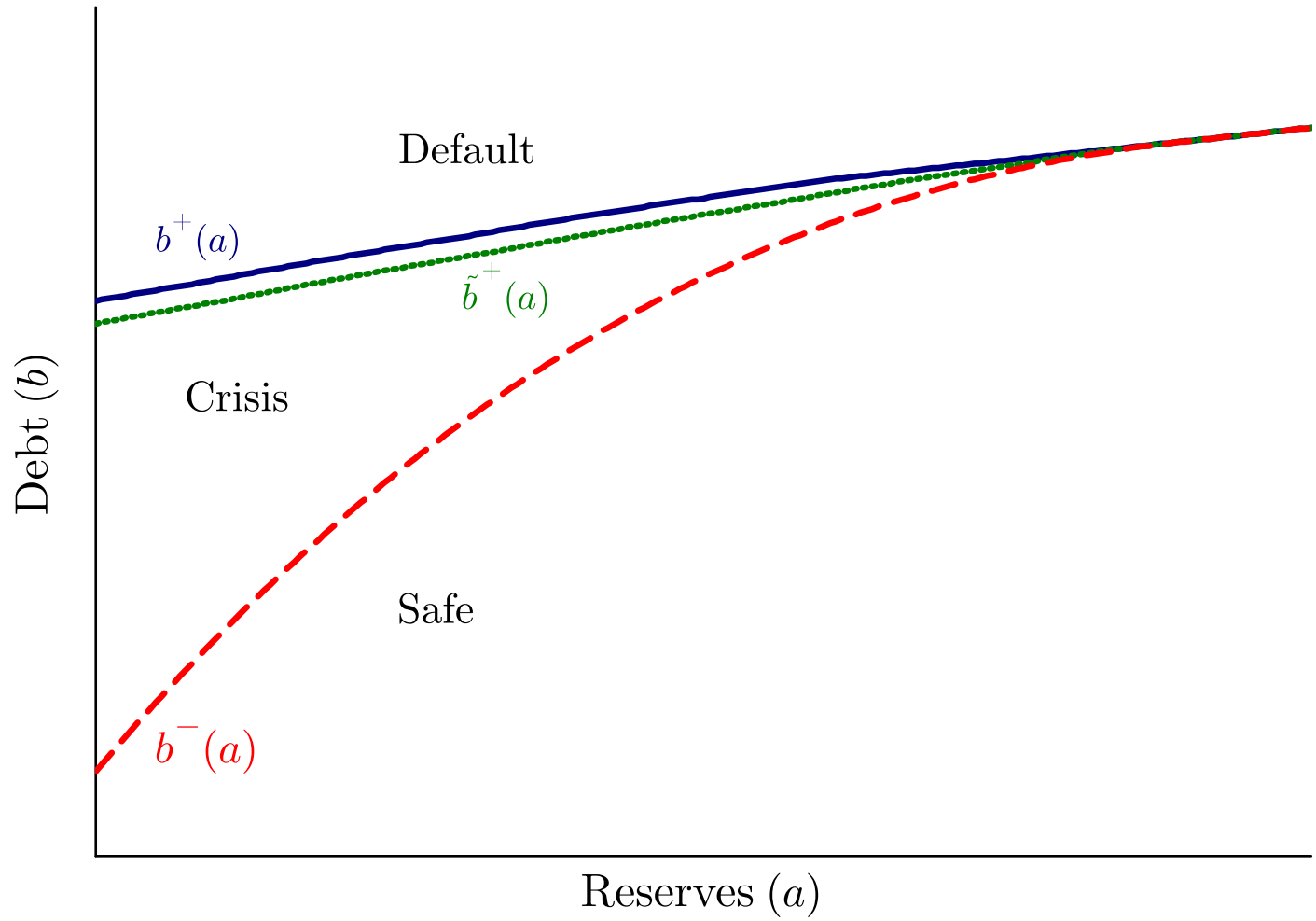


Default zone expands

back



Data: increasing reserves and debt lowers spreads (preliminary)

back

Dep. Variable:	log(Spread)		
	(0)	(1)	(2)
Reserves	−2.39***		
	(0.11)		
Sov.Debt	1.25***	−1.13***	1.58***
	(0.10)	(0.14)	(0.20)
NFA_public		−2.39***	−2.69***
		(0.11)	(0.11)
$(\text{Sov.Debt})^2$			−5.48***
			(0.31)
Num.Obs.	4497	4497	4497
R2	0.791	0.791	0.997

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

All specs. include country FEs, year dummies and additional macro controls (as in Sosa-Padilla and Sturzenegger, 2023).