International Equity Flows, Monetary Policy, and Time Consistency*

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Abstract

This paper studies optimal monetary policy in an open economy with international equity flows. Foreign ownership creates a time-inconsistency problem: once equity is held abroad, the central bank has an ex post incentive to deviate from the natural allocation by raising real wages and compressing profits paid to foreign shareholders. Anticipating this discretionary policy response, foreign investors require a discount when purchasing domestic equity, reducing domestic wealth. Because households do not internalize that their collective equity sales exacerbate the central bank's future incentive to compress profits, they over-divest, generating multiple Pareto-ranked equilibria. In the bad equilibrium, equity valuations are depressed, and employment is lower. Unlike existing theories of capital-flow management, our model provides a rationale for capital controls on equity inflows.

Keywords: Monetary policy, exchange rates, equity flows
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1 Introduction

Over the past three decades, cross-border equity investment has become a defining feature of the international financial landscape. As Figure 1 shows, the combined value of foreign direct investment (FDI) and foreign ownership of equity (FOE) has increased from roughly 20% of GDP in 1995 to nearly 90% in 2024. While a common view holds that cross-border equity positions promote risk-sharing and improve welfare, their growing scale raises important questions about their implications for macroeconomic policy and aggregate welfare.

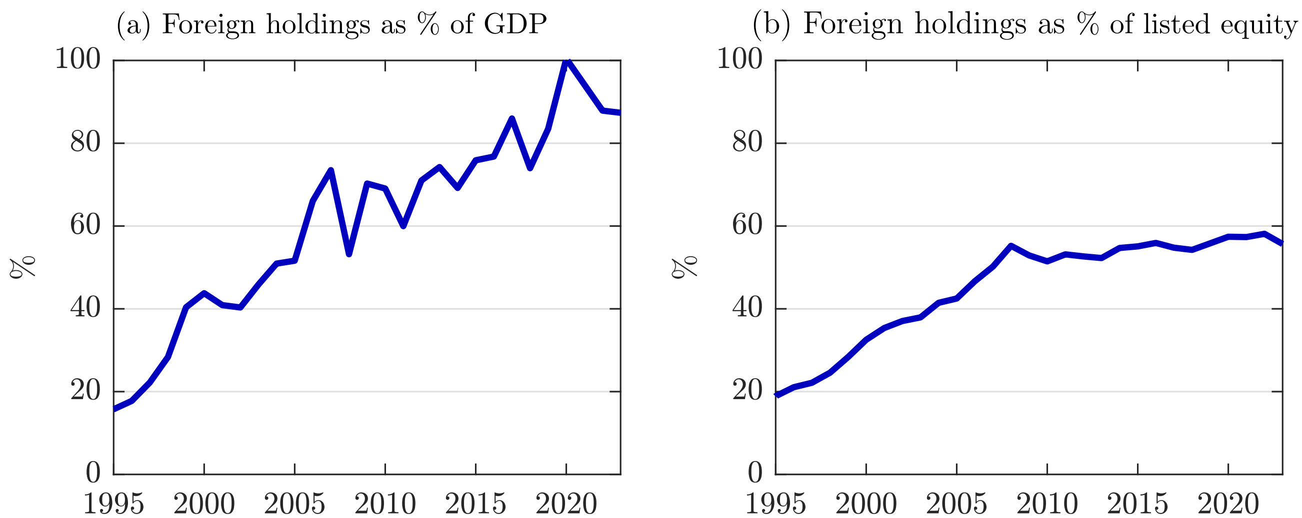

Figure 1: Foreign ownership of domestic equity

Notes: Panel (a) shows the stock of domestic equity owned by foreigners as a share of GDP, and panel (b) shows it as a share of the domestic equity market. Foreign ownership of domestic equity is measured as the sum of foreign direct investment (FDI) and foreign ownership of non-financial and bank domestic equity stocks (FOE). The series represent a simple average across the following countries: Austria, Belgium, Brazil, Bulgaria, Canada, Chile, Colombia, Croatia, the Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Israel, Italy, Japan, South Korea, Mexico, Norway, Poland, Portugal, the Slovak Republic, Slovenia, Spain, Sweden, Turkey, the United Kingdom, and the United States. Data sources: External Wealth of Nations (EWN) database (Lane and Milesi-Ferretti, 2018) and OECD National Accounts Statistics.

This paper explores the implications of international equity flows for monetary policy and welfare. We show that foreign ownership of domestic equity introduces a time inconsistency problem for monetary policy, as the central bank has an incentive to depart ex post from the natural allocation in order to reduce firms' profits and erode the value of equity claims held by foreign investors. Moreover, we show that opening to international equity flows exposes the economy to a Pareto-inferior equilibrium in which households sell excessive amounts of domestic equity ex ante. This arises because households do not internalize that greater foreign ownership exacerbates the central bank's ex post temptation to reduce profits, which depresses the equity price at which they sell the equity claims.

We formalize these results in a tractable open-economy New Keynesian framework in which households trade domestic equity with foreign investors, in addition to foreign bonds. To transparently illustrate our results, the baseline model abstracts from uncertainty, imperfections in capital markets, and domestic heterogeneity.

In the case where households own the entire share of domestic equity, optimal monetary policy attains the natural allocation—equivalently, a zero labor wedge or zero output gap—by targeting a level of employment such that the marginal rate of substitution between consumption and leisure equals the marginal product of labor.

When households own the entire domestic equity stock, an increase in the real wage—holding employment fixed—has no effect on aggregate resources: the induced changes in labor income and profits exactly offset in the representative household's budget constraint. Hence, the split between labor income and profits is irrelevant for allocations. This irrelevance no longer holds when foreign investors own domestic equity. In that case, a higher real wage reduces the profits accruing to foreigners, while increasing households' income. Although domestic households also receive lower profits per equity claim, their benefit from higher wages exceeds the loss in equity payouts since they hold only a fraction of the total equity. The split between labor income and profits now matters.

With nominal rigidities, monetary policy can affect real wages; thus, a central bank has an incentive to raise real wages when foreign investors own domestic equity. In our baseline model with nominal wage rigidity, we show that the central bank finds it optimal to over-appreciate the exchange rate relative to the full-employment allocation. The logic is that starting from an allocation with a zero labor wedge, there is a first-order gain from reducing employment because the disutility from working exceeds the marginal utility value of the additional resources, as a share of the output revenue is, in effect, leaked abroad. Accordingly, foreign ownership leads the central bank to appreciate the exchange rate to induce a positive labor wedge, and reallocate resources from foreign investors toward domestic households.

Our key mechanism can be summarized by the central bank's intratemporal optimality condition,

$$-U_h \;=\; \lambda\big(F_h - \theta_1^* \Pi_h\big),$$

where U_h is the marginal utility of labor, F_h is the marginal product of labor, Π_h is the marginal effect of employment on equilibrium profits, λ is the shadow value of relaxing the country's intertemporal resource constraint, and θ_1^* is the foreign-owned share of domestic equity at the time of the nominal rigidity. With full domestic ownership ($\theta_1^* = 0$), the condition reduces to the standard efficiency rule that equates the marginal rate of substitution between consumption and leisure to the marginal product of labor. With foreign ownership ($\theta_1^* > 0$), the central bank internalizes that part of the marginal revenue created by higher employment accrues as profits paid abroad. In effect, the domestic marginal benefit of production is reduced by a "leakage" term proportional to foreign equity ownership.

The time-inconsistency problem arises because foreign investors anticipate the central bank's ex post incentive to erode their profits and therefore purchase equity only at a discount. This lack of commitment reduces the wealth of domestic households. When households sell equity, they fail to internalize that greater foreign ownership intensifies the central bank's incentive to compress future profits, which depresses equity valuations today and lowers welfare. As a result, households collectively sell too much equity, generating an ex post distortion in output without delivering ex ante benefits.

Moreover, we show that multiple equilibria arise quite naturally. When households sell more assets, future dividends fall and equity prices decline, with deep-pocketed foreign investors taking the other side in the market. While individual investors are indifferent between holding and not holding equity, when they collectively sell more equity, these sales reduce domestic wealth. Interestingly, the presence of multiple equilibria is not a result of fire sales and amplification through financial constraints, as in previous literature.

A key policy implication is that restricting foreign purchases of domestic equity is optimal. When domestic households initially own the entire equity stock, the optimal policy is to shut down all equity flows. This is because any sales of equity result in lower allocative efficiency and financial losses, as investors buy the equity at discounted prices. When foreign investors already hold some equity, there is a benefit from buying back the equity claims from foreign investors, as this leads to a less distorted economy in the future. However, as households increase their holdings of domestic equity, they purchase back the equity at a higher price and suffer a financial loss. The government thus trades off closing the output gap against capital gains accruing to foreigners. At the optimum, the government induces households to repurchase a limited share of the equity held by foreigners.

In an extension of the baseline model, we incorporate an exogenous limit on external borrowing. We consider a scenario in which this limit unexpectedly tightens, interpreting it as a reluctance of foreign investors to continue holding the external debt of the SOE (i.e., a sudden stop). We show that, in this setting, the SOE finds it optimal to relax restrictions on foreign purchases of domestic equity relative to the baseline, which in our framework depresses equity prices. Interestingly, prices decline not because of inherent differences in valuations between domestic and foreign investors, but due to distortionary incentives induced by equity inflows in future exchange rate policy.

In another extension, we incorporate non-tradable goods into the baseline model. In this environment, the central bank's incentive to deviate from the natural allocation may depend on the sectoral composition of foreign equity ownership. As in the baseline model, a nominal exchange rate appreciation raises real wages and lowers profits in the tradable sector. However, when tradable and non-tradable goods are complements—which is the empirically relevant case (Mendoza, 1995; Stockman and Tesar, 1995; Gonzalez-Rozada et al., 2003; Lorenzo et al., 2005)—the resulting increase in the relative price of non-tradables may raise profits in the non-tradable sector. The central bank may then have incentives to over-appreciate or over-depreciate the exchange rate depending on whether foreign investors predominantly hold equity in the tradable or non-tradable sector. Moreover, an appropriately balanced composition of foreign equity ownership between the sectors can eliminate the distortionary incentives in future exchange rate policy, despite the presence of foreign ownership. This result has important policy implications for the design of controls on equity inflows across firms in the tradable and non-tradable sectors.

Related literature. Our paper belongs to the literature on monetary policy in open economies. As in closed-economy settings, a central theme in this literature is macroeconomic stabilization: when prices are rigid, monetary policy can help align relative prices and achieve the natural allocation (e.g., Friedman, 1953; Mundell, 1960; Schmitt-Grohé and Uribe, 2001). There are also studies that examine channels by which a departure from the natural allocation may be optimal in environments where trade in financial assets is restricted to bonds. One strand of this literature is concerned with terms of trade manipulation. When countries have market power in the tradable goods they produce, central banks can use monetary policy to tilt the terms of trade in their favor (e.g., Obstfeld and Rogoff, 1995 and Corsetti and Pesenti, 2001). Another strand of this literature considers the case when bonds are in domestic currency and analyzes how variations in exchange rates can generate valuation effects on the net foreign asset position. For instance, Fanelli (2023) shows how the optimal monetary policy under commitment balances macro-stabilization and insurance while Ottonello and Perez (2019), Du, Pflueger and Schreger (2020) and Engel and Park (2022) study the resulting inflationary bias when the government cannot commit. In contrast to these contributions, we consider a model with trade in equity claims and show how this gives rise to a novel source of time inconsistency with distinct implications for policy.

Our paper is also related to the literature on international portfolios. A central theme in this literature is understanding the diversification of equity portfolios (e.g., Engel and Matsumoto, 2009; Coeurdacier, Kollmann and Martin, 2010; Kollmann, 2006; Baxter and

Jermann, 1997; Devereux and Sutherland, 2006, Tille and Van Wincoop, 2010; Heathcote and Perri, 2013). In contrast to the existing literature, our paper takes a normative perspective and uncovers an aggregate welfare cost from such diversification. In addition, most of these studies are in the context of real models, thus abstracting from the role of monetary policy. A notable exception is Engel and Matsumoto (2009), who show how a portfolio of home and foreign equities and a forward position in foreign exchange can achieve the complete-market allocation to first order. However, they do not consider optimal monetary policy.[1]

Our work also relates to recent studies examining the implications of the rise in cross-country asset ownership. Atkeson, Heathcote and Perri (2022) document that the U.S. "exorbitant privilege" from valuation effects largely disappeared after 2007. They show the deterioration is due to the surge in U.S. corporate equity values, driven by a rise in markups. Quadrini and Ríos-Rull (2024) analyze optimal taxation in a setting where multinationals engage in profit shifting. In contrast, our focus is on studying the implications of the rise in cross-country asset ownership for monetary policy.[2]

Our paper is also related to the literature on monetary unions—in particular, those emphasizing how joining a monetary union can deal with time inconsistency problems in monetary policy. In Alesina and Barro (2002), joining a monetary union reduces the inflationary bias generated by the time inconsistency problem of monetary policy, as stressed by Barro and Gordon (1983). Fornaro (2022) presents an environment where the central bank has incentives to depreciate the exchange rate to reduce the real value of non-tradable collateral that foreign investors can seize from defaulting firms ex post. Because of this time inconsistency problem, he shows that joining a monetary union can help increase bond inflows. In contrast to these studies, which focus on bond positions and valuation effects, we show that international equity flows give rise to a distinct source of time inconsistency.

Our paper is also related to the literature on monetary policy with heterogeneous agents in closed economies. An important insight in this literature is that that the incidence of profit income across households shapes the transmission of monetary policy (Bilbiie, 2008 and Broer et al., 2020). Bilbiie (2008) also shows that, around an efficient allocation with equalized marginal utilities, limited asset-market participation changes the welfare tradeoff between inflation and output stabilization because variations in profits affect consumption dispersion. Bhandari, Evans, Golosov and Sargent (2021) and Acharya, Challe and Dogra

[1] Of course, models of optimal monetary policy with complete markets implicitly allow trade in equity claims. However, much of this literature (e.g., Clarida et al., 2002; Devereux and Engel, 2003; Obstfeld and Rogoff, 1995; Corsetti et al., 2010) studies optimal policy under commitment and/or limited international cooperation, settings in which the mechanism we highlight is absent.

[2] Empirical contributions include Lane and Milesi-Ferretti (2005); Coppola et al. (2021); Beck et al. (2024).

(2020) study Ramsey optimal monetary policy in a HANK economy with rich heterogeneity, with potentially unevenly distributed dividends, focusing on how monetary policy provide insurance against aggregate shocks.[3] Our mechanism is distinct: because the government cannot tax foreign shareholders and places no weight on their welfare, the marginal effect of policy on profits remitted abroad creates a first-order motive to depart from the natural allocation. Under sticky wages, we show that this motive induces the central bank to tighten monetary policy, thereby reducing output.

Our paper also relates to the literature on capital controls and capital flow management policies. This literature has developed a rationale for taxes on debt flows, based on pecuniary externalities (e.g., Bianchi, 2011) and aggregate demand externalities (e.g., Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016). See Bianchi and Lorenzoni (2021) for a review of the literature. In contrast, we provide a rationale for capital controls on equity flows.

Outline. The remainder of the paper is organized as follows. Section 2 introduces the environment. Section 3 studies the optimal exchange rate policy. Section 4 examines the role for capital controls. Section 5 analyzes extensions of the baseline framework. Section 6 concludes.

2 The Model

Consider a small open economy (SOE) with time indexed by $t \in \{0, 1, 2, \dots\}$. The SOE is composed of a continuum of identical households and firms, and a monetary authority. There is an international financial market with a risk-free bond that pays an exogenous real return. Our key departure from the standard framework is that foreign investors may also trade equity claims on domestic firms.

We restrict attention to a deterministic environment where nominal rigidities will be present at $t = 1$. In addition, our baseline model assumes that households do not face any borrowing limits, except for a no-Ponzi condition. This will allow us to highlight the key mechanism at play and characterize analytically the equilibrium. We next describe the decision problems of households, firms, and foreign investors, before analyzing the competitive equilibrium and optimal policies.

[3] Other normative contributions on optimal monetary policy with rich heterogeneity include Dávila and Schaab (2023) who consider a setup where profits are zero in equilibrium.

2.1 Firms

There is a unit mass of identical firms, which produce a single final good using labor, according to a strictly increasing and strictly concave production function $F(h)$. We assume that the production function satisfies $F(0) = 0, \lim_{h \to 0} F'(h) = \infty$, and $\lim_{h \to \infty} F'(h) = 0$.

We assume that the final good is tradable in the international market and the law of one price holds. Normalizing the foreign-currency price to one implies that the domestic-currency price of the good is $P_t = E_t$, where E_t denotes the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency.

The firms' problem consists of choosing employment to maximize profits:

$$\max_{h_t \geq 0} \{E_t F(h_t) - W_t h_t\} , \tag{1}$$

where W_t is the nominal wage in domestic currency.

Profit maximization implies that labor demand satisfies

$$F'(h_t) = w_t \ , \ \forall t \geq 0, \tag{2}$$

where $w_t \equiv W_t / E_t$ is the real wage. We denote by π_t the firms' profits in units of foreign currency.

2.2 Households

There is a continuum of identical households of measure one. Their preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - v(\ell_t)] ,$$

where $\beta \in (0, 1)$ is the discount factor, c_t denotes consumption, and ℓ_t denotes hours worked. The utility function $u(\cdot)$ is strictly increasing and strictly concave, and the disutility from working is strictly increasing and convex. We assume the usual Inada conditions: $\lim_{c \to 0} u'(c) = \infty$, $\lim_{c \to \infty} u'(c) = 0$, $\lim_{\ell \to 0} v'(\ell) = 0$, and $\lim_{\ell \to \infty} v'(\ell) = \infty$.

Households receive labor income, consume, and trade domestic firms' shares and bonds in international financial markets.[4] We let $\theta_t \geq 0$ and b_t respectively denote the holdings of firms' shares and bonds carried to period t (chosen at $t - 1$). Bonds are assumed to be

[4]Our results would be unchanged if households held foreign equities in addition to bonds. The reason is that a small open economy's policy does not affect outcomes in foreign markets.

one-period and denominated in foreign currency or, equivalently, units of consumption. Firms' shares entitle their owner to a fraction of the firm's profits. Given a constant amount of firms' shares, which we normalize to one, a household that owns θ_t units of equity receives $\pi_t\theta_t$ in period t.

Households' budget constraint is given by

$$c_t = w_t\ell_t + (\pi_t + q_t)\theta_t - q_t\theta_{t+1} + (1+r)b_t - b_{t+1}\,, \tag{3}$$

where q_t is the price of equity (in units of foreign currency) and r is the constant interest rate on bonds. We highlight that while r is exogenously given, the price of equity q_t is endogenously determined in equilibrium. Moreover, we index the price by t to capture that aggregate states will fluctuate over time.

For our baseline model, we assume households do not face borrowing limits, except for a no-Ponzi condition:

$$\lim_{t\to\infty} \frac{b_{t+1} + q_t\theta_{t+1}}{(1+r)^t} \geq 0\,.$$

Wage rigidity. In period $t = 1$, the nominal wage W_1 is rigid. We assume that at that period, labor is demand-determined, which means that households must supply the hours of labor that firms demand. We focus attention on the case where hours are evenly distributed across households. Using (2), we then have that the number of hours that each household works in equilibrium must be given by

$$\ell_1 = F'^{-1}\left(\frac{\overline{W}_1}{E_1}\right), \tag{4}$$

where $\overline{W}_1$ denotes the exogenous rigid wage.[5]

Household problem. At every period, households choose consumption, asset portfolios, and labor supply (except at period 1, when the number of labor hours is determined by (4)). We let $V_t(b_t, \theta_t)$ denote the value at the beginning of the period for a household with an initial individual asset portfolio (b_t, θ_t), where we index the value function by t because the aggregate portfolio and thus prices vary over time. The households' optimization problem can be represented recursively as follows:

$$V_t\left(b_t, \theta_t\right) = \max_{c_t, \ell_t, b_{t+1}, \theta_{t+1} \geq 0} \left\{u\left(c_t\right) - v\left(\ell_t\right) + \beta V_{t+1}\left(b_{t+1}, \theta_{t+1}\right)\right\}\,, \tag{5}$$

[5]Our modeling of period-1 wage rigidity is motivated by expositional simplicity and can be extended to allow for wage setting in period 0 with differentiated labor, subject to pricing frictions.

subject to the budget constraint

$$c_t = w_t \ell_t + (\pi_t + q_t)\theta_t - q_t \theta_{t+1} + (1+r)b_t - b_{t+1}\,,$$

the labor-supply schedule (4) for $t = 1$, and the no-Ponzi condition.

Household optimality requires that

$$u'(c_t)w_t = v'(\ell_t)\,,$$

for all $t \neq 1$. It also requires the Euler equations

$$u'(c_t) = \beta(1+r)u'(c_{t+1})\,, \tag{6}$$

$$q_t\, u'(c_t) \geq \beta(\pi_{t+1} + q_{t+1})u'(c_{t+1})\,, \tag{7}$$

for all $t \geq 0$, where the second condition holds with equality if $\theta_{t+1} > 0$.

2.3 Foreign investors

The key departure from a standard framework is that foreign investors can buy and sell equity of domestic firms. We assume that foreign investors have deep pockets—that is, they face no binding wealth or borrowing constraints—and that they face a no-short-selling constraint in domestic equity, so their holdings must be non-negative, $\theta^*_{t+1} \geq 0$. It follows from investors' optimality and (6)-(7) that for the market of equity to clear, it must be that the return on bonds equals the return on firms' shares.[6] That is,

$$1 + r = \frac{\pi_{t+1} + q_{t+1}}{q_t}. \tag{8}$$

Equation (8) implies that both households and foreign investors are indifferent about the relative holdings of equity and bonds in their portfolios.

2.4 Competitive equilibrium

We can now define a competitive equilibrium for a given exchange rate policy. We restrict attention to a symmetric equilibrium, in which all of the households choose the same portfolios.

[6] Foreign investors' portfolio condition is $(1+r)q_t \geq (\pi_{t+1} + q_{t+1})$, with equality if $\theta^*_{t+1} > 0$. Likewise, (6)-(7) yield $(1+r)q_t \geq (\pi_{t+1} + q_{t+1})$ with equality if $\theta_{t+1} > 0$. For $\theta_{t+1} + \theta^*_{t+1} = 1$, (8) must hold.

Definition 1. Given an initial portfolio (b_0, θ_0), a world real interest rate r, a rigid wage $\overline{W}_1$, and an exchange rate policy, $\{E_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is a sequence of allocations $\{c_t, h_t\}_{t=0}^{\infty}$, portfolios $\{b_{t+1}, \theta_{t+1}, \theta_{t+1}^*\}_{t=0}^{\infty}$, profits π_t, and prices $\{W_t, q_t\}_{t=0}^{\infty}$ such that:

(i) Households optimize. That is, consumption, labor, and the portfolios solve problem (5).

(ii) Firms choose employment optimally. That is, (2) holds.

(iii) The price of firms' shares satisfies condition (8) and $\theta_{t+1} + \theta_{t+1}^* = 1$.

Profits, GNI, and balance-of-payments accounting. A key equilibrium variable in our analysis is firms' profits. Combining (1) and (2), we get that equilibrium profits are a function of aggregate employment only:

$$\pi_t = F(h_t) - F'(h_t)h_t \equiv \Pi(h_t),$$

where the profit function $\Pi(\cdot)$ satisfies $\Pi(0) = 0$ and $\Pi'(h) = -F''(h)h > 0$ for $h > 0$.

Following standard balance-of-payments accounting, we define gross national income (GNI), the current account (CA), and the net foreign asset position (NFA) as

$$\begin{aligned} GNI_t &\equiv F(h_t) + rb_t - \theta_t^*\Pi(h_t), \\ CA_t &\equiv GNI_t - c_t, \\ NFA_t &\equiv b_t - q_t\theta_t^*. \end{aligned}$$

GNI is the income accruing to domestic residents: $F(h_t)$ is domestic output, $\theta_t^*\Pi(h_t)$ is profit income paid out to foreign shareholders, and rb_t is interest received on the country's net foreign bond position.[7] The CA measures the economy's net saving vis-a-vis the rest of the world: income accruing to residents minus domestic absorption. The NFA position is the value of foreign assets b_t net of the market value $q_t\theta_t^*$ of domestic equity held by foreigners, which represents a liability for the small open economy.

[7] Notice that we can write the GNI equivalently as

$$GNI_t = \theta_t F(h_t) + (1 - \theta_t)F'(h_t)h_t + rb_t,$$

an average of total output and labor income (weighted by the fraction of equity owned by domestic households) plus interest on foreign bond holdings.

Using the household budget constraint, market clearing for equity claims $\theta_t^* = 1 - \theta_t$, and the expression for GNI yields the balance-of-payments condition: a current-account deficit must be financed either by reducing bond holdings b_t or by issuing additional domestic equity to foreign investors (raising θ_t^*):

$$CA_t = (b_{t+1} - b_t) - q_t(\theta_{t+1}^* - \theta_t^*).$$

Given this expression, and our definition of the NFA, we can express its evolution as

$$NFA_{t+1} - NFA_t = CA_t - \theta_{t+1}^*(q_{t+1} - q_t).$$

That is, the change in the NFA position equals the current account plus a valuation term $-\theta_{t+1}^*(q_{t+1} - q_t)$, which captures capital gains and losses on the outstanding stock of equity claims held by foreign investors.

3 International Equity Flows and Exchange Rate Policy

In this section, we characterize equilibria and analyze the implications of international equity flows for the optimal exchange rate policy. To streamline the analysis, we assume $\beta(1+r) = 1$, so the real interest rate equals the households' discount rate and consumption is constant.

Road map. We proceed with our analysis as follows: First, we characterize the steady-state equilibrium for $t \geq 2$. Second, we analyze the competitive equilibrium at $t = 1$ and the optimal exchange rate policy. Finally, we study the scope for capital controls in period $t = 0$.

3.1 Steady-state equilibrium for $t \geq 2$

The assumption that nominal wages are flexible after $t \geq 2$ ensures monetary neutrality in this continuation equilibrium. Moreover, given that $\beta(1 + r) = 1$, we have that all allocations are constant. We use the variables c and h to denote steady-state values. Lemma 1 characterizes this continuation equilibrium for any given exchange rate policy and for any initial portfolio of assets and liabilities that characterizes the economy's NFA position (b_2, θ_2^*). We restrict attention to continuation equilibria that satisfy the following no-bubble condition for the equity price:

$$\lim_{t\to\infty} \frac{q_{t+1}}{(1+r)^t} = 0\,. \tag{9}$$

Lemma 1 (Steady-state equilibrium). *Given an initial portfolio* (b_2, θ_2^*)*, the continuation equilibrium for* $t \geq 2$ *features constant consumption and labor. In particular, the steady-state values,* h *and* c*, are given by*

$$\frac{v'(h)}{u'(c)} = F'(h)\,, \tag{10}$$

$$c = F(h) + rb_2 - \Pi(h)\theta_2^*\,, \tag{11}$$

and the sequence of portfolios $\left\{b_{t+1}, \theta_{t+1}^*\right\}_{t\geq 2}$ *and the steady-state price of equity,* q*, satisfy*

$$b_{t+1} - q\,\theta_{t+1}^* = b_2 - q\,\theta_2^* \tag{12}$$

and

$$q = \frac{F(h) - F'(h)h}{r}. \tag{13}$$

Proof. In Appendix A.1. □

Condition (10) is the standard condition equating the marginal rate of substitution between consumption and labor to the marginal product. For the analysis that follows, it will be useful to define $h = \mathcal{H}(c)$, as the level of employment consistent with (10), where $\mathcal{H}'(\cdot) < 0$, which reflects the standard wealth effect on labor supply.[8]

Condition (11) says that in steady state—when the current account is balanced—consumption equals gross national income (GNI). Condition (12) states that any portfolio of bonds and domestic equity that keeps the NFA constant is consistent with equilibrium, reflecting that households are indifferent across portfolios that deliver the same level of consumption. Condition (13) states the asset price equals the present discounted value of profits.

3.2 Continuation equilibrium at period $t = 1$

In this section, we characterize the equilibrium starting from $t = 1$, given an initial portfolio (b_1, θ_1^*). We use the employment function $\mathcal{H}(\cdot)$, defined above, in the following lemma, which characterizes the equilibrium outcome in period 1 for any given exchange rate E_1 and initial

[8] To see that $\mathcal{H}'(c) < 0$, we differentiate $v'(\mathcal{H}(c)) = F'(\mathcal{H}(c))u'(c)$ and obtain

$$\mathcal{H}'(c) = \frac{u''(c)F'(\mathcal{H}(c))}{v''(\mathcal{H}(c)) - u'(c)F''(\mathcal{H}(c))} < 0,$$

where the inequality follows from $u'' < 0, F'' < 0, v'' > 0$.

portfolio (b_1, θ_1^*).

Lemma 2 (Equilibrium for $t \geq 1$)**.** *Given an initial portfolio (b_1, θ_1^*), a rigid wage $\overline{W}_1$, an exchange rate E_1, the equilibrium for $t \geq 1$ is such that*

i) h_1 is given by

$$\frac{\overline{W}_1}{E_1} = F'(h_1); \tag{14}$$

ii) $h_t = \mathcal{H}(c)$ for $t > 1$, and $c_t = c$ for $t \geq 1$, with c given by

$$c = \frac{r}{1+r}\Big[F(h_1) + rb_1 - \Pi(h_1)\theta_1^*\Big] + \frac{1}{1+r}\Big[F\big(\mathcal{H}(c)\big) + rb_1 - \Pi(\mathcal{H}(c))\theta_1^*\Big]; \tag{15}$$

iii) $q_t = q$, with q given by (13), and the sequence of portfolios is such that

$$(b_2 - q\,\theta_2^*) \;=\; (b_1 - q\,\theta_1^*) + \frac{F(h_1) - \Pi(h_1)\theta_1^* - \big[F\left(\mathcal{H}(c)\right) - \Pi\left(\mathcal{H}(c)\right)\theta_1^*\big]}{1+r} \tag{16}$$

and $\left\{b_{t+1}, \theta_{t+1}^*\right\}_{t\geq 2}$ satisfy (12).

Proof. In Appendix A.2. □

Condition (14) captures that when the nominal wage is preset, the exchange rate pins down the real wage, and employment is determined by firms' demand at that real wage. Condition (15) reflects that consumption must be constant over time, given that $\beta(1+r) = 1$ and that preferences between consumption and labor are separable. The expression for consumption follows from the intertemporal budget constraint: the two terms are the annuity value of period-1 GNI and of future GNI, respectively. Recall that returns on bonds and equity are equal in equilibrium, which implies that households are indifferent across any portfolio of bonds and domestic equity that supports the equilibrium level of consumption.[9] Condition (16) describes the evolution of the NFA. Notice that, because in general period-1 GNI differs from future GNI, the current account in period 1 is not necessarily balanced; hence, the NFA can vary between periods 1 and 2.

[9]Moreover, this implies that future GNI can be expressed as $F(\mathcal{H}(c)) + rNFA_1$.

3.3 Optimal exchange rate policy

We now study the problem of a central bank that chooses monetary policy in period 1 to maximize households' welfare in the competitive equilibrium. Notice that by setting the period-1 exchange rate, the central bank can effectively choose labor in period 1. Using Lemma 2, we can formulate this problem as choosing a period-1 level of employment h_1, a constant level of consumption c, and an associated steady-state level of employment $\mathcal{H}(c)$, subject to a country budget constraint. That is, the optimal policy problem is

$$V_1(b_1, \theta_1^*) = \max_{c, h_1} \left\{ u(c) - v(h_1) + \frac{u(c) - v\left(\mathcal{H}(c)\right)}{r} \right\}, \tag{17}$$

subject to:

$$\frac{1+r}{r} c = F(h_1) + rb_1 - \Pi(h_1)\theta_1^* + \frac{F\left(\mathcal{H}(c)\right) + rb_1 - \Pi\left(\mathcal{H}(c)\right)\theta_1^*}{r}. \tag{18}$$

Denoting by $\lambda > 0$ the Lagrange multiplier on the intertemporal country budget constraint (18), we have the following optimality condition with respect to period-1 employment h_1:

$$v'(h_1) = \lambda \left[F'(h_1) - \Pi'(h_1)\theta_1^*\right]. \tag{19}$$

Condition (19) equates the marginal disutility of working an additional unit of time in the period to the marginal benefits, which are given by the additional period-1 resources times the marginal benefit of relaxing the country budget constraint. Crucially, the change in period-1 resources from a marginal increase in employment is the increase in *output net of the rise in profits*—that is, $F'(h_1) - \theta_1^*\Pi'(h_1)$. The latter term captures the additional profits accruing to foreign shareholders, and their effect on domestic resources is

$$-\theta_1^*\Pi'(h_1) = \theta_1^* F''(h_1)h_1 \leq 0,$$

where the inequality is strict whenever $\theta_1^* > 0$ because the production function is strictly concave: $F''(h) < 0$. That is, higher employment raises output but lowers wages—as the central bank needs to depreciate the exchange rate to stimulate labor demand—and thus increases firms' profits. Because a share θ_1^* of firms' profits accrue abroad, a more stimulative policy generates an increase in the effective liabilities that the SOE faces. This is why stimulating employment carries a higher marginal cost than in the case without foreign ownership.

The optimality condition with respect to consumption c yields

$$u'(c) + \frac{1}{1+r}\Big[-v'\big(\mathcal{H}(c)\big) + \lambda\big(F'\big(\mathcal{H}(c)\big) - \Pi'\big(\mathcal{H}(c)\big)\theta_1^*\big)\Big]\mathcal{H}'(c) = \lambda. \tag{20}$$

Equation (20) states that the marginal benefit of increasing consumption must equal the shadow cost of the additional resources required to finance it. The first term on the left-hand side is the direct marginal utility of an additional permanent unit of consumption. The second term captures the indirect effect of higher consumption on welfare through its impact on steady-state employment: because $\mathcal{H}'(c) < 0$, an increase in c reduces future hours, which in turn lowers both the disutility from working $v'\big(\mathcal{H}(c)\big)$ and the annuity value of future output and profits $F'\left(\mathcal{H}(c)\right) - \Pi'\left(\mathcal{H}(c)\right)\theta_1^*$.

Combining optimality conditions (19) and (20), we obtain

$$\underbrace{F'(h_1)u'(c) - v'(h_1)}_{\text{Labor wedge}} = \frac{\Pi'(h_1)u'(c) - \frac{1}{1+r}\Big[\Pi'(h_1)\, v'\big(\mathcal{H}(c)\big) - \Pi'(\mathcal{H}(c))\, v'(h_1)\Big]\,\mathcal{H}'(c)}{1 - \frac{1}{1+r}F'(\mathcal{H}(c))\mathcal{H}'(c)}\theta_1^*. \tag{21}$$

The left-hand side of (21) represents the labor wedge. When foreign investors do not own equity claims on domestic firms, $\theta_1^* = 0$, the right-hand side of (21) is zero, and the central bank sets the wedge to zero, which replicates the flexible-wage (natural) allocation. By contrast, when $\theta_1^* > 0$, the right-hand side of (21) is generally different from zero, so the optimal policy features a non-zero labor wedge, and the central bank finds it optimal to depart from the natural allocation. The intuition is that the central bank uses the exchange rate to compress profits accruing to foreign shareholders, thereby reducing the value of the economy's foreign liabilities. As discussed above, a reduction in period-1 employment h_1 reduces contemporaneous profits, which induces the central bank to generate a positive labor wedge, as shown by the first term of the numerator on the right side. However, at the same time, a lower h_1 reduces period-1 total income and the net foreign asset position, which generates a negative wealth effect that induces households to supply more labor in steady state, raising long-run profits. In an economy with GNI exhibiting positive but decreasing marginal returns to labor hours, for instance, this indirect effect is dominated by the contemporaneous benefit of higher wages in period 1. This implies that the right-hand side of (21) is positive and that the optimal labor wedge is strictly positive whenever $\theta_1^* > 0$—as shown in Proposition 1.

Assumption 1 (GNI with decreasing marginal returns). *Production function $F(\cdot)$ is such that $GNI(h) \equiv F(h) + rb - \theta^*\Pi(h)$ satisfies:*

$$GNI'(h) > 0 \text{ and } GNI''(h) < 0, \text{ for any given } \theta^* \in [0,1]. \tag{22}$$

Let the labor wedge induced under optimal exchange rate policy be given by

$$\zeta_1(b_1, \theta_1^*) \equiv u'(c(b_1, \theta_1^*))F'(h_1(b_1, \theta_1^*)) - v'(h_1(b_1, \theta_1^*)),$$

where consumption $c(b_1, \theta_1^*)$ and period-1 employment $h_1(b_1, \theta_1^*)$ are jointly determined by (18) and (21), given international portfolio (b_1, θ_1^*).

Proposition 1 (Positive labor wedge). *Suppose that Assumption 1 holds. Then, $\zeta_1(b_1, \theta_1^*) \geq 0$, with strict inequality if $\theta_1^* > 0$.*

Proof. In Appendix A.3 □

Comparative statics. Next, we study how the labor wedge responds to marginal changes in the share of equity owned by foreigners. Naturally, everything else fixed, changes in foreign equity ownership affect the wealth of the SOE and, through a resulting wealth effect, the labor wedge. To control for these wealth effects, we restrict attention to perturbations to the overall international portfolio that in the absence of changes in the price of domestic equities, keep the NFA fixed. Formally, let $q_0(b_1, \theta_1^*)$ denote the equilibrium price of domestic equities in period 0 given international portfolio (b_1, θ_1^*). This price is given by

$$q_0(b_1, \theta_1^*) = \frac{1}{1+r}\left[\Pi(h_1(b_1, \theta_1^*)) + \frac{1}{r}\Pi(\mathcal{H}(c(b_1, \theta_1^*)))\right]. \tag{23}$$

We consider perturbations of the portfolios such that

$$\hat{\theta}_1^* = \theta_1^* + \hat{\varepsilon} \text{ and } \hat{b}_1 = b_1 + q_0(b_1, \theta_1^*)\,\hat{\varepsilon},$$

where $\hat{\varepsilon} > 0$ is positive, but arbitrarily small.[10] If wealth effects on labor supply exhibit decreasing marginal returns—and Assumption 1 holds—then the labor wedge is increasing in these perturbations, as shown in Proposition 2.

[10]Note that $NFA_1 = \hat{b}_1 - q_0(b_1, \theta_1^*)\,\hat{\theta}_1^* = b_1 - q_0(b_1, \theta_1^*)\,\theta_1^*$ for any $\hat{\varepsilon} > 0$, which implies that the NFA would remain constant if equity was evaluated at the prevailing price before the perturbation. Importantly, in these perturbations, we restrict attention to a case with $\theta_1^* < 1$.

Assumption 2 (Wealth effects with decreasing marginal returns)**.** *Functions $\{F(\cdot), u(\cdot), v(\cdot)\}$ are such that $\mathcal{H}(c)$ satisfies:*

$$\mathcal{H}''(c) = \frac{F'\big(\mathcal{H}(c)\big)u'''(c) + 2F''\big(\mathcal{H}(c)\big)u''(c)\mathcal{H}'(c) - \big[v'''\big(\mathcal{H}(c)\big) - F'''\big(\mathcal{H}(c)\big)u'(c)\big]\left(\mathcal{H}'(c)\right)^2}{v''\big(\mathcal{H}(c)\big) - F''\big(\mathcal{H}(c)\big)u'(c)} > 0. \tag{24}$$

Proposition 2 (Increasing labor wedge)**.** *Suppose that both Assumption 1 and Assumption 2 hold. Then, if $\theta_1^* < 1$, the following holds:*

$$\left.\frac{dh_1(\hat{b}_1, \hat{\theta}_1^*)}{d\hat{\varepsilon}}\right|_{\hat{\varepsilon}=0} < 0, \quad \left.\frac{dc(\hat{b}_1, \hat{\theta}_1^*)}{d\hat{\varepsilon}}\right|_{\hat{\varepsilon}=0} < 0, \quad \left.\frac{d\zeta_1(\hat{b}_1, \hat{\theta}_1^*)}{d\hat{\varepsilon}}\right|_{\hat{\varepsilon}=0} > 0\,.$$

Proof. In Appendix A.4 □

This proposition shows that an increase in foreign equity leads the central bank to induce lower employment and consumption and a higher labor wedge. That is, if wealth is held constant, higher holdings of domestic equity claims by foreign investors lead the central bank to follow a tighter monetary policy to reduce payments to foreign investors. We note that Assumption 1 and Assumption 2 are sufficient to guarantee the result, but not necessary.

3.4 Quantitative results

In this section, we provide a simple quantification to assess the role of international equity flows in shaping the optimal exchange rate policy and the welfare implications. To do so, we parametrize $\{u(\cdot), v(\cdot), F(\cdot)\}$ with isoelastic functional forms, as follows:

$$u(c) = \frac{1}{1-\frac{1}{\sigma}}c^{1-\frac{1}{\sigma}} \qquad v(\ell) = \chi\frac{1}{1+\psi}\ell^{1+\psi}\,, \quad F(h) = h^{\alpha}\,,$$

with $\sigma > 0$, $\chi > 0$, $\psi > 0$ and $\alpha \in (0,1)$. Under these functional forms,

$$GNI(h) = \big[1 - (1-\alpha)\theta^*\big]h^{\alpha} + rb \;\; \text{and} \;\; \mathcal{H}(c) = \left(\frac{\alpha}{\chi}c^{-1/\sigma}\right)^{\frac{1}{1+\psi-\alpha}}\,,$$

which implies that both Assumption 1 and Assumption 2 hold.[11]

The time period is a year. We set the intertemporal elasticity of substitution to $\sigma = 1$, the inverse labor supply elasticity to $\psi = 1$, the labor share to $\alpha = 0.65$, and the interest rate to $r = 0.04$. These values are standard in the literature. We also normalize the disutility from labor χ so that output equals 1 under flexible prices.

We set the initial NFA to -60% of output, roughly the U.S. value in 2025. A reference value for θ_1^*, following Figure 1, is $\theta_1^* = 0.5$. In the simulations below, we consider values for $\theta_1^* \in [0, 1]$ and adjust the initial bond holdings so that the net foreign asset position remains constant in the flexible price allocation. In particular, we vary b_1 so that $b_1 - \tilde{q}\,\theta_1^* = -60\%$, where $\tilde{q}$ denotes the price of equity under flexible wages.

Figure 2 presents the results for the optimal exchange rate policy at $t = 1$. The top panels plot employment, the labor wedge, and profits relative to the natural allocation; the bottom panels display the exchange rate, consumption, and welfare gains. All variables are expressed as percentage deviations from an economy without foreign equity ownership (or, equivalently, flexible prices).[12]

When $\theta_1^* = 0$, the optimal policy sets the labor wedge to zero (panel b), reproducing the flexible-wage allocation. As foreign holdings increase, the central bank appreciates the exchange rate (panel e), and this leads to lower employment (panel a), profits (panel c), and consumption (panel d). We can see that the deviations from the flexible-price allocation are sizable. For $\theta_1^* = 0.5$, the monetary authority appreciates the exchange rate by 4%, employment falls by about 10%, and consumption declines by 0.15% relative to the case without foreign equity ownership.

Panel (f) reports the consumption-equivalent welfare gains from adopting the optimal exchange rate policy relative to a benchmark that maintains a zero labor wedge (i.e., the natural allocation). The figure indicates substantial gains from departing from the natural allocation. In the limiting case in which foreign investors own 100% of firms, households would be willing to forgo 1.75% of period-1 consumption to switch to the optimal policy.

[11] Note that the factor $(1-\alpha)\theta^*$ captures the wedge between domestic production and domestic income induced by foreign equity ownership. Because the monetary authority internalizes, when choosing employment, that a portion of firms' profits goes to foreign equity owners, it behaves as if the economy's effective productivity were lower.

[12] We normalize the labor wedge by $u'(c)F'(h)$ so that it can be interpreted as a labor income tax.

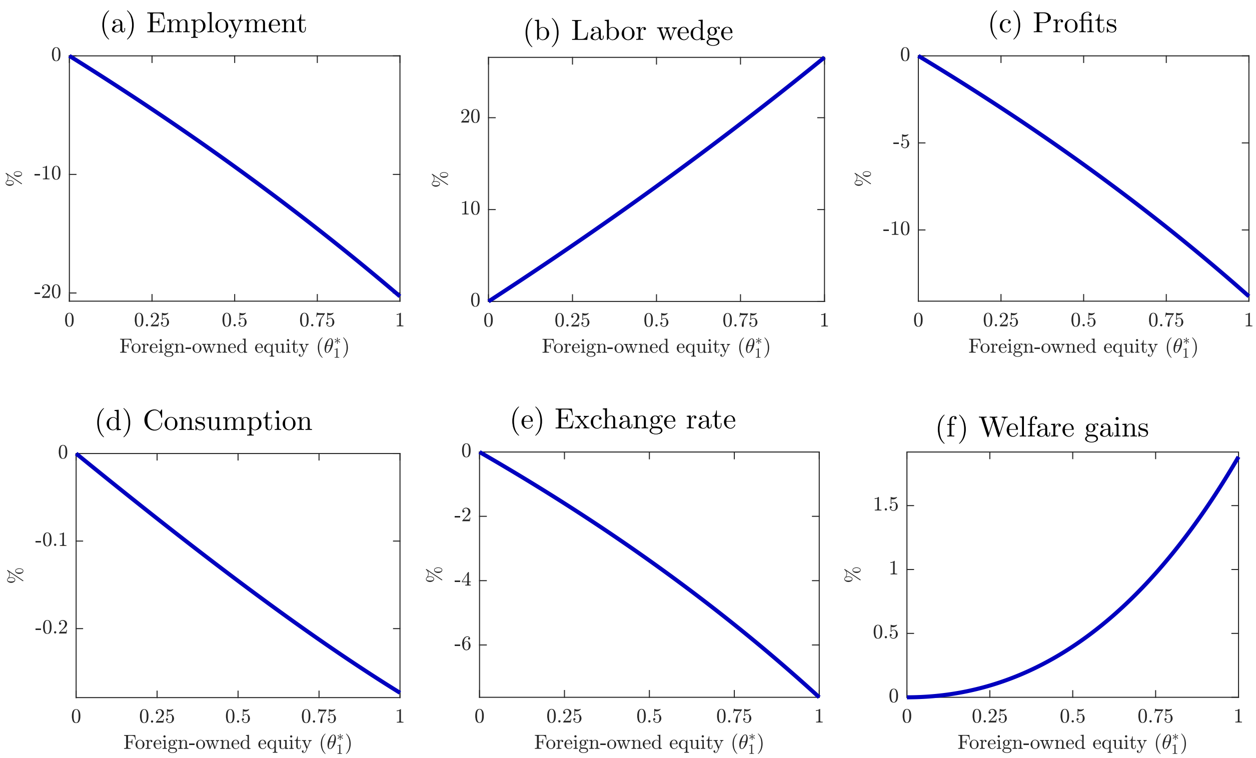

Figure 2: Equilibrium under optimal exchange rate policy

Notes: The figure plots the continuation equilibrium at period $t = 1$ under the optimal exchange rate as a function of the foreign-owned fraction of domestic firms' shares.

Takeaway. This section highlighted the implications of foreign equity ownership for the determination of the optimal exchange rate. The key insight is that when foreigners own equity, the central bank has incentives to appreciate the exchange rate beyond the natural allocation to boost real wages and reduce firm profits.

4 International Equity Flows and Capital Controls

We now turn to the determination of the initial portfolio $\{b_1, \theta_1^*\}$ and explore the implications of foreign equity ownership for the desirability of capital controls.

4.1 Equilibria for $t \geq 0$

We study the equilibrium for $t \geq 0$ when it is expected that the central bank will follow at period 1 the optimal exchange rate policy $E_1(b_1, \theta_1^*)$. The lemma below shows that there is a continuum of equilibria with different allocations and welfare implications.

Lemma 3 (Equilibrium)**.** *Given* (b_0, θ_0^*) *and the period-1 optimal exchange rate policy* $E_1(b_1, \theta_1^*)$*, there exists a continuum of Pareto-ranked competitive equilibria indexed by* (b_1, θ_1^*)*, where the set of possible equilibrium values for* (b_1, θ_1^*) *is characterized by*

$$c(b_1, \theta_1^*) = F(\mathcal{H}(c(b_1, \theta_1^*))) - \Pi(\mathcal{H}(c(b_1, \theta_1^*)))\theta_0^* + q_0(b_1, \theta_1^*)\,(\theta_1^* - \theta_0^*) + (1+r)b_0 - b_1\,, \quad (25)$$

where $q_0(b_1, \theta_1^*)$ *is given by* (23)*.*

Proof. In Appendix A.5. □

As in the continuation equilibria in Section 3, households are indifferent across any portfolio of bonds and equity shares that implement the same level of permanent consumption. However, while the indeterminacy in the portfolio was inconsequential in the continuation equilibria, the indeterminacy at period 0 gives rise to multiple equilibria with different prices and allocations. This happens because the portfolio chosen in period 0 influences the period-1 optimal exchange rate, which in turn influences the period-0 equity price and the country's budget constraint (25). In particular, following Proposition 2, when households choose to sell more of their equity claims, the central bank chooses in period 1 a more appreciated exchange rate, and firm profits are lowered as a result. Since investors perfectly anticipate the central bank's decision, they are willing to buy the equity claims only at a lower price, which ultimately erodes the wealth of the SOE and tightens the country's budget constraint.

Figure 3 illustrates the multiplicity of equilibria for the isoelastic case, where we use the same parameter values as in subsection 3.4, and assume for simplicity that $\theta_0^* = 0$. Panels (a) and (b) respectively present the equity price and consumption in period 0 as a function of the equity claims that households choose to sell in equilibrium. Panel (c) presents the welfare losses—in terms of period-0 consumption—relative to the equilibrium in which households continue to hold all the equity. The worst possible equilibrium is the one in which households sell all equity, where the welfare losses are about 2%. In that case, as the figure illustrates, the equity price falls by about 0.4%, and permanent consumption falls by roughly 0.3%.

Time inconsistency and multiple equilibria. The multiplicity of Pareto-ranked equilibria reflects a coordination problem among domestic households. Individually, households are indifferent between holding equity and bonds, because both assets deliver the same return in equilibrium. Collectively, however, larger equity sales raise foreign ownership, which exacerbates the central bank's *ex post* incentive to compress profits. Anticipating lower future profits, foreign investors are willing to purchase domestic equity only at a lower price in

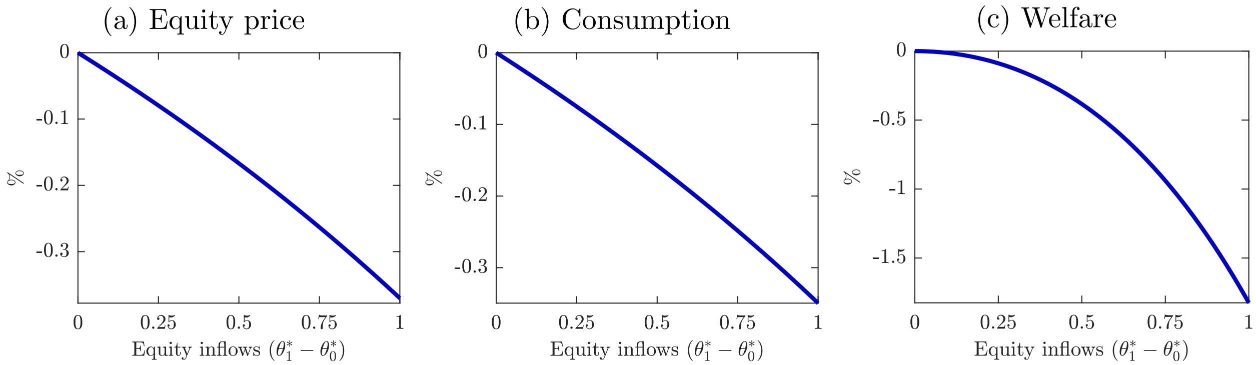

Figure 3: Equilibrium as a function of equity inflows

Notes: The figure plots the equilibrium outcome as a function of the possible equity inflows for a case in which the SOE initially owns all equity shares (i.e., $\theta_0^* = 0$).

period 0. This price decline reduces domestic wealth and lowers welfare. At the heart of the coordination failure is limited commitment: once domestic equity is held abroad, the central bank finds it optimal *ex post* to reduce profits, and this validates the lower equity valuation *ex ante*.

By contrast, if the central bank were instead able to commit, the competitive equilibrium would feature unique values of consumption and labor in all periods, regardless of the portfolio composition chosen by households in period 0. The proposition below formalizes this result, and shows additionally that if $\theta_0^* = 0$, the equilibrium corresponds to the natural allocation.

Proposition 3 (Equilibrium under commitment)**.** *Suppose that the central bank at $t = 0$ can commit to a period-1 exchange rate E_1. Then, given international portfolio (b_0, θ_0^*), the competitive equilibrium under the optimal exchange rate features unique values of consumption and labor in all periods. Moreover, if $\theta_0^* = 0$, the optimal period-1 exchange rate policy implements the natural allocation in period 1.*

Proof. In Appendix A.6. □

The coordination failure suggests that limiting foreign equity ownership could be desirable, a point we turn next.

4.2 Capital controls on international equity flows

We consider a benevolent domestic financial regulatory authority, which can set capital controls on international equity flows in period 0 to maximize the welfare of the households in the SOE. Crucially, the regulatory authority takes into account that monetary policy, in period 1, is set in a time-consistent manner. We assume that in period 0, the regulatory authority chooses the portfolio position directly on behalf of households and lets households choose consumption, labor, and bonds.[13]

The problem of the regulatory authority can be expressed as

$$V_0(b_0, \theta_0^*) = \max_{c_0, b_1, \theta_1^* \in [0,1]} \left\{ u(c_0) - v(\mathcal{H}(c_0)) + \frac{1}{1+r} V_1(b_1, \theta_1^*) \right\}, \tag{26}$$

subject to:

$$c_0 = F(\mathcal{H}(c_0)) - \Pi(\mathcal{H}(c_0))\theta_0^* + q_0(b_1, \theta_1^*)\left(\theta_1^* - \theta_0^*\right) + (1+r)b_0 - b_1\,, \tag{27}$$

$$c_0 = c(b_1, \theta_1^*)\,, \tag{28}$$

where the country's budget constraint (27) already incorporates that the price at which households sell depends on the endogenous portfolio. Equilibrium constraint (28) reflects that because households can choose their bond holdings b_1 in the initial period, for any policy $\theta_1^* = 1 - \theta_1$, perfect consumption smoothing must hold.

The proposition below characterizes the optimal restrictions on foreign ownership of domestic equity.

Proposition 4 (Positive capital controls)**.** *Suppose that both Assumption 1 and Assumption 2 hold. Then, the solution to problem* (26) *is such that*

$$\theta_1^* = \begin{cases} 0 & \text{if } \theta_0^* = 0, \\ \in [0, \theta_0^*) & \text{if } \theta_0^* > 0. \end{cases}$$

Proof. In Appendix A.7. □

Consider first the case with $\theta_0^* = 0$. When households initially own all of the equity shares, the regulatory authority restricts households from selling any equity claims. The reason is as follows. Keeping the same portfolio induces the monetary authority to implement the natural allocation. Moreover, any sales of equity claims would lead the monetary authority to depart from the natural allocation and reduce profits. However, because the reduction in profits is priced ex ante, this implies a financial loss for households. Any sales of equity claims, therefore, result in a distorted allocation and a financial loss.

[13] Without loss of generality, we assume that the regulatory authority does not intervene in future periods.

When $\theta_0^* > 0$, the regulatory authority finds it optimal to induce *equity outflows*. As in the case with $\theta_0^* = 0$, allowing households to sell domestic equity claims is detrimental because the economy ends up with more distorted allocations and with financial losses. Moreover, now there is a benefit from buying back the equity claims from foreign investors, as this leads to a higher allocative efficiency in period 1. However, as households buy more equity, this raises the price of their purchases, leading again to a financial loss. In other words, as households buy back more equity from foreign investors, the foreign investors effectively appropriate part of the higher allocative efficiency gains.[14]

5 Extensions

In this section, we extend the theory in two directions. First, we allow for binding financial constraints. Second, we incorporate a non-tradable good in the model.

5.1 The case with a binding debt limit

In our baseline model, the SOE is not subject to borrowing constraints. This implies that independently of any restriction imposed on equity inflows by the regulatory authority, households are able to smooth consumption by increasing external debt. Next, we consider a situation in which households may face limits to external borrowing.

We assume that in period 0 bond holdings b_1 must satisfy

$$b_1 \geq -\underline{b}\,,$$

where $\underline{b} > 0$ is an exogenous limit to external debt. We interpret this constraint as capturing the limited enforceability of international debt contracts. For simplicity, we assume that the borrowing constraint applies only at period 0.

[14] In our analysis, we have assumed that the regulatory authority controls only equity flows. If the regulatory authority also chooses households' bond position b_1 in addition to equity holdings, the consumption-smoothing constraint $c_0 = c(b_1, \theta_1^*)$ no longer applies. We can show that the optimal policy then leaves foreign equity holdings unchanged, $\theta_1^* = \theta_0^*$ (no equity repurchases or sales), and sets higher initial consumption, $c_0 > c(b_1, \theta_1^*)$. This higher c_0 generates a positive wealth effect that shifts period-0 labor supply inward, raising real wages $w_0 = F'(\mathcal{H}(c_0))$ and reducing firm profits $\Pi(\mathcal{H}(c_0))$. In other words, control of bond and equity flows allows the SOE to avoid any financial losses, but the period-1 allocation remains distorted as this allows the regulatory authority to extract resources from the initial investors.

The regulatory authority now faces a problem that can be expressed as

$$V_0(b_0, \theta_0^*) = \max_{c_0, b_1, \theta_1^* \in [0,1]} \left\{ u(c_0) - v(\mathcal{H}(c_0)) + \frac{1}{1+r} V_1(b_1, \theta_1^*) \right\}, \tag{29}$$

subject to:

$$c_0 = F(\mathcal{H}(c_0)) - \Pi(\mathcal{H}(c_0))\theta_0^* + q_0(b_1, \theta_1^*)\,(\theta_1^* - \theta_0^*) + (1+r)b_0 - b_1\,, \tag{30}$$

$$c_0 \le c(b_1, \theta_1^*)\,, \tag{31}$$

$$b_1 \ge -\underline{b}\,, \tag{32}$$

$$0 = (c_0 - c(b_1, \theta_1^*))\,(b_1 + \underline{b})\,, \tag{33}$$

where condition (31) allows consumption in period 0 to be below consumption in period 1, and where equilibrium constraint (33) specifies that if that is the case, the debt limit (i.e., condition (32)) must be binding. Constraint (33) is also needed because if the solution were such that $c_0 < c(b_1, \theta_1^*)$, households would rather deviate from it, by reducing b_1 to narrow the gap between c_0 and $c(b_1, \theta_1^*)$.

We restrict attention to a situation in which $b_0 < -\underline{b}$. This implies that foreign investors are no longer willing to hold the initial debt. We interpret this situation as a sudden stop.

Proposition 5 (Optimal control with binding debt limit)**.** *Suppose that both Assumption 1 and Assumption 2 hold. Then, if $\theta_0^* = 0$, $\theta_1^* > \theta_0^*$ is optimal. More generally, in a case with $\theta_0^* < 1$, if the solution to (29) is such that $c_0 < c\,(b_1, \theta_1^*)$, it must satisfy $\theta_1^* > \theta_0^*$.*

Proof. In Appendix A.8 □

If the SOE initially owns all the equity (i.e., $\theta_0^* = 0$), a sudden stop does not allow it to simultaneously retain the equity and achieve the consumption path of the natural allocation. This is because, in this case, reducing debt without selling equity shares forces the SOE to reduce consumption in period 0 below its level in period 1. The regulatory authority then finds it optimal to resolve this trade-off by selling a fraction of the equity shares to foreign investors. The reason is that not selling shares would require a relatively large contraction in period-0 consumption, which would significantly reduce welfare, as shown in Figure 4. By contrast, if $\theta_0^* > 0$, a sudden stop may not necessarily induce the SOE to sell equity shares. This is because, as shown in Proposition 4, absent the sudden stop, the optimal policy would actually be to repurchase equity shares. The regulatory authority therefore finds it optimal to sell equity shares only when the sudden stop is sufficiently severe: for instance, if the sudden stop is such that smoothing consumption perfectly between period 0 and period 1 is not feasible, then selling equity shares is desirable. In general, selling all the equity shares to foreign investors is never optimal.

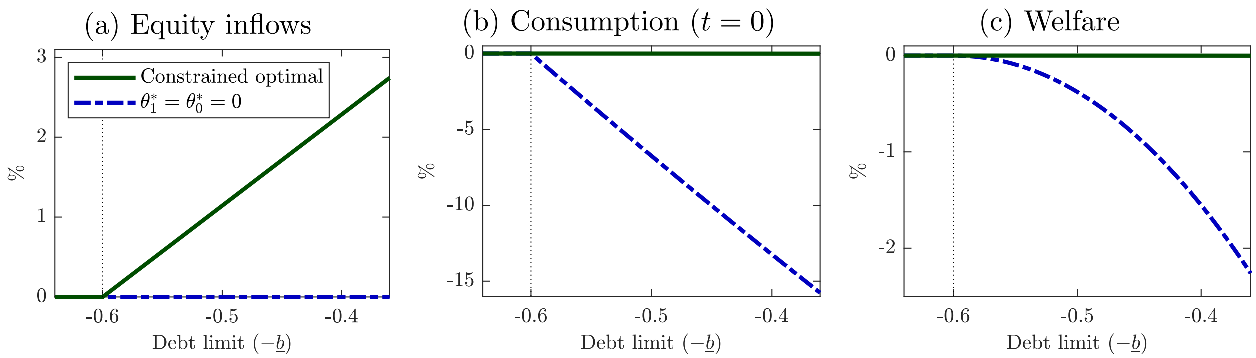

Figure 4: Equilibrium under constrained optimal capital control

Notes: The figure plots the equilibrium outcome under the constrained optimal capital control for a case in which the SOE initially owns all equity shares. The initial debt is $b_0 = -0.6$ (i.e., $NFA = -60\%$ of output). Values represent deviations from the natural allocation.

The key takeaway from this extension is that even in a situation where equity inflows are socially desirable, the regulatory authority still has incentives to curb the amount of inflows. Notice that the optimal equity inflows restriction helps support equity prices, which in turn reduces the SOE's need to sell domestic equity and mitigates a vicious cycle—usually referred to as "fire sales"—between drops in equity prices and the need for further equity sales. Importantly, in this environment, equity prices fall not because of inherent differences in valuations between domestic and foreign investors, but because of distortionary incentives induced by equity inflows in future exchange rate policy.

5.2 Model with tradable and non-tradable goods

Until now, we have considered an environment with a single final good, assumed to be tradable internationally. In this section, we extend the baseline model by introducing a non-tradable good, and analyze the implications for monetary policy of international equity flows to firms in both the tradable and non-tradable sectors.

Households. Preferences are given again by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(\ell_t\right)\right],$$

but now $c_t = \tilde{c}(c_t^T, c_t^N)$ is a linearly homogeneous aggregator of the tradable good, c_t^T, and the non-tradable good, c_t^N, and $\ell_t = \ell_t^T + \ell_t^N$ is the total number of hours households work in the tradable sector, ℓ_t^T, and in the non-tradable sector, ℓ_t^N. For convenience, we will use $U(c_t^T, c_t^N) \equiv u(\tilde{c}(c_t^T, c_t^N))$ to denote utility as a function of the two goods.

The households' budget constraint, again in units of foreign currency (or the tradable good), is given by

$$c_t^T + p_t^N c_t^N = w_t \ell_t + (\pi_t^T + q_t^T)\theta_t^T + (\pi_t^N + q_t^N)\theta_t^N - q_t^T \theta_{t+1}^T - q_t^N \theta_{t+1}^N + (1+r)b_t - b_{t+1} \,,$$

where q_t^T and q_t^N are the equity prices of firms in the tradable and the non-tradable sector, respectively, and p_t^N is the relative price of the non-tradable good.

In addition to the optimality conditions presented in Section 2, we now have a condition equating the marginal rate of substitution between the goods to their relative price,

$$\frac{U_N(c_t^T, c_t^N)}{U_T(c_t^T, c_t^N)} = p_t^N \,, \tag{34}$$

where U_T and U_N respectively denote the marginal utility with respect to the tradable and the non-tradable goods.

As in our baseline model, the nominal wage in units of domestic currency is fixed at a level $\overline{W}_1$ in period 1.[15] Thus, hours worked in each sector are determined by firms' labor demand.

Firms. The profits of firms in the tradable sector are now given by π_t^T, with

$$\pi_t^T \equiv \max_{h_t^T \geq 0} \left\{ F_T\left(h_t^T\right) - w_t h_t^T \right\}, \tag{35}$$

whereas those of firms in the non-tradable sector are given by π_t^N, with

$$\pi_t^N \equiv \max_{h_t^N \geq 0} \left\{ p_t^N F_N\left(h_t^N\right) - w_t h_t^N \right\} \,. \tag{36}$$

Note that the price of the non-tradable good, p_t^N, can differ from 1 since in equilibrium it is domestically determined.

Foreign investors. Lastly, foreign investors can hold equity shares of firms in either sector, but they cannot short-sell the shares. Put formally, $\theta_{t+1}^{*T} \geq 0$ and $\theta_{t+1}^{*N} \geq 0$. As in the model

[15]An analysis with sticky prices for non-tradables is available upon request.

presented in Section 2, for markets of equity shares to clear, it must be that the return on the shares equals the return on bonds. That is,

$$1+r=\frac{\pi_{t+1}^{j}+q_{t+1}^{j}}{q_{t}^{j}} \text{ for } j\in\{N,T\}. \tag{37}$$

Definition 2 (Competitive equilibrium with non-tradables)**.** Given initial conditions $(b_0, \theta_0^{*T}, \theta_0^{*N})$, a rigid wage $\overline{W}_1$, and an exchange rate policy, $\{E_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is a sequence of allocations $\{c_t^T, c_t^N, h_t^T, h_t^N\}_{t=0}^{\infty}$, portfolios $\{b_{t+1}, \theta_{t+1}^T, \theta_{t+1}^{*T}, \theta_{t+1}^N, \theta_{t+1}^{*N}\}_{t=0}^{\infty}$, profits $\{\pi_t^T, \pi_t^N\}_{t=0}^{\infty}$, and prices $\left\{w_t, p_t^N, q_t^T, q_t^N\right\}_{t=0}^{\infty}$ such that:

(i) Households and firms optimize.

(ii) The market for the non-tradable good domestically clears. That is, $c_t^N = F_N\left(h_t^N\right)$.

(iii) The prices of firms' shares satisfy (37) and $\theta_{t+1}^j + \theta_{t+1}^{*j} = 1$ for both $j \in \{N, T\}$.

As in Section 2, we assume $\beta(1+r) = 1$ to ensure stationarity in the continuation equilibria from period 2 onward.

Employment and Profits. Consider for simplicity the case with isoelastic functions. Formally:

$$U\left(c^T, c^N\right) = \frac{1}{1-\frac{1}{\sigma}}\left[\omega (c^T)^{1-\frac{1}{\gamma}} + (1-\omega)(c^N)^{1-\frac{1}{\gamma}}\right]^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\gamma}}},$$

$$v\left(\ell\right) = \chi\frac{1}{1+\psi}\ell^{1+\psi} \qquad F_j\left(h\right) = h^{\alpha_j},$$

where $\omega \in (0,1)$ is the utility weight of the tradable good relative to that of the non-tradable good, $\gamma > 0$ is the elasticity of substitution between the two goods, and the rest of the parameters are interpreted in the same manner as described in Section 3.4. In this case, from combining (34) and (36) together with $c_t^N = F_N\left(h_t^N\right)$, one gets that in equilibrium employment and profits in the non-tradable sector are related according to

$$\pi_t^N = (1-\alpha_N)\,\frac{1-\omega}{\omega}\left(\frac{F_N\left(h_t^N\right)}{c_t^T}\right)^{-1/\gamma} F_N\left(h_t^N\right).$$

Moreover, from (35), one obtains that employment and profits in the tradable sector are related as follows:

$$\pi_t^T = (1 - \alpha_T) F_T \left(h_t^T\right) \, .$$

These expressions show that in the tradable sector, employment and profits are always positively related, while in the non-tradable sector—all else equal—they are positively or inversely related depending on whether the goods are substitutes or complements. Specifically, if the goods are substitutes (i.e., $\gamma > 1$), the relationship is positive, whereas if they are complements (i.e., $\gamma < 1$) it is negative. This happens because the elasticity of substitution modulates the strength of the relationship between price and quantity consumed, making it more or less than inversely proportional depending on whether γ is above or below one. Opposite signs in the employment-profits relationship between the sectors critically matter for monetary policy. For instance, if foreign equity ownership is such that the responses to employment of foreign-owned profits in the tradable and in the non-tradable sector offset, then the central bank finds it optimal not to distort the allocation, despite the foreign ownership.

Optimal labor wedge. The proposition below formalizes this discussion by characterizing the set of international portfolios $\{b_1, \theta_1^{*T}, \theta_1^{*N}\}$ that removes the central bank's incentive to distort the allocation in period 1. Portfolios outside this set induce either a positive or a negative labor wedge. This result is important for period-0 policy because it identifies how the composition of tradable and non-tradable foreign equity holdings must be balanced to avoid future monetary distortion.

Proposition 6 (Zero optimal labor wedge)**.** *The set of international portfolios $\{b_1, \theta_1^{*T}, \theta_1^{*N}\}$ that induce a zero labor wedge in period 1 is characterized by*

$$\Psi_T \left(c^T\right) \, \theta_1^{*T} + \Psi_N \left(c^T\right) \, \theta_1^{*N} = 0 \, , \tag{38}$$

where $\Psi_j \left(c^T\right)$ is given by (A.51), and c^T solves (A.48).

Proof. In Appendix A.9. □

Figure 5 illustrates the set for the isoelastic case. We set the elasticity of substitution between the goods to $\gamma = 0.45$, which is consistent with empirical estimates by Stockman and Tesar (1995). The intensity of labor in the non-tradable sector is set equal to $\alpha^N = 0.75$ and that in the tradable sector is set equal to $\alpha^T = 0.25$. The former value is in line with Schmitt-Grohé and Uribe (2016) while the latter is chosen so that the aggregate labor share of output equals 0.65. The weight of the tradable good in the CES aggregator, i.e. $\omega = 0.18$, is chosen so that under flexible nominal wages, the share of tradable output in total output equals 20%, in line with data. Lastly, the rest of the parameters are determined as described in Section 3.4. Their values are given by $\sigma = 1$, $\chi = 0.015$, $\psi = 1$, and $r = 4\%$. The NFA is set equal to -60% of output. This value is kept fixed for all the international portfolios displayed in the figure.

The set of efficiency-inducing portfolios is plotted with the solid blue line. In the region below the line, the central bank finds it optimal to induce a positive labor wedge, whereas in the region above, it finds it optimal to induce a negative labor wedge. The slope of the line is approximately $\theta_1^{*N}/\theta_1^{*T} \simeq 6.4$. This implies that foreign investors must hold approximately six and a half units of firms' shares in the non-tradable sector per unit of firms' shares held in the tradable sector not to tempt the central bank to deviate from a zero labor wedge. Put equivalently, because the ratio of equity prices between the sectors is approximately $q_1^N/q_1^T \simeq 4/3$, the value of the former shares in their equity portfolio must be approximately $8.5 \simeq 4/3 * 6.4$ the value of the latter, or in terms of portfolio weights, the former must account for approximately 90% of the portfolio and the latter for approximately 10%. Overall, starting from a situation in which the SOE initially owns all the equity, the set of efficiency-inducing portfolios allows equity inflows to the SOE of up to approximately $\left((1/6.40)\, q_1^T + q_1^N\right) / \left(F_T\left(h_1^T\right) + p_1^N F_N\left(h_1^N\right)\right) \simeq 6.6$ times GDP without inducing distortionary incentives for the central bank.

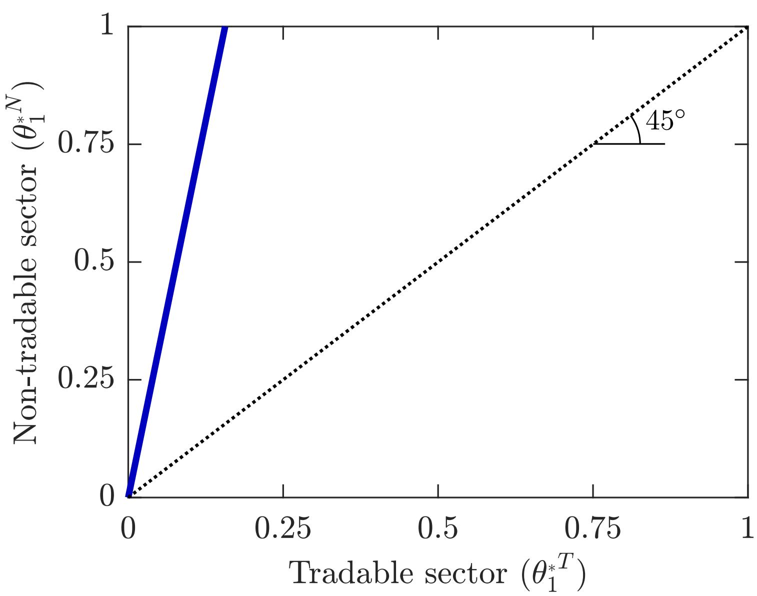

Figure 5: Set of efficiency-inducing international portfolios

Notes: The figure plots the set of international equity portfolios that eliminates the incentives of the central bank to distort the allocation when foreign investors own equity shares in period 1.

An important takeaway from this extension is that the composition of equity flows across sectors critically matters for monetary policy. In particular, a central bank may want to depreciate or appreciate the exchange rate depending on whether foreign investors predominantly hold firms' shares in sectors with a negative or a positive relationship between firm profits and employment. Moreover, a regulatory authority that wants to reduce the temptation of the central bank to erode investors' rents in the future does not necessarily have to restrict equity inflows. Rather, it must curb the inflows appropriately across sectors with opposite signs in the sensitivity of profits to employment.

6 Conclusion

In this paper, we have examined the implications of international equity flows for monetary policy in a small open economy. We develop a tractable New Keynesian framework in which foreign investors can hold domestic equity claims and the central bank sets the exchange rate under nominal wage rigidity. We show that when foreign investors own domestic equity, the central bank has an incentive to deviate from the natural allocation by appreciating the exchange rate, raising real wages, and compressing the profits that accrue to foreign investors. In effect, foreign ownership acts like a reduction in productivity: from the perspective of the domestic planner, a share of the output leaks in the form of dividends paid to foreigners, and the central bank responds by choosing lower employment than in the flexible-wage allocation.

Foreign ownership creates a novel source of time inconsistency for monetary policy. Anticipating lower returns, foreign investors purchase equity at discounted prices, which reduces domestic households' wealth and gives rise to multiple Pareto-ranked equilibria in which the bad equilibrium features larger equity inflows, depressed asset prices, and higher unemployment. We show that to mitigate this time inconsistency problem, it is optimal to manage equity flows. In particular, it is optimal to limit equity inflows ex ante, to reduce the temptation of the central bank to over-appreciate the exchange rate relative to the natural allocation ex post.

Taken together, our results highlight that the globalization of equity portfolios has important implications for monetary policy and for capital flow management. To highlight the key mechanisms at play, we have presented a parsimonious model without risk, frictions on financial intermediaries, or heterogeneity, and with a simple, temporary nominal rigidity. For future work, it would be valuable to extend the framework along these dimensions and to confront it with data, in order to further analyze the normative implications of foreign equity ownership.

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A Proofs

A.1 Proof of Lemma 1

Proof. Consider the problem (5). Together with the envelope conditions, the first-order conditions with respect to $c_t, \ell_t, b_{t+1}, \theta_{t+1}$ yield

$$w_t = \frac{v'(\ell_t)}{u'(c_t)}, \tag{A.1}$$

$$u'(c_t) = \beta(1+r)\,u'(c_{t+1}) \tag{A.2}$$

$$u'(c_t)q_t = \beta(\pi_{t+1} + q_{t+1})u'(c_{t+1}) \tag{A.3}$$

where the latter condition already incorporates that any $\theta_{t+1} \geq 0$ is optimal, a result that follows directly from (A.2) and (8). Using that $\beta(1+r) = 1$ we obtain from (A.2) that $c_t = c_{t+1} = c$ for all t. From (A.2),(A.3) and (8), it is also immediate that the portfolio between bonds and equity is undetermined.

Given $c_t = c$, equations (A.1) and (2) together with $\ell_t = h_t$, imply that $\ell_t = h_t = h$ and $w_t = w$ for all $t \geq 2$. We thus have that profits are constant as well and given by $\pi = F(h) - F'(h)h$. The expression (13) in the lemma then follows by iterating forward on (8) and using the no-bubble condition (9).

Iterating forward on the budget constraint (3) and using the no-Ponzi condition, we obtain

$$c = w\ell + \pi\theta_2 + rb_2,$$

Combining this expression with (2) and market clearing conditions $\ell_t = h_t$ and $\theta_t + \theta_t^* = 1$ we arrive to (11). Finally, replacing c in the budget constraint (3), we obtain (12).

□

A.2 Proof of Lemma 2

Proof. First, note that for $t \geq 2$, the economy is in a steady-state equilibrium. Therefore, we have labor, wages, profits, and prices for $t \geq 2$, as characterized in Lemma 1.

At $t = 1$, households solve

$$V_1(b_1, \theta_1) = \max_{c_1, b_2, \theta_2 \geq 0} \left\{ u(c_1) - v(\ell_1) + \frac{1}{1+r} V(b_2, \theta_2) \right\} \tag{A.4}$$

subject to:

$$c_1 + b_2 + q\,\theta_2 = \frac{\bar{W}_1}{E_1}\ell_1 + (1+r)b_1 + (\pi_1 + q)\,\theta_1 \;, \tag{A.5}$$

where we have used that $q_1 = q$ and that the value function is stationary for $t \geq 2$, a result that follows because in a steady-state equilibrium, wages, profits, and equity prices are constant. Note that we have also used that $\beta(1+r) = 1$.

The value for $t \geq 2$ is given by

$$V(b_2, \theta_2) = \frac{1+r}{r}\left[u(c) - v(\ell)\right] \;,$$

with $c = w\ell + \pi\theta_2 + rb_2$ and $w = v'(\ell)/u'(c)$.

From optimality with respect to b_2 and using the envelope condition, we obtain $c_1 = c$. Using that consumption, wages, and hours are constant and that q satisfies (8), we obtain by iterating forward on the budget constraint that

$$c = \frac{r}{1+r}\left(\frac{\bar{W}_1}{E_1}\ell_1 + \pi_1\theta_1\right) + \frac{1}{1+r}\left(w\ell + \pi\theta_1\right) + rb_1 \;.$$

Combining this expression with (2), market clearing conditions $\ell_t = h_t$ and $\theta_t + \theta_t^* = 1$, and mapping $h = \mathcal{H}(c)$, we arrive to (15). Finally, replacing c_1 together with (2), $\ell_t = h_t$, $\theta_t + \theta_t^* = 1$, and $h = \mathcal{H}(c)$ in the budget constraint (A.5), we obtain (16).

□

A.3 Proof of Proposition 1

Proof. Let $\mathcal{L}$ denote the Lagrangian associated with problem (17). The Lagrangian is:

$$\begin{aligned}\mathcal{L} = u(c) - v(h_1) + \frac{u(c) - v(\mathcal{H}(c))}{r} + \\ + \lambda\left[F(h_1) + rb_1 - \Pi(h_1)\theta_1^* + \frac{F(\mathcal{H}(c)) + rb_1 - \Pi(\mathcal{H}(c))\,\theta_1^*}{r} - \frac{1+r}{r}c\right].\end{aligned}$$

The derivatives of the Lagrangian with respect to the choice variables c and h_1 are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c} &= \frac{1+r}{r}u'(c) + \frac{1}{r}\Big[-v'\big(\mathcal{H}(c)\big) + \lambda\big(F'\big(\mathcal{H}(c)\big) - \Pi'\big(\mathcal{H}(c)\big)\theta_1^*\big)\Big]\mathcal{H}'(c) - \frac{1+r}{r}\lambda\,, \\
\frac{\partial \mathcal{L}}{\partial h_1} &= -v'(h_1) + \lambda\left[F'(h_1) - \Pi'(h_1)\theta_1^*\right]\,,
\end{aligned} \tag{A.6}$$

which are set to zero at an interior optimum, together with the resource constraint in (17). We work with (A.6) as a derivative below, since the argument turns on its sign away from the optimum.

Condition (22) implies that, given $\lambda > 0$, derivative $\partial\mathcal{L}/\partial h_1$ is strictly decreasing in h_1. Combining first-order condition $\partial\mathcal{L}/\partial c = 0$ with $v'(\mathcal{H}(c)) = F'(\mathcal{H}(c))u'(c)$, one gets:

$$\lambda = \left[1 + \frac{-\Pi'\big(\mathcal{H}(c)\big)\mathcal{H}'(c)}{(1+r) - \left[F'\big(\mathcal{H}(c)\big) - \Pi'\big(\mathcal{H}(c)\big)\theta_1^*\right]\mathcal{H}'(c)}\theta_1^*\right]u'(c). \tag{A.7}$$

Substituting (A.7) into (A.6) and evaluating the resulting derivative at point $h_1 = \mathcal{H}(c)$, one obtains:

$$\left.\frac{\partial \mathcal{L}}{\partial h_1}\right|_{h_1=\mathcal{H}(c)} = -\frac{(1+r)\Pi'\big(\mathcal{H}(c)\big)u'(c)}{(1+r) - \left[F'\big(\mathcal{H}(c)\big) - \Pi'\big(\mathcal{H}(c)\big)\theta_1^*\right]\mathcal{H}'(c)}\theta_1^* \le 0\,,$$

where the inequality follows from the fact that condition (22) implies that $F'\big(\mathcal{H}(c)\big) - \Pi'\big(\mathcal{H}(c)\big)\theta_1^* > 0$.

If $\theta_1^* > 0$, then condition $h_1 < \mathcal{H}(c)$ must hold at the optimum. Substituting this result into $F'(h_1)u'(c) - v'(h_1)$, one gets that $\zeta_1(b_1, \theta_1^*) > 0$. If instead $\theta_1^* = 0$, $h_1 = \mathcal{H}(c)$ is optimal, which implies that $\zeta_1(b_1, \theta_1^*) = 0$.

□

A.4 Proof of Proposition 2

Proof. Applying the Implicit Function Theorem to $\{(18),(21)\}$ to derive the partial derivatives of $\{h_1(b_1, \theta_1^*), c(b_1, \theta_1^*)\}$ with respect to b_1 and θ_1^*, one obtains:

$$\frac{\partial h_1}{\partial b_1}q_0 + \frac{\partial h_1}{\partial \theta_1^*} = -\frac{\mathcal{A}_{\theta_1^*}}{\Delta}\mathcal{Z}_c\,, \tag{A.8}$$

$$\frac{\partial c}{\partial b_1}q_0 + \frac{\partial c}{\partial \theta_1^*} = \frac{\mathcal{A}_{\theta_1^*}}{\Delta}\mathcal{Z}_{h_1}\,, \tag{A.9}$$

with

$$\mathcal{A}_{\theta_1^*} = -\Pi'(h_1)u'(c) - \frac{1}{1+r}\Big(- \Pi'(h_1)v'(\mathcal{H}(c)) + \Pi'(\mathcal{H}(c))v'(h_1)\Big)\mathcal{H}'(c)\,,$$

$$\mathcal{Z}_c = \frac{1}{1+r}\Big[F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*\Big]\,\mathcal{H}'(c) - 1\,,$$

$$\mathcal{Z}_{h_1} = \frac{r}{1+r}\Big[F'(h_1) - \Pi'(h_1)\theta_1^*\Big]\,,$$

$$\begin{aligned}\mathcal{A}_c = {} & [F'(h_1) - \Pi'(h_1)\theta_1^*]u''(c) + \\ & -\frac{1}{1+r}\Bigg[[F'(h_1) - \Pi'(h_1)\theta_1^*]\,v''(\mathcal{H}(c))[\mathcal{H}'(c)]^2 - [F''(\mathcal{H}(c)) - \Pi''(\mathcal{H}(c))\theta_1^*]\,v'(h_1)[\mathcal{H}'(c)]^2 + \\ & + \Big[[F'(h_1) - \Pi'(h_1)\theta_1^*]v'(\mathcal{H}(c)) - [F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*]v'(h_1)\Big]\,\mathcal{H}''(c)\Bigg]\,,\end{aligned}$$

$$\begin{aligned}\mathcal{A}_{h_1} = {} & [F''(h_1) - \Pi''(h_1)\theta_1^*]u'(c) - v''(h_1) + \\ & -\frac{1}{1+r}\Bigg[[F''(h_1) - \Pi''(h_1)\theta_1^*]\,v'(\mathcal{H}(c)) - [F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*]\,v''(h_1)\Bigg]\mathcal{H}'(c)\,,\end{aligned}$$

$$\Delta = \mathcal{A}_{h_1}\mathcal{Z}_c - \mathcal{A}_c\mathcal{Z}_{h_1}\,.$$

Condition (22) implies that $\mathcal{Z}_c < 0$ and $\mathcal{Z}_{h_1} > 0$. Condition (21) together with Proposition 1 imply that $\mathcal{A}_{\theta_1^*} < 0$. Conditions (22) and (24) together with $h_1 \leq \mathcal{H}(c)$ imply that $\mathcal{A}_c < 0$ and $\mathcal{A}_{h_1} < 0$. These results, in turn, imply that $\Delta > 0$. Therefore,

$$\frac{\partial h_1}{\partial b_1}q_0 + \frac{\partial h_1}{\partial \theta_1^*} < 0\,, \quad \frac{\partial c}{\partial b_1}q_0 + \frac{\partial c}{\partial \theta_1^*} < 0\,, \quad \frac{\partial \zeta_1}{\partial b_1}q_0 + \frac{\partial \zeta_1}{\partial \theta_1^*} > 0\,. \tag{A.10}$$

which, in turn, implies that

$$\frac{\partial \zeta_1}{\partial b_1}q_0 + \frac{\partial \zeta_1}{\partial \theta_1^*} = [F''(h_1)u'(c) - v''(h_1)]\left(\frac{\partial h_1}{\partial b_1}q_0 + \frac{\partial h_1}{\partial \theta_1^*}\right) + F'(h_1)u''(c)\left(\frac{\partial c}{\partial b_1}q_0 + \frac{\partial c}{\partial \theta_1^*}\right) > 0.$$

As a remark, to be used later on in Proof A.7 and in Proof A.8, note that

$$\begin{aligned}\frac{\partial q_0}{\partial b_1}q_0(b_1,\theta_1^*) + \frac{\partial q_0}{\partial \theta_1^*} = & \frac{1}{1+r}\left[\Pi'(h_1)\left[\frac{\partial h_1}{\partial b_1}q_0 + \frac{\partial h_1}{\partial \theta_1^*}\right] + \frac{1}{r}\,\Pi'(\mathcal{H}(c))\,\mathcal{H}'(c)\left[\frac{\partial c}{\partial b_1}q_0 + \frac{\partial c}{\partial \theta_1^*}\right]\right]\,, \\ = & \frac{1}{1+r}\frac{\mathcal{A}_{\theta_1^*}}{\Delta}\left[-\Pi'(h_1)\,\mathcal{Z}_c + \frac{1}{r}\,\Pi'(\mathcal{H}(c))\,\mathcal{H}'(c)\mathcal{Z}_{h_1}\right] < 0\,, \end{aligned} \tag{A.11}$$

where the second equality results from substituting (A.8) and (A.9) into the first one, and where the inequality in the second line follows from the facts that $\mathcal{A}_{\theta_1^*} < 0$, $\Delta > 0$, and the bracketed term is positive.

□

A.5 Proof of Lemma 3

Proof. At $t = 0$, households solve

$$V_0(b_0, \theta_0) = \max_{c_0, b_1, \theta_1 \geq 0} \left\{ u(c_0) - v(\ell_0) + \frac{1}{1+r} V_1(b_1, \theta_1) \right\}$$

subject to:

$$c_0 + b_1 + q_0\theta_1 = w_0\ell_0 + (1+r)b_0 + (\pi_0 + q_0)\theta_0 \,, \tag{A.12}$$

where $q_0 = \frac{1}{1+r}(\pi_1 + q)$ already uses $q_1 = q$ and where recall that for $t \geq 2$ the economy is in a steady-state equilibrium. From problem (A.4) it follows that

$$\frac{\partial V_1}{\partial b_1} = (1+r)\,\lambda_1(b_1, \theta_1) \quad \text{and} \quad \frac{\partial V_1}{\partial \theta_1} = (1+r)\,\lambda_1(b_1, \theta_1)\, q_0 \,.$$

These conditions together with optimality at $t = 0$ imply that $c_0 = c$. Substituting this result, optimality condition (2), market clearing conditions $\ell_t = h_t$ and $\theta_t + \theta_t^* = 1$, and mapping $h = \mathcal{H}(c)$ into (A.12), we arrive to (25). Condition (20) together with $v'(\mathcal{H}(c)) = F'(\mathcal{H}(c))u'(c)$ and $\theta_1 + \theta_1^* = 1$ imply that Lagrange multiplier $\lambda_1(b_1, \theta_1^*)$ satisfies

$$\lambda_1(b_1, \theta_1^*) = \frac{1 - \frac{1}{1+r} F'(\mathcal{H}(c(b_1, \theta_1^*)))\mathcal{H}'(c(b_1, \theta_1^*))}{1 - \frac{1}{1+r}\Big[F'\big(\mathcal{H}(c(b_1, \theta_1^*))\big) - \Pi'\big(\mathcal{H}(c(b_1, \theta_1^*))\big)\theta_1^*\Big]\mathcal{H}'(c(b_1, \theta_1^*))} u'(c(b_1, \theta_1^*)) \,,$$

where note that given $c(b_1, \theta_1^*)$, θ_1^* is the only variable that affects the right-hand side, doing so in a strictly increasing manner. This then implies that a continuum of Pareto-ranked competitive equilibria exists.

□

A.6 Proof of Proposition 3

Proof. The proof has two parts. For the first part, observe first that (6) implies that consumption is constant, and in addition, we have $h_t = h = \mathcal{H}(c)$ for all $t \neq 1$. Using that

$$q_0 = \frac{1}{1+r}\left[\Pi\left(h_1\right) + \frac{1}{r}\Pi\left(\mathcal{H}(c)\right)\right].$$

and iterating forward on the budget constraint, and using the transversality condition we have that c solves

$$\left(\frac{1+r}{r}\right)c = F(\mathcal{H}(c)) - \Pi(\mathcal{H}(c))\theta_0^* + (1+r)b_0 - q_0\theta_0^* + \frac{1}{1+r}\left(F(h_1) + \frac{F\big(\mathcal{H}(c)\big)}{r}\right) \quad \text{(A.13)}$$

where h_1 is given by

$$\frac{\overline{W}_1}{E_1} = F'(h_1)\,, \quad \text{(A.14)}$$

Differentiating the right-hand side of (A.13) with respect to c (with h_1 fixed by (A.14)) and using $F'(\mathcal{H}(c)) - \theta_0^*\Pi'(\mathcal{H}(c)) = GNI'(\mathcal{H}(c))$ gives

$$\frac{d\,\text{RHS}}{dc} = \mathcal{H}'(c)\,GNI'\big(\mathcal{H}(c)\big)\left[1 + \frac{1}{r(1+r)}\right] < 0,$$

where the sign follows from $\mathcal{H}'(c) < 0$ together with $GNI'(\mathcal{H}(c)) > 0$ under Assumption 1. (When $\theta_0^* > 0$, the term $-\Pi(\mathcal{H}(c))\theta_0^*$ and the valuation term $-q_0\theta_0^*$ would each be increasing in c in isolation; the monotonicity part of Assumption 1 is precisely what guarantees they do not overturn the sign.) Because the left-hand side of (A.13) is increasing in c, the solution c—and hence h—is unique.

For the second part, we use that c satisfies (A.13) for any period-1 exchange rate policy and using that $\theta_0^* = 0$, we can express the central bank problem as

$$V_0(b_0, 0) = \max_{c,b_1,\theta_1^*,h_1}\left\{u(c) - v(\mathcal{H}(c)) + \frac{u(c) - v\left(h_1\right)}{1+r} + \frac{1}{1+r}\frac{u(c) - v\left(\mathcal{H}(c)\right)}{r}\right\},$$

subject to:

$$\left(\frac{1+r}{r}\right)c \leq F(\mathcal{H}(c)) - \Pi(\mathcal{H}(c))\theta_0^* + (1+r)b_0 + \frac{1}{1+r}\left(F(h_1) + \frac{F\big(\mathcal{H}(c)\big)}{r}\right)$$

Optimality with respect to h_1 and c yields

$$v'(h_1) = u'(c)F'(h_1)$$

which implies a zero labor wedge in period 1. □

A.7 Proof of Proposition 4

Proof. Let $\mathcal{L}_0$ denote the Lagrangian associated with problem (26), where λ_0 and μ_0 are the multipliers on the equality constraints (27) and (28), and $\underline{\nu}_0$ and $\overline{\nu}_0$ are the non-negative multipliers on the bounds $\theta_1^* \geq 0$ and $\theta_1^* \leq 1$. The Lagrangian is:

$$\begin{aligned}\mathcal{L}_0 = {} & u(c_0) - v(\mathcal{H}(c_0)) + \frac{1}{1+r} V_1(b_1, \theta_1^*) + \\ & + \lambda_0 \Big[F(\mathcal{H}(c_0)) - \Pi(\mathcal{H}(c_0))\theta_0^* + q_0(b_1, \theta_1^*)\,(\theta_1^* - \theta_0^*) + (1+r)b_0 - b_1 - c_0 \Big] + \\ & + \mu_0 \Big[c(b_1, \theta_1^*) - c_0 \Big] + \underline{\nu}_0\, \theta_1^* + \overline{\nu}_0\, (1 - \theta_1^*)\,.\end{aligned}$$

The first-order conditions with respect to the choice variables c_0, b_1 and θ_1^* are:

$$0 = u'(c_0) - v'\left(\mathcal{H}(c_0)\right)\mathcal{H}'(c_0) + \lambda_0 \Big[[F'(\mathcal{H}(c_0)) - \Pi'(\mathcal{H}(c_0))\theta_0^*]\mathcal{H}'(c_0) - 1 \Big] - \mu_0, \tag{A.15}$$

$$0 = \frac{1}{1+r}\frac{\partial V_1}{\partial b_1} + \lambda_0 \left[\frac{\partial q_0}{\partial b_1}\,(\theta_1^* - \theta_0^*) - 1 \right] + \mu_0 \frac{\partial c}{\partial b_1}\,, \tag{A.16}$$

$$0 = \frac{1}{1+r}\frac{\partial V_1}{\partial \theta_1^*} + \lambda_0 \left[\frac{\partial q_0}{\partial \theta_1^*}\,(\theta_1^* - \theta_0^*) + q_0(b_1, \theta_1^*) \right] + \mu_0 \frac{\partial c}{\partial \theta_1^*} + \underline{\nu}_0 - \overline{\nu}_0\,, \tag{A.17}$$

where, from problem (17),

$$\frac{\partial V_1}{\partial b_1} = (1+r)\,\lambda_1(b_1, \theta_1^*)\,, \tag{A.18}$$

$$\frac{\partial V_1}{\partial \theta_1^*} = -\,(1+r)\,\lambda_1(b_1, \theta_1^*)\,\frac{1}{1+r}\left[\Pi(h_1(b_1, \theta_1^*)) + \frac{1}{r}\Pi(\mathcal{H}(c(b_1, \theta_1^*))) \right]. \tag{A.19}$$

These conditions hold together with constraints (27) and (28) and the complementary slackness conditions

$$\underline{\nu}_0\, \theta_1^* = 0, \qquad \overline{\nu}_0\,(1 - \theta_1^*) = 0, \qquad \underline{\nu}_0, \overline{\nu}_0 \geq 0. \tag{A.20}$$

Using that q_0 is determined by (23), and substituting (A.18) and (A.19) into first-order conditions (A.16) and (A.17), one gets:

$$0 = \lambda_1(b_1, \theta_1^*) + \lambda_0 \left[\frac{\partial q_0}{\partial b_1} (\theta_1^* - \theta_0^*) - 1 \right] + \mu_0 \frac{\partial c}{\partial b_1}\,, \tag{A.21}$$

$$0 = -\lambda_1(b_1, \theta_1^*) q_0(b_1, \theta_1^*) + \lambda_0 \left[\frac{\partial q_0}{\partial \theta_1^*} (\theta_1^* - \theta_0^*) + q_0(b_1, \theta_1^*) \right] + \mu_0 \frac{\partial c}{\partial \theta_1^*} + \underline{\nu}_0 - \overline{\nu}_0\,.$$

Summing up these two conditions—after multiplying both sides of the first one by $q_0(b_1, \theta_1^*)$—one obtains:

$$0 = \lambda_0 \left[\frac{\partial q_0}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial q_0}{\partial \theta_1^*} \right] (\theta_1^* - \theta_0^*) + \mu_0 \left[\frac{\partial c}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial c}{\partial \theta_1^*} \right] + \underline{\nu}_0 - \overline{\nu}_0\,. \tag{A.22}$$

Note that

$$\lambda_0 > 0\,. \tag{A.23}$$

This holds because following an increase in b_0, it is possible to increase both c_0 and b_1 in a manner that $c_0 = c(b_1, \theta_1^*)$ holds and θ_1^* remains unchanged.

Assume (22) and (24) hold. Then, as shown in Proof A.4, (A.10) and (A.11) hold, which implies that

$$\frac{\partial c}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial c}{\partial \theta_1^*} < 0 \quad \text{and} \quad \frac{\partial q_0}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial q_0}{\partial \theta_1^*} < 0. \tag{A.24}$$

Consider first the case with $\theta_0^* > 0$. Conjecture that $\mu_0 = 0$. Then, inequalities (A.23) and (A.24) together with condition (A.22) and complementary slackness (A.20) imply that $\theta_1^* = \theta_0^*$ must be optimal. Moreover, condition (A.15) implies that

$$\lambda_0 = u'(c_0) - v'\left(\mathcal{H}(c_0)\right) \mathcal{H}'(c_0) + \lambda_0 [F'(\mathcal{H}(c_0)) - \Pi'(\mathcal{H}(c_0))\theta_0^*] \mathcal{H}'(c_0)\,, \tag{A.25}$$

and condition (A.21) together with $\theta_1^* = \theta_0^*$ imply that

$$\lambda_1(b_1, \theta_1^*) = \lambda_0\,. \tag{A.26}$$

Conditions (A.25) and (A.26) together with conditions (20) and $v'(\mathcal{H}(c)) = F'(\mathcal{H}(c))u'(c)$

imply that

$$\frac{u'(c_0)}{u'(c(b_1,\theta_1^*))} = \frac{1-[F'(\mathcal{H}(c_0))-\Pi'(\mathcal{H}(c_0))\theta_0^*]\mathcal{H}'(c_0)}{1-\frac{1}{1+r}\Big[F'\big(\mathcal{H}(c(b_1,\theta_1^*))\big)-\Pi'\big(\mathcal{H}(c(b_1,\theta_1^*))\big)\theta_0^*\Big]\mathcal{H}'(c(b_1,\theta_1^*))}\times$$
$$\times\ \frac{1-\frac{1}{1+r}F'\big(\mathcal{H}(c(b_1,\theta_1^*))\big)\mathcal{H}'(c(b_1,\theta_1^*))}{1-F'\left(\mathcal{H}(c_0)\right)\mathcal{H}'(c_0)}.$$

Thus, $c_0 > c(b_1,\theta_1^*)$, but this result is not consistent with the restriction that $c_0 = c(b_1,\theta_1^*)$, which implies that conjecture $\mu_0 = 0$ is false. Result $c_0 > c(b_1,\theta_1^*)$ if $\mu_0 = 0$ also implies that $\mu_0 > 0$ must hold. This, together with inequalities (A.23) and (A.24), condition (A.22), and complementary slackness (A.20), in turn, imply that $\theta_1^* < \theta_0^*$ is optimal when $\theta_0^* > 0$.

Consider now the case with $\theta_0^* = 0$. Note that $\theta_1^* = \theta_0^* = 0$, $b_1 = b_0$, $c_0 = c(b_0,\theta_0^*)$, $\lambda_0 = \lambda_1(b_0,\theta_0^*)$ and $\mu_0 = 0$ satisfy first-order conditions (A.15)–(A.17), constraints (27) and (28), and complementary slackness (A.20).

□

A.8 Proof of Proposition 5

Proof. Let us express the Lagrangian of the problem of the regulatory authority as follows

$$\begin{aligned}
\mathcal{L}_0 = & u(c_0) - v(\mathcal{H}(c_0)) + \frac{1}{1+r}V_1(b_1,\theta_1^*)+ \\
& + \lambda_0\Big[F(\mathcal{H}(c_0)) - \Pi(\mathcal{H}(c_0))\theta_0^* + q_0(b_1,\theta_1^*)\left(\theta_1^*-\theta_0^*\right) + (1+r)b_0 - b_1 - c_0\Big]+ \\
& + \mu_0\Big[c(b_1,\theta_1^*) - c_0\Big] \\
& + \kappa_0\left(\underline{b} + b_1\right) \\
& + \xi_0\Big[c(b_1,\theta_1^*) - c_0\Big]\left(\underline{b} + b_1\right) \\
& + \underline{\nu}_0\,\theta_1^* + \overline{\nu}_0\left(1-\theta_1^*\right),
\end{aligned}$$

where λ_0 and ξ_0 are the multipliers on the equality constraints (30) and (33), and μ_0, κ_0, $\underline{\nu}_0$ and $\overline{\nu}_0$ are the non-negative multipliers on the inequality constraints (31), (32), $\theta_1^* \geq 0$ and $\theta_1^* \leq 1$.

The first-order conditions with respect to the choice variables c_0, b_1 and θ_1^* are:

$$0 = u'(c_0) - v'(\mathcal{H}(c_0))\,\mathcal{H}'(c_0) + \lambda_0\left[[F'(\mathcal{H}(c_0)) - \Pi'(\mathcal{H}(c_0))\theta_0^*]\mathcal{H}'(c_0) - 1\right] - \mu_0 - \xi_0\,(\underline{b} + b_1)\ ,$$

$$0 = \frac{1}{1+r}\frac{\partial V_1}{\partial b_1} + \lambda_0\left[\frac{\partial q_0}{\partial b_1}(\theta_1^* - \theta_0^*) - 1\right] + \mu_0\frac{\partial c}{\partial b_1} + \kappa_0 + \xi_0\left[\frac{\partial c}{\partial b_1}(\underline{b} + b_1) + c(b_1,\theta_1^*) - c_0\right]\ ,$$

$$0 = \frac{1}{1+r}\frac{\partial V_1}{\partial \theta_1^*} + \lambda_0\left[\frac{\partial q_0}{\partial \theta_1^*}(\theta_1^* - \theta_0^*) + q_0(b_1,\theta_1^*)\right] + \mu_0\frac{\partial c}{\partial \theta_1^*} + \xi_0\frac{\partial c}{\partial \theta_1^*}(\underline{b} + b_1) + \underline{\nu}_0 - \overline{\nu}_0\,.$$

These conditions hold together with constraints (30) – (33) and the complementary slackness conditions

$$\mu_0\,[c(b_1,\theta_1^*) - c_0] = 0, \quad \kappa_0\,(\underline{b} + b_1) = 0, \quad \underline{\nu}_0\,\theta_1^* = 0, \quad \overline{\nu}_0\,(1-\theta_1^*) = 0, \quad \mu_0,\kappa_0,\underline{\nu}_0,\overline{\nu}_0 \geq 0. \tag{A.27}$$

In the above, recall that from problem (17):

$$\frac{\partial V_1}{\partial b_1} = (1+r)\,\lambda_1(b_1,\theta_1^*)\,, \tag{A.28}$$

$$\frac{\partial V_1}{\partial \theta_1^*} = -\,(1+r)\,\lambda_1(b_1,\theta_1^*)\,\frac{1}{1+r}\left[\Pi(h_1(b_1,\theta_1^*)) + \frac{1}{r}\Pi(\mathcal{H}(c(b_1,\theta_1^*)))\right]. \tag{A.29}$$

Using that q_0 is determined by (23), and substituting (A.28) and (A.29) into first-order conditions $\partial\mathcal{L}_0/\partial b_1 = 0$ and $\partial\mathcal{L}_0/\partial\theta_1^* = 0$, one gets:

$$0 = \lambda_1(b_1,\theta_1^*) + \lambda_0\left[\frac{\partial q_0}{\partial b_1}(\theta_1^* - \theta_0^*) - 1\right] + \mu_0\frac{\partial c}{\partial b_1} + \kappa_0 + \xi_0\left[\frac{\partial c}{\partial b_1}(\underline{b} + b_1) + c(b_1,\theta_1^*) - c_0\right]\ ,$$

$$0 = -\lambda_1(b_1,\theta_1^*)q_0(b_1,\theta_1^*) + \lambda_0\left[\frac{\partial q_0}{\partial \theta_1^*}(\theta_1^* - \theta_0^*) + q_0(b_1,\theta_1^*)\right] + \mu_0\frac{\partial c}{\partial \theta_1^*} + \xi_0\frac{\partial c}{\partial \theta_1^*}(\underline{b} + b_1) + \underline{\nu}_0 - \overline{\nu}_0\,,$$

and summing up these two conditions—after multiplying both sides of the first one by $q_0(b_1,\theta_1^*)$—one obtains:

$$\begin{aligned} 0\ &= \lambda_0\left[\frac{\partial q_0}{\partial b_1}q_0(b_1,\theta_1^*) + \frac{\partial q_0}{\partial \theta_1^*}\right](\theta_1^* - \theta_0^*) + [\mu_0 + \xi_0\,(\underline{b} + b_1)]\left[\frac{\partial c}{\partial b_1}q_0(b_1,\theta_1^*) + \frac{\partial c}{\partial \theta_1^*}\right] + \\ &\quad + \left[\kappa_0 + \xi_0\,[c(b_1,\theta_1^*) - c_0]\right]q_0(b_1,\theta_1^*) + \underline{\nu}_0 - \overline{\nu}_0 \end{aligned} \tag{A.30}$$

Note that

$$\lambda_0 > 0\,. \tag{A.31}$$

This holds because following an increase in b_0, it is possible to increase both c_0 and b_1 in a manner that $c_0 \leq c(b_1, \theta_1^*)$ holds and θ_1^* remains unchanged.

As shown in Proof A.4, if (22) and (24) hold, then

$$\frac{\partial q_0}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial q_0}{\partial \theta_1^*} < 0 \text{ and } \frac{\partial c}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial c}{\partial \theta_1^*} < 0\,. \tag{A.32}$$

Suppose the optimum features $c_0 < c(b_1, \theta_1^*)$. Complementary slackness (A.27) then gives $\mu_0 = 0$, and constraint (33) gives $\underline{b} + b_1 = 0$, so the term in $[\mu_0 + \xi_0\,(\underline{b} + b_1)]$ drops out of (A.30), leaving

$$0 = \lambda_0 \left[\frac{\partial q_0}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial q_0}{\partial \theta_1^*}\right](\theta_1^* - \theta_0^*) + \Big[\kappa_0 + \xi_0\,[c(b_1, \theta_1^*) - c_0]\Big]\, q_0(b_1, \theta_1^*) + \underline{\nu}_0 - \overline{\nu}_0\,. \tag{A.33}$$

Here $\kappa_0 + \xi_0\,[c(b_1, \theta_1^*) - c_0] > 0$, because a higher $\underline{b}$ relaxes the borrowing constraint, while the coefficient on $(\theta_1^* - \theta_0^*)$ is strictly negative by (A.31) and (A.32). Together with (A.27), this requires $\theta_1^* > \theta_0^*$ whenever $\theta_0^* < 1$.

Finally, let $\theta_0^* = 0$ and conjecture that $\theta_1^* = 0$ is optimal. Conditions (18), (21) and (30) then imply $c_0 < c(b_1, 0)$ for all $b_1 \geq -\underline{b}$, so (33) gives $b_1 = -\underline{b}$. The premise of the previous paragraph therefore holds, and (A.33) implies $\theta_1^* > 0$, contradicting the conjecture.

□

A.9 Proof of Proposition 6

Proof. The proof proceeds in three steps as follows. First, we derive the continuation equilibrium for any period $t \geq 2$. Then, we solve the continuation equilibrium at period 1. Lastly, we lay out the problem of the central bank in period 1, and derive its FOCs.

Step 1. Given an initial portfolio $\left(b_2, {\theta_2^*}^T, {\theta_2^*}^N\right)$, the continuation equilibrium for any $t \geq 2$ features constant quantities and prices over time. Throughout, we denote profits in sector $j \in \{N, T\}$, expressed in units of the good produced in that sector and as a function of sectoral employment, by $\Pi_j(h) \equiv F_j(h) - F_j'(h)h$. Quantities $\{h^T, h^N, c^T\}$ are jointly determined by

$$\frac{F_T'(h^T)}{F_N'(h^N)} = \frac{U_N(c^T, F_N(h^N))}{U_T(c^T, F_N(h^N))}\,, \tag{A.34}$$

$$F_T'(h^T) = \frac{v'(h^T + h^N)}{U_T(c^T, F_N(h^N))}\,, \tag{A.35}$$

$$c^T = F_T(h^T) + rb_2 - \left[\Pi_T(h^T)\theta_2^{*T} + p^N \Pi_N(h^N)\theta_2^{*N}\right]\,,$$

the sequence of portfolios $\left\{b_{t+1}, \theta_{t+1}^{*T}, \theta_{t+1}^{*N}\right\}_{t\geq 2}$ satisfies

$$b_{t+1} - \left(q^T\,\theta_{t+1}^{*T} + q^N\,\theta_{t+1}^{*N}\right) = b_2 - \left(q^T\,\theta_2^{*T} + q^N\,\theta_2^{*N}\right) \tag{A.36}$$

equity prices $\{q^T, q^N\}$ are determined by

$$q^T = \frac{1}{r}\,\Pi_T(h^T)\,, \tag{A.37}$$

$$q^N = \frac{1}{r}\,p^N\,\Pi_N(h^N)\,, \tag{A.38}$$

and the price of the non-tradable good p^N is given by

$$p^N = \frac{U_N(c^T, F_N(h^N))}{U_T(c^T, F_N(h^N))}\,. \tag{A.39}$$

Note that condition (A.34) follows from combining household optimality with firm optimality. Define the mappings

$$h^T = \mathcal{H}_T\left(c^T\right), \qquad h^N = \mathcal{H}_N\left(c^T\right), \qquad p^N = \mathcal{P}_N\left(c^T\right)\,, \tag{A.40}$$

where $\mathcal{H}_T(\cdot)$ and $\mathcal{H}_N(\cdot)$ solve $\{$(A.34),(A.35)$\}$ for any given c^T, and $\mathcal{P}_N(\cdot)$ then follows from (A.39). These mappings will be useful for characterizing the continuation equilibrium at period 1.

Step 2. Given an initial portfolio $\left(b_1, \theta_1^{*T}, \theta_1^{*N}\right)$, a rigid wage $\overline{W}_1$, an exchange rate E_1, the equilibrium for $t \geq 1$ is such that

i) h_1^T and h_1^N are respectively determined by

$$\frac{\overline{W}_1}{E_1} = F_T'\left(h_1^T\right) \text{ and } \frac{\overline{W}_1}{E_1} = p_1^N\,F_N'\left(h_1^N\right)\,, \tag{A.41}$$

where, as in (A.39), the period-1 relative price of the non-tradable good is $p_1^N =$

$U_N(c_1^T, F_N(h_1^N))/U_T(c_1^T, F_N(h_1^N))$, and with c_1^T being given by

$$\frac{U_T(c_1^T, F_N(h_1^N))}{U_T\left(c^T, F_N\left(\mathcal{H}_N\left(c^T\right)\right)\right)} = 1 \tag{A.42}$$

ii) $h_t^j = \mathcal{H}_j(c^T)$ and $c_t^T = c^T$ for $t > 1$, with c^T being determined by

$$0 = \frac{r}{1+r}\left[F_T(h_1^T) + rb_1 - \left[\Pi_T\left(h_1^T\right)\theta_1^{*T} + p_1^N \Pi_N(h_1^N)\theta_1^{*N}\right] - c_1^T\right] + \\ \frac{1}{1+r}\left[F_T\big(\mathcal{H}_T\left(c^T\right)\big) + rb_1 - \left[\Pi_T\left(\mathcal{H}_T\left(c^T\right)\right)\theta_1^{*T} + \mathcal{P}_N\left(c^T\right)\Pi_N\left(\mathcal{H}_N\left(c^T\right)\right)\theta_1^{*N}\right] - c^T\right]$$

iii) $q_t^j = q^j$ for $j \in \{N, T\}$, with q^T and q^N respectively given by (A.37) and (A.38), and the sequence of portfolios is such that

$$b_2 - \left(q^T\theta_2^{*T} + q^N\theta_2^{*N}\right) = b_1 - \left(q^T\theta_1^{*T} + q^N\theta_1^{*N}\right) + \\ \frac{F_T(h_1^T) - \left[\Pi_T\left(h_1^T\right)\theta_1^{*T} + p_1^N\Pi_N(h_1^N)\theta_1^{*N}\right]}{1+r} - \\ \frac{F_T\big(\mathcal{H}_T\left(c^T\right)\big) - \left[\Pi_T\left(\mathcal{H}_T\left(c^T\right)\right)\theta_1^{*T} + \mathcal{P}_N\left(c^T\right)\Pi_N\left(\mathcal{H}_N\left(c^T\right)\right)\theta_1^{*N}\right]}{1+r}$$

and $\left\{b_{t+1}, \theta_{t+1}^{*T}, \theta_{t+1}^{*N}\right\}_{t\geq 2}$ satisfy (A.36).

Combining the two equations in (A.41) one gets

$$\frac{F_T'(h_1^T)}{F_N'(h_1^N)} = \frac{U_N(c_1^T, F_N(h_1^N))}{U_T(c_1^T, F_N(h_1^N))} \tag{A.43}$$

Define mappings $\tilde{\mathcal{C}}\left(\cdot\right)$ and $\tilde{\mathcal{H}}\left(\cdot\right)$ as the solution to $c_1^T = \tilde{\mathcal{C}}\left(h_1^T, c^T\right)$ and $h_1^N = \tilde{\mathcal{H}}\left(h_1^T, c^T\right)$ as a function of $\left(h_1^T, c^T\right)$ in system of equations $\{$(A.42), (A.43)$\}$. Additionally define

$$\tilde{\mathcal{P}}\left(h_1^T, c^T\right) \equiv \frac{U_N\left(\tilde{\mathcal{C}}\left(h_1^T, c^T\right), F_N\left(\tilde{\mathcal{H}}\left(h_1^T, c^T\right)\right)\right)}{U_T\left(\tilde{\mathcal{C}}\left(h_1^T, c^T\right), F_N\left(\tilde{\mathcal{H}}\left(h_1^T, c^T\right)\right)\right)}.$$

Note that $\tilde{\mathcal{C}}\left(\mathcal{H}_T\left(c^T\right), c^T\right) = c^T$ and $\tilde{\mathcal{H}}\left(\mathcal{H}_T\left(c^T\right), c^T\right) = \mathcal{H}_N\left(c^T\right)$. Note also that $\tilde{\mathcal{P}}\left(\mathcal{H}_T\left(c^T\right), c^T\right) = \mathcal{P}_N\left(c^T\right)$. These mappings will be useful for laying out the problem of the central bank in period 1.

Step 3. The problem of the central bank in period 1 can be expressed as

$$\max_{h_1^T, c^T} \Bigg\{ U\left(\tilde{\mathcal{C}}\left(h_1^T, c^T\right), F_N\left(\tilde{\mathcal{H}}\left(h_1^T, c^T\right)\right)\right) - v\left(h_1^T + \tilde{\mathcal{H}}\left(h_1^T, c^T\right)\right) + \\ + \frac{1}{r}\left[U\left(c^T, F_N\left(\mathcal{H}_N\left(c^T\right)\right)\right) - v\left(\mathcal{H}_T\left(c^T\right) + \mathcal{H}_N\left(c^T\right)\right)\right] \Bigg\},$$

subject to

$$\tilde{\mathcal{C}}\left(h_1^T, c^T\right) + \frac{1}{r}c^T = F_T(h_1^T) + rb_1 - \left[\Pi_T\left(h_1^T\right)\theta_1^{*T} + \tilde{\mathcal{P}}\left(h_1^T, c^T\right)\,\Pi_N(\tilde{\mathcal{H}}\left(h_1^T, c^T\right))\theta_1^{*N}\right] + \\ \frac{1}{r}\left[F_T(\mathcal{H}_T\left(c^T\right)) + rb_1 - \left[\Pi_T\left(\mathcal{H}_T\left(c^T\right)\right)\theta_1^{*T} + \mathcal{P}_N\left(c^T\right)\,\Pi_N\left(\mathcal{H}_N\left(c^T\right)\right)\theta_1^{*N}\right]\right] \tag{A.44}$$

Let λ denote the Lagrange multiplier associated with the constraint. The Lagrangian of the problem is

$$\mathcal{L} = U\left(\tilde{\mathcal{C}}\left(h_1^T, c^T\right), F_N\left(\tilde{\mathcal{H}}\left(h_1^T, c^T\right)\right)\right) - v\left(h_1^T + \tilde{\mathcal{H}}\left(h_1^T, c^T\right)\right) + \\ + \frac{1}{r}\left[U\left(c^T, F_N\left(\mathcal{H}_N\left(c^T\right)\right)\right) - v\left(\mathcal{H}_T\left(c^T\right) + \mathcal{H}_N\left(c^T\right)\right)\right] + \\ + \lambda\Bigg[F_T(h_1^T) + rb_1 - \left[\Pi_T\left(h_1^T\right)\theta_1^{*T} + \tilde{\mathcal{P}}\left(h_1^T, c^T\right)\,\Pi_N(\tilde{\mathcal{H}}\left(h_1^T, c^T\right))\theta_1^{*N}\right] - \tilde{\mathcal{C}}\left(h_1^T, c^T\right) + \\ + \frac{1}{r}\left[F_T(\mathcal{H}_T\left(c^T\right)) + rb_1 - \left[\Pi_T\left(\mathcal{H}_T\left(c^T\right)\right)\theta_1^{*T} + \mathcal{P}_N\left(c^T\right)\,\Pi_N\left(\mathcal{H}_N\left(c^T\right)\right)\theta_1^{*N}\right]\right] - \frac{1}{r}c^T\Bigg].$$

The FOC with respect to h_1^T is

$$0 = \left(U_T - \lambda\right)\tilde{\mathcal{C}}_1 + \left[U_N F_N' - v'\right]\tilde{\mathcal{H}}_1 - v' + \lambda\left[F_T' - \Pi_T'\,\theta_1^{*T} - \left[\tilde{\mathcal{P}}_1\,\Pi_N + \tilde{\mathcal{P}}\,\Pi_N'\,\tilde{\mathcal{H}}_1\right]\theta_1^{*N}\right] \tag{A.45}$$

and the FOC with respect to c^T is

$$
\begin{aligned}
0 = (U_T - \lambda)\,\tilde{\mathcal{C}}_2 + [U_N\,F'_N - v']\,\tilde{\mathcal{H}}_2 + \frac{1}{r}\,[U_T - v'\,\mathcal{H}'_T\,] + \\
+\,\lambda\Bigg[-\frac{1}{r} - \left[\tilde{\mathcal{P}}_2\,\Pi_N + \tilde{\mathcal{P}}\,\Pi'_N\,\tilde{\mathcal{H}}_2\right]\theta_1^{*N} + \\
+\,\frac{1}{r}\left[F'_T\,\mathcal{H}'_T\, - \Pi'_T\,\mathcal{H}'_T\,\theta_1^{*T} - [\mathcal{P}'_N\,\Pi_N + \mathcal{P}_N\,\Pi'_N\,\mathcal{H}'_N\,]\,\theta_1^{*N}\right]\Bigg]
\end{aligned}
\tag{A.46}
$$

where we omit the arguments in the functions to reduce clutter and where, in the latter FOC, the result $\frac{1}{r}\,[U_N F'_N - v']\,\mathcal{H}'_N = 0$ has already been substituted away. This result follows from (A.34) and (A.35).

A zero labor wedge is equivalent to $h_1^T = \mathcal{H}_T\left(c^T\right)$. In conditions (A.44), (A.45) and (A.46), under a zero labor wedge, the following holds:

$$
\tilde{\mathcal{C}} = c^T\,, \tag{A.47a}
$$

$$
\begin{aligned}
\tilde{\mathcal{H}} &= \mathcal{H}_N\,,\\
U_N\,F'_N - v' &= 0\,,\\
U_T\,F'_T - v' &= 0\,,\\
\tilde{\mathcal{P}} &= \mathcal{P}_N\,,\\
\tilde{\mathcal{P}}_1\mathcal{H}'_T + \tilde{\mathcal{P}}_2 &= \mathcal{P}'_N\,,\\
\tilde{\mathcal{H}}_1\mathcal{H}'_T + \tilde{\mathcal{H}}_2 &= \mathcal{H}'_N\,,
\end{aligned}
$$

$$
\tilde{\mathcal{C}}_1\mathcal{H}'_T + \tilde{\mathcal{C}}_2 = 1\,. \tag{A.47b}
$$

Substituting (A.47a) – (A.47b) into (A.44), one gets that steady-state tradable consumption solves

$$
c^T = F_T(\mathcal{H}_T\left(c^T\right)) + rb_1 - \left[\Pi_T(\mathcal{H}_T\left(c^T\right))\theta_1^{*T} + \mathcal{P}_N\left(c^T\right)\Pi_N(\mathcal{H}_N\left(c^T\right))\theta_1^{*N}\right]\,, \tag{A.48}
$$

where the mappings $\mathcal{H}_T(\cdot)$, $\mathcal{H}_N(\cdot)$ and $\mathcal{P}_N(\cdot)$ are those defined in (A.40). Moreover, substituting (A.47a) – (A.47b) into (A.45) and (A.46) and summing up the resulting two conditions after multiplying the former by $\mathcal{H}'_T$, one obtains:

$$
\lambda = \frac{1 - F'_T\,\mathcal{H}'_T}{1 - F'_T\,\mathcal{H}'_T + \Pi'_T\mathcal{H}'_T\theta_1^{*T} + [\mathcal{P}'_N\,\Pi_N + \mathcal{P}_N\,\Pi'_N\,\mathcal{H}'_N]\,\theta_1^{*N}}U_T\,. \tag{A.49}
$$

Lastly, substituting (A.49) together with (A.47a)–(A.47b) into (A.45), one gets:

$$\Pi_T'\theta_1^{*T} + \left[\left(\tilde{\mathcal{P}}_1 + \tilde{\mathcal{P}}_2\frac{F_T' - \tilde{\mathcal{C}}_1}{\tilde{\mathcal{C}}_2}\right)\Pi_N + \tilde{\mathcal{P}}\,\Pi_N'\left(\tilde{\mathcal{H}}_1 + \tilde{\mathcal{H}}_2\frac{F_T' - \tilde{\mathcal{C}}_1}{\tilde{\mathcal{C}}_2}\right)\right]\theta_1^{*N} = 0\,, \qquad \text{(A.50)}$$

where subscripts 1 and 2 denote partial derivatives with respect to the first and second argument.

For convenience, let

$$\Psi_T\left(c^T\right) \equiv \Pi_T'\,,$$
$$\Psi_N\left(c^T\right) \equiv \left(\tilde{\mathcal{P}}_1 + \tilde{\mathcal{P}}_2\frac{F_T' - \tilde{\mathcal{C}}_1}{\tilde{\mathcal{C}}_2}\right)\Pi_N + \tilde{\mathcal{P}}\,\Pi_N'\left(\tilde{\mathcal{H}}_1 + \tilde{\mathcal{H}}_2\frac{F_T' - \tilde{\mathcal{C}}_1}{\tilde{\mathcal{C}}_2}\right)\,.$$

With this notation, (A.50) is exactly condition (38) in the paper: the depreciation leaves the central bank indifferent precisely when the two sectors' foreign-profit sensitivities offset, $\Psi_T\theta_1^{*T} + \Psi_N\theta_1^{*N} = 0$.

□