Bank Runs, Fragility, and Regulation

Manuel Amador   Javier Bianchi

Motivation

What is the optimal banking regulation when there is a risk of runs?

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- Should regulators impose minimum capital requirements?

- Standard view: restrict leverage because banks do not fully bear downside losses
  - bailouts, deposit insurance
- In this paper:
  - A theory of banking regulation in general equilibrium
  - Banks **over-leverage**, even absent bailouts

What we do

- General equilibrium model of banks runs (Amador and Bianchi, 2024)
  - Default is strategic (Cole-Kehoe)
  - Runs can happen despite liquid assets ⇒ require equilibrium profits
  - Endogenous asset prices

    + Risky leverage choice Ex-ante efficiency?

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In the absence of runs:

  - Competitive equilibria are constrained efficient [Benchmark result]

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- Competitive equilibria are constrained efficient [Benchmark result]

With runs:

- Banks are over-leveraged

Related Literature

- **Bank runs and policy interventions.** Diamond and Dybvig (1983); Cooper and Ross (1998); Ennis and Keister (2009); Keister (2016); Dávila and Goldstein (2023); Kashyap, Tsomocos and Vardoulakis (2024); Gertler and Kiyotaki (2012); Amador and Bianchi (2024).

  This paper: regulation in a general eqm. model of runs with endogenous asset prices

- **Efficiency with incomplete markets/limited commitment:** Hart (1975); Stiglitz (1982); Geanakoplos and Polemarchakis (1985); Kehoe and Levine (1993); Alvarez-Jermann (2000).

  This paper: welfare analysis in an environment where default occurs in equilibrium.

- **Macroprudential policy and pecuniary externalities.** Caballero and Krishnamurthy (2003); Lorenzoni (2008); Bianchi (2011); Stein (2012); Dávila and Korinek (2018).

  This paper: externality due to self-fulfilling runs and equilibrium default

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- Three periods $t = 0, 1, 2$
- One final consumption good and one factor (capital)
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- Production is given by

individual productivity

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$$\underbrace{z}_{\text{individual productivity}} \cdot k$$

- Can default at $t = 1, 2$ (Cole-Kehoe timing)
- Creditors: linear utility and discount rate R

Roadmap

1. Bank problem in partial equilibrium, for given price of capital $\{p_t\}$
   - Characterize "region of vulnerability"
2. General equilibrium: market clearing for capital, determination of $\{p_t\}$
3. Normative analysis: socially optimal ex-ante leverage choices

Banks' Preferences and budget constraints

- Preferences

$$u(c_0) + \beta \mathbb{E} u(c_1) + \beta^2 \mathbb{E} u(c_2), \quad \text{where} \quad u = \log \quad [\text{tractability}]$$

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- Budget constraints under **repayment**

Bond price schedule

$$c_0 = (z + p_0)k_0 - Rb_0 + q_0(b_1, k_1)b_1 - p_0 k_1,$$

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- Period 1 value

$$V_1^D(k_1, z_1^D) = u(z_1^D k_1) + \beta u(z_2^D k_1)$$

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V^D independent of prices and increasing in k

Period 2

Simple static problem

$$V_2(b_2, k_2) = \max_{d_2 \in \{0,1\}} \left\{ (1 - d_2) u(z k_2 - R b_2) + d_2 u(z_2^D k_2) \right\}$$

Default choice:

$$d_2(b_2, k_2) = \begin{cases} 1 & \text{if } \; R b_2 > \phi k_2, \text{where } \phi \equiv z - z_2^D \\ 0 & \text{otherwise,} \end{cases}$$

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Borrowing limit at $t = 1$ $\quad Rb_2 \leq \phi k_2$

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Without a run

$$\text{s.t.} \quad c_1 = n_1 + b_2 - p_1 k_2$$

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Value functions indexed by p_1

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Default Thresholds

- Given p_1, two default thresholds

$$\text{Fundamental:} \quad V_1^R(n_1) = V_1^D(k_1, \hat{z}^F)$$

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$$\hat{z}^F(l_1) \geq \hat{z}^{Run}(l_1), \qquad \text{strict if } \frac{z}{p_1} > R.$$

If $\frac{z}{p_1} \geq R$:

$$V_1^{Run}(n_1) = A + (1+\beta)\log(n_1) + \beta\log\left(\frac{z}{p_1}\right)$$

cannot borrow

If $\frac{z}{p_1} \geq R$:

excess return from leverage

$$V_1^R(n_1) = A + (1+\beta)\log(n_1) + \beta \log\left(\frac{z-\phi}{p_1 - \phi/R}\right)$$

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$\Rightarrow V_1^{R}(n_1) = V_1^{Run}(n_1)$

If $\frac{z}{p_1} > R$:

excess return from leverage

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Vulnerability to runs despite assets being liquid (Amador and Bianchi, 2024)

Self-Fulfilling Runs

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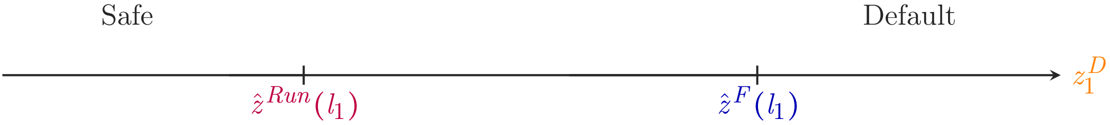

$V_1^{\text{Run}}(n_1) \geq V_1^D(k_1, z_1^D)$

$V_1^D(k_1, z_1^D) > V_1^R(n_1)$

[Run & repay is an off-equilibrium event]

Self-Fulfilling Runs

Safe | **Vulnerable** | Default

$\hat{z}^{Run}(l_1)$ $\quad$ $\hat{z}^{F}(l_1)$ $\quad$ z_1^D

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- **Sunspot:** a **vulnerable** bank faces a run with probability λ.

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$\Rightarrow$ bond price schedule is given by

$$q_0(l_1) = (1-\lambda)F(\hat{z}^F(l_1)) + \lambda F(\hat{z}^{Run}(l_1))$$

Period 0: Value and Leverage Choice

$$V_0(n_0) = \max_{c_0 \geq 0,\ k_1 \geq 0,\ l_1} u(c_0) + \beta \Bigg[\underbrace{\Big((1-\lambda) F(\hat{z}^F(l_1)) + \lambda F(\hat{z}^{Run}(l_1)) \Big) V_1^R(n_1)}_{\text{Repayment}}$$

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subject to

$$c_0 = n_0 + (q_0(l_1) l_1 - p_0) k_1,$$
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Leverage induces higher c_0 and expected profits but more likely default

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1. Bank problem in partial equilibrium, for given asset prices $\{p_t\}$
    - Characterize "region of vulnerability"
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Competitive Equilibrium

Usual optimization + symmetry + Aggregate demand for capital equals K for $t \in \{0, 1\}$

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Definition

Given B_0, and a run probability, λ, a *symmetric competitive equilibrium* consists of $\left\{p_0, p_1, q_0, \hat{z}^F, \hat{z}^{Run}, d_1, d_2, V_1^R, V_1^D, b_1, k_1, b_2, k_2\right\}$ such that:

(a) Banks choose portfolios and repayment optimally

(b) Investors break even

$$q_0(l_1) = (1 - \lambda) F(\hat{z}^F(l_1)) + \lambda F(\hat{z}^{Run}(l_1))$$

(d) The market for capital clears.

- Aggregate demand for capital equals K at $t = 0, 1$.

Equilibrium at $t = 1$

- Characterization in terms of **aggregate leverage** $L_1 = B_1/K$

- Share of banks defaulting is increasing in L_1:

$$\underbrace{\left[1 - F(\hat{z}^F(L_1 \mid p_1))\right]}_{\text{Fundamentals}} + \underbrace{\lambda\left[F(\hat{z}^F(L_1 \mid p_1)) - F(\hat{z}^{Run}(L_1 \mid p_1))\right]}_{\text{Runs}}$$

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$$K_2 = \frac{\beta}{(1+\beta)(p_1 - \phi/R)} N_1 \qquad .$$

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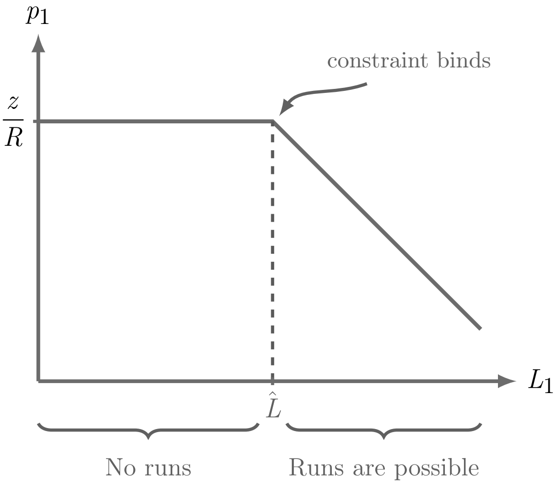
$$\mathcal{P}_1(L_1) \equiv \begin{cases} \frac{z}{R} & \text{if } L_1 \leq \hat{L}, \\ \beta z + (1+\beta)\frac{\phi}{R} - \beta R L_1 & \text{if } L_1 \in (\hat{L}, \overline{L}). \end{cases}$$

Equilibrium Asset Prices at $t = 1$



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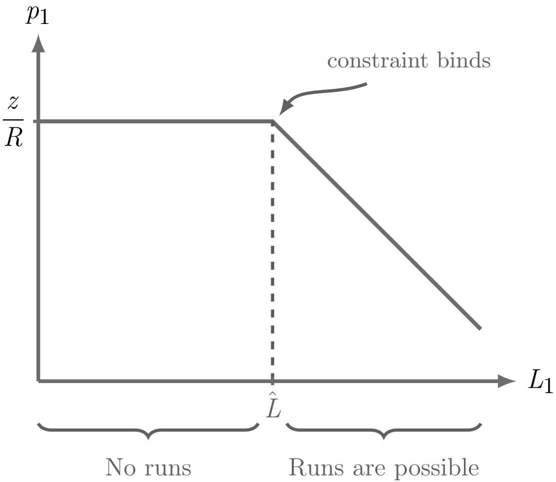

Price of capital p_1 is decreasing in L_1 when banks are constrained.

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Constrained-Efficiency

- Planner chooses L_1 and banks retain all other decisions
  - Market for capital clears competitively
  - Banks choose default decisions at $t = 1, 2$

Constrained-Efficiency

$$\max_{c_0 \geq 0, L_1, p_1} u(c_0) + \beta \Bigg[\underbrace{\Big((1-\lambda) F(\hat{z}^F(L_1|p_1)) + \lambda F(\hat{z}^{Run}(L_1|p_1)) \Big) V_1^R(n_1)}_{\text{Repayment}}$$

$$\underbrace{+ (1-\lambda) \int_{\hat{z}^F(L_1|p_1)}^{\overline{z}} V_1^D(k_1, \tilde{z}) \, dF(\tilde{z}) + \lambda \int_{\hat{z}^{Run}(L_1|p_1)}^{\overline{z}} V_1^D(k_1, \tilde{z}) \, dF(\tilde{z})}_{\text{Default}} \Bigg].$$

subject to:

$$c_0 = zK - RB_0 + q_0(L_1|p_1) L_1 K,$$

and where:

GE effect

$$n_1 = (z + p_1) K - R L_1 K, \quad p_1 = \mathcal{P}_1(L_1)$$

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GE effect

Creditors remain indifferent

Analysis without Runs

Proposition (Constrained-efficiency)

Suppose $\lambda = 0$. *Any competitive equilibrium is constrained efficient.*

Preliminary Lemma

Lemma: Consider any aggregate leverage L_1 and its associated price $p_1 = \mathcal{P}_1(L_1)$

- Take a bank with $l_1 = L_1$: the value goes up if the price deviates from eqm. one

(i) $V_1^R((z + p_1)K - RKL_1|p_1) \leq V_1^R((z + \hat{p}_1)K - RKL_1|\hat{p}_1)$;

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Proof of Constrained-Efficiency with $\lambda = 0$

Let L^E and L^P be the compet. eqm. and planner's leverage

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$\Rightarrow$ Banks can achieve weakly higher utility than planner.

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But planner can also choose L^E.

$\Rightarrow$ L^E must solve the planner's problem

Uniqueness and Existence

Proposition (Uniqueness)

Suppose that: (i) there is a unique solution to the planner problem, or (ii) there exists a competitive equilibrium with leverage $L_1 = B_1/K > \hat{L}$.

Then, there is at most one (symmetric pure-strategy) competitive equilibrium.

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Proposition (Existence)

Suppose that Assumption 2 holds and

i) f *is continuous and such that* $f(\underline{z}) = f(\overline{z}) = 0$.

ii) $\left[\frac{1-F(z)}{1+\beta} + \frac{f(z)}{F(z)} z\right]$ *is decreasing in* z *for any* $z \in [\underline{z}, \overline{z}]$.

Then, there exists a competitive equilibrium.

Taking stock so far

- Absent runs, competitive equilibria are constrained efficient [Benchmark result]
  - No need for minimum capital requirements

Economy with runs $\lambda > 0$

Thresholds as a Function of Aggregate Leverage

Consider a reduction in $L_1 \Rightarrow \hat{p}_1 = \mathcal{P}(L_1 - \Delta) > \mathcal{P}(L_1) = p_1$

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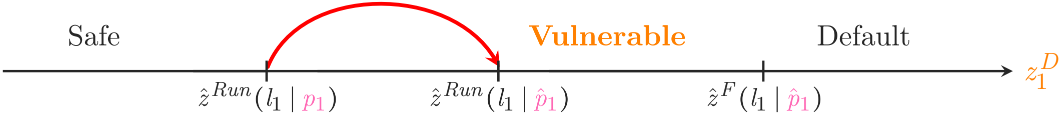
- Zero *first-order* effects on $\hat{z}^F$
- Increase in $\hat{z}^{Run}$: $\dfrac{\partial \hat{z}^{Run}}{\partial p_1} = \hat{z}^{Run} \dfrac{(1+\beta)\phi}{R(z + p_1 - RL_1)p_1} > 0$ $\Leftarrow$ banks are net sellers in a run



Over-leverage with $\lambda > 0$: Individual Euler Equation

$$\frac{1}{c_0} - \frac{\beta R}{c_1} = -\frac{(1-\lambda) f(\hat{z}^F)\frac{\partial \hat{z}^F}{\partial l_1} + \lambda f(\hat{z}^{Run})\frac{\partial \hat{z}^{Run}}{\partial l_1}}{q_0}\frac{l_1}{c_0}$$

$$- \frac{\lambda f(\hat{z}^{Run})\frac{\partial \hat{z}^{Run}}{\partial l_1}}{q_0}\frac{\beta}{K}\left[V_1^R(n_1 \mid p_1) - V_1^D(K, \hat{z}^{Run})\right].$$

Higher L_1 lowers q_0 and raises the probability of runs. These effects are internalized by individual banks.

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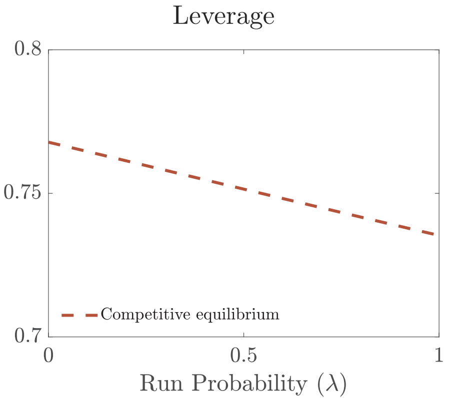
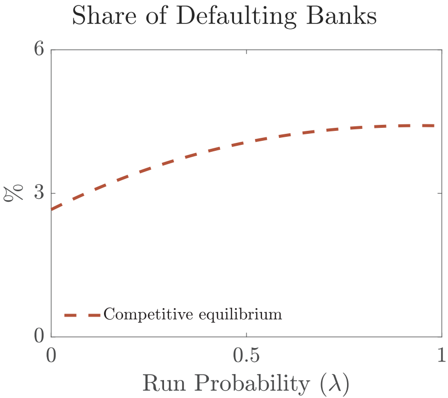
In the paper:

- Extension with outside option V^D, a function of prices [details]
- Productivity shocks to repayment z [details]

Competitive Eqm. vs. Constrained Efficient





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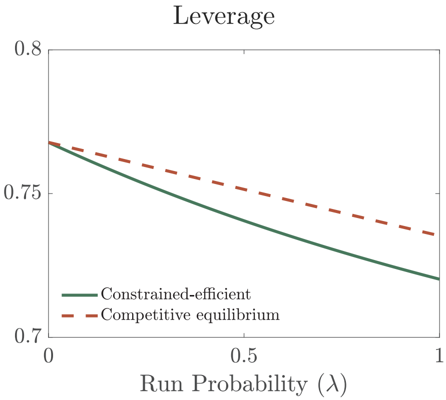

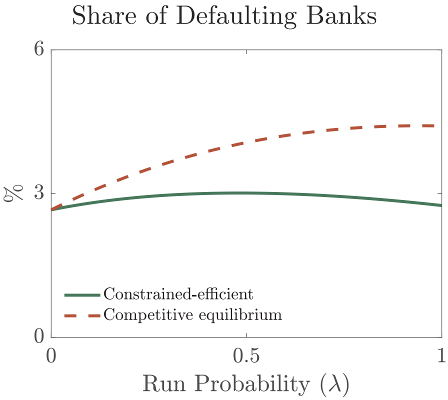

Some Connections

- Bank runs: Diamond and Dybvig (1983); Ennis and Keister 2009; Diamond and Rajan 2012; Allen and Gale 2000; Gertler and Kiyotaki 2012; Kashyap et al. 2024, etc.

  Here: self-fulfilling runs on individual banks in **general equilibrium.**

  - Scope for macroprudential policy from GE effects on asset prices

- Macroprudential policies: Caballero and Krishnamurthy 2003; Lorenzoni 2007; Bianchi 2011; Stein 2012; Davila and Korinek 2018; He and Kondor 2016; etc.

  Here: inefficiency without fire-sales, redistributive effects, or collateral constraints.

  - Bank runs + equilibrium default

Conclusions

- Foundations for macroprudential banking regulation under self-fulfilling runs
- Individual banks do not internalize that by raising leverage, they make other banks more vulnerable to runs
- Capital requirements are desirable even absent bailouts

EXTRAS

Endogenous V^D BACK

$$
\begin{aligned}
0 = {} & \frac{1}{c_0}\left[q_0(L_1 \mid \mathcal{P}_1(L_1)) + \left(\frac{\partial q_0(L_1 \mid p_1)}{\partial L_1} + \frac{\partial q_0(L_1 \mid p_1)}{\partial p_1} \mathcal{P}_1'(L_1) \right) L_1 \right] K \\
& + \beta\lambda \left[V_1^R(L_1, p_1) - V_1^D(K, \hat{z}^{Run} \mid p_1) \right] f(\hat{z}^{Run}) \left[\frac{\partial \hat{z}^{Run}}{\partial L_1} + \frac{\partial \hat{z}^{Run}}{\partial p_1} \mathcal{P}_1'(L_1) \right] \\
& + \beta \left[\lambda F(\hat{z}^{Run}) + (1-\lambda) F(\hat{z}^F) \right] \frac{\partial V_1^R(L_1, p_1)}{\partial L_1} \\
& + \beta \mathcal{P}_1'(L_1) \left[q_0(L_1 \mid \mathcal{P}_1(L_1)) \frac{\partial V_1^R(L_1, p_1)}{\partial p_1} \right. \\
& \qquad \left. + \lambda \int_{\hat{z}^{Run}}^{\hat{z}^F} \frac{\partial V_1^D(K, z_1^D \mid p_1)}{\partial p_1} \, dF(z_1^D) + \int_{\hat{z}^F}^{\bar{z}} \frac{\partial V_1^D(K, z_1^D \mid p_1)}{\partial p_1} \, dF(z_1^D) \right].
\end{aligned}
$$

Shock to Repayment Productivity BACK

$$\frac{1}{c_0}\left[q_0(L_1\mid p_1)-\left((1-\lambda)f\left(\hat z^F(L_1\mid p_1)\right)\frac{\partial \hat z^F(L_1\mid p_1)}{\partial L_1}+\lambda f\left(\hat z^{Run}(L_1\mid p_1)\right)\frac{\partial \hat z^{Run}(L_1\mid p_1)}{\partial L_1}\right)L_1\right]$$

$$=\beta R(1-\lambda)\int_{\hat z^F}^{\hat z^{Run}}\frac{1}{c_1(n_1)}\,\mathrm{d}F(z_1)+\beta R\int_{\hat z^{Run}}^{\overline{z}}\frac{1}{c_1(n_1)}\,\mathrm{d}F(z_1)$$

$$+\lambda\frac{\beta}{K}f(\hat z^{Run})\frac{\partial \hat z^{Run}}{\partial L_1}\left[V_1^R\big(n_1(\hat z^{Run})\big)-V_1^D(K)\right]$$

$$+\underbrace{\lambda f\left(\hat z^{Run}(L_1\mid p_1)\right)\frac{\partial \hat z^{Run}(L_1\mid p_1)}{\partial p_1}\mathcal{P}_1'(L_1)\left[\frac{L_1}{c_0}+\frac{\beta}{K}\left(V_1^R\big(n_1(\hat z^{Run})\big)-V_1^D(K)\right)\right]}_{\substack{\text{General eqm. effect}\\ \text{on run threshold}}}$$

$$+\underbrace{(1-\lambda)f\left(\hat z^F(L_1\mid p_1)\right)\frac{\partial \hat z^F(L_1\mid p_1)}{\partial p_1}\mathcal{P}_1'(L_1)\frac{L_1}{c_0}}_{\substack{\text{General eqm. effect}\\ \text{on fundamental threshold}}}$$