

The Prudential Use of Capital Controls and Foreign Currency Reserves

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Advances in Theory and Practice

- Capital flow management has become part of accepted toolbox
- Growing literature on second-best use of capital controls
 - Pecuniary and aggregate demand externalities
(Bianchi 2011; Schmitt-Grohé-Urbe 2016; Farhi-Werning 2016; etc)
- Focus on “prudential use”
- Similar developments for foreign currency reserves

This Paper

- Revisits literature using a unified framework
- A few themes:
 - Monetary policy dilemma for emerging economies
 - Pecuniary and aggregate demand externalities
 - Capital controls ex ante and ex post
 - Role of upward supply of funds
 - A reconciliation of two views on reserve accumulation
 - Controls on outflows as crisis management
- Also in the chapter:
 - Review of stylized facts, effectiveness, chronology of policies

Model Ingredients:

- Small open economy
- Tradable/Non-Tradable structure
- Incomplete financial markets
- Sticky wages
- Upward supply of funds from international investors
- Fear of floating

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Framework related to Basu, Boz, Gopinath, Roch, and Unsal (2020)

The Model

Preferences and Technology

- Representative consumer:

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t^T, c_t^N)$$

$$U(c_t^T, c_t^N) = \frac{1}{1-\sigma} \left(\phi^\rho (c_t^T)^{1-\rho} + (1-\phi)^\rho (c_t^N)^{1-\rho} \right)^{\frac{1-\sigma}{1-\rho}}$$

- ρ : inv-elasticity of subst. T-NT
- σ : inv-intertemp. elasticity

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 - Law of one price: $p_t^T = e_t p_t^*$ and $p_t^* = 1$

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- T Endowment: y_t^T
 - Law of one price: $p_t^T = e_t p_t^*$ and $p_t^* = 1$
- N output: $y_t^N = n_t$
 - Supply of hours \bar{n} (no labor disutility)

Consumers' Budget Constraint

- Position in pesos a_t (zero domestic net supply)
- Long position in dollars a_t^*
- Borrowing in dollars b_t^*

In eqm.: either $b_t^* = 0$ or $a_t^* = 0$

$$p_t^T c_t^T + p_t^N c_t^N + \frac{1}{1+i_t} a_{t+1} + \frac{1}{1+i_t^*} e_t a_{t+1}^* + e_t b_t^* \leq e_t y_t^T + w_t n_t + a_t + e_t a_t^* + \frac{1}{1+\hat{i}_t^*} e_t b_{t+1}^*$$

- Interest rates:
 - i_t set by central bank; i_t^* exogenous; \hat{i}_t^* endogenous

Firms and Nominal Rigidity

- Firms' problem: $\max_n p_t^N n - w_t n$

$$\Rightarrow p_t^N = w_t \text{ in equilibrium}$$

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- Downward nominal wage rigidity (Schmitt-Grohé and Uribe)
- Employment demand determined if market clearing wage above \bar{w}

$$w_t \geq \underline{w}, \quad n_t \leq \bar{n},$$

with at least one equality

Prelude: Equilibrium Employment

Household optimality

$$\frac{p_t^N}{e_t} = \left(\frac{1 - \phi}{\phi} \frac{c_t^T}{c_t^N} \right)^\rho$$

Market clearing

$$c_t^N = y_t^N$$

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Market clearing

$$c_t^N = y_t^N$$

Assuming binding wage rigidity and using $w_t = p_t$

$$n_t = \frac{1 - \phi}{\phi} \left(\frac{w}{e_t} \right)^{-\frac{1}{\rho}} c_t^T$$

Employment increasing in e and c^T

Supply of Funds

- International investors face cost $\frac{1}{\omega} \Phi(\cdot)$ of taking dollar position in the country
- Borrow at rate $1 + i^*$ and lend at rate $1 + \hat{i}^*$
- Profits at $t + 1$ from buying \hat{b}_{t+1}^* loans:

$$\hat{b}_{t+1}^* - \frac{1 + i_t^*}{1 + \hat{i}_t^*} \hat{b}_{t+1}^* - \frac{1}{\omega_t} \Phi \left(\frac{\hat{b}_{t+1}^*}{1 + \hat{i}_t^*} \right).$$

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- With quadratic Φ , optimality:

$$\frac{\hat{b}_{t+1}^*}{1 + \hat{i}_t^*} = \omega_t \left(\hat{i}_t^* - i_t^* \right)$$

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Study equilibrium where households borrow $\Rightarrow \hat{i}_t^* > i_t^*$

Definition of Equilibrium

A competitive equilibrium, given an initial asset position $\{b_{t-1}^*, a_{t-1}^*, a_{t-1}\}$ and a process for the nominal interest rate $\{i_t\}_{s=t-1}^{\infty}$, is a sequence of prices $\{p_{s-1}^N, e_{s-1}\}_{s=t}^{\infty}$, allocations $\{c_s^T, c_s^N, n_s\}_{s=t-1}^{\infty}$, bond positions for consumers $\{b_{s+1}^*, a_{s+1}^*, a_{s+1}\}_{s=t}^{\infty}$ and investors $\{\hat{b}_{s+1}^*\}_{s=t}^{\infty}$ such that:

- (i) Consumers, firms, and investors optimize
- (ii) Market clearing for non-tradables

$$c_s^N = y_s^N$$

and for domestic bonds and foreign bonds

$$a_s = 0, \quad b_s^* = \hat{b}_s^*$$

- (iii) Labor market: $n_t \leq \bar{n}$, $w_t \geq \underline{w}$, $(n_t - \bar{n})(w_t - \underline{w}) = 0$

Simplifying Assumptions

- Constant tradable endowment y^T
- Economy starts at $t - 1$ (with zero assets).
- Capital flight shock ω at time t
- From $t + 1, t + 2, \dots$ all prices flexible, $\omega_t = \infty$ and $i_t^* = \frac{1}{\beta} - 1$

Steady state equilibrium in period $t + 1$

- $c_{t+1}^N = c_{t+2}^N \dots = \bar{n}, \quad c_{t+1}^T = c_{t+2}^T \dots = y^T - (1 - \beta)b_{t+1}^*$

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Steady state equilibrium in period $t + 1$

- $c_{t+1}^N = c_{t+2}^N \dots = \bar{n}, \quad c_{t+1}^T = c_{t+2}^T \dots = y^T - (1 - \beta)b_{t+1}^*$
- Monetary policy neutral for $t + 1 \dots$
 - Assume exchange is kept constant for $t + 1 \dots$ at \bar{e}

Demand for Funds

- Intertemporal Euler

$$U_T(c_t^T, c_t^N) = \beta (1 + \hat{i}_t^*) U_T(c_{t+1}^T, \bar{n})$$

- Resource constraint for T

$$c_t^T = y^T + a_t^* - b_t^* + \frac{b_{t+1}^*}{1 + \hat{i}_t^*}, \quad c_{t+1}^T = y^T - (1 - \beta) b_{t+1}^*.$$

Demand for Funds

- Intertemporal Euler

$$U_T(c_t^T, c_t^N) = \beta (1 + \hat{i}_t^*) U_T(c_{t+1}^T, \bar{n})$$

- Resource constraint for T

$$(1 - \beta) (c_t^T - y^T - a_t^* + b_t^*) + \frac{1}{1 + \hat{i}_t^*} (c_{t+1}^T - y^T) = 0$$

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- Intertemporal Euler

$$U_T \left(c_t^T \right) = \beta \left(1 + \hat{i}_t^* \right) U_T \left(c_{t+1}^T \right)$$

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- Under separable prefs., borrowing demand simplifies.

Demand for Funds

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- Denoting demand for funds by $\mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*) = \frac{b_{t+1}^*}{1 + \hat{i}_t^*}$

Demand for Funds

- Intertemporal Euler

$$U_T \left(c_t^T \right) = \beta \left(1 + \hat{i}_t^* \right) U_T \left(c_{t+1}^T \right)$$

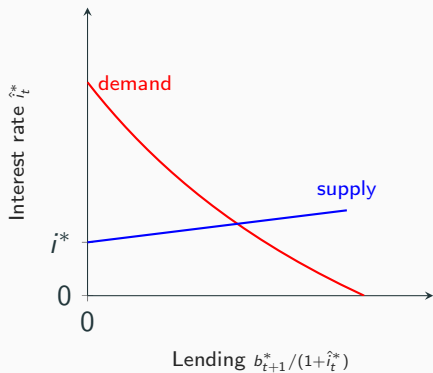
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- Denoting demand for funds by $\mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*) = \frac{b_{t+1}^*}{1 + \hat{i}_t^*}$

$$\mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*) = \frac{\left[1 - \left[\beta \left(1 + \hat{i}_t^* \right) \right]^{1/\sigma} \right] y^T - \left[\beta \left(1 + \hat{i}_t^* \right) \right]^{1/\sigma} (a_t^* - b_t^*)}{(1 - \beta) \left(1 + \hat{i}_t^* \right) + \left[\beta \left(1 + \hat{i}_t^* \right) \right]^{1/\sigma}}$$

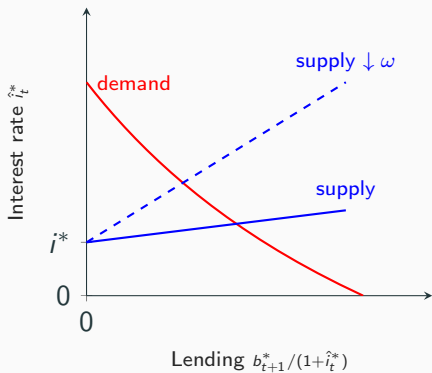
International Loan Market



$$\mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*) = \omega_t (\hat{i}_t^* - i_t^*)$$

International Loan Market

Capital Flight



$$\mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*) = \omega_t (\hat{i}_t^* - i_t^*)$$

Equilibrium in Period t

- Given monetary policy, system is block recursive in (\hat{i}_t^*, e_t, n_t)
 - Non-separable case ▶ General

$$\mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*) = \omega_t (\hat{i}_t^* - i_t^*) \quad \text{international loan market}$$

$$\frac{\bar{e}}{e_t} (1 + \hat{i}_t^*) = 1 + i_t \quad \text{domestic/internat. borrowing indifference}$$

$$n_t = \frac{1 - \phi}{\phi} \left(\frac{w_t}{e_t} \right)^{-\frac{1}{\rho}} \left[y^T + a_t^* - b_t^* + \mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*) \right] \quad \text{domestic demand}$$

Policy Analysis

Fear of Floating Ingredient

- Policy maker's objective:

$$(1 - \beta) \left(U \left(c_t^T, c_t^N \right) - \Psi \left(e_t \right) \right) + \beta U \left(c_{t+1}^T, c_{t+1}^N \right)$$

where $\Psi \left(e_t \right)$ is a convex function with a minimum at \bar{e}

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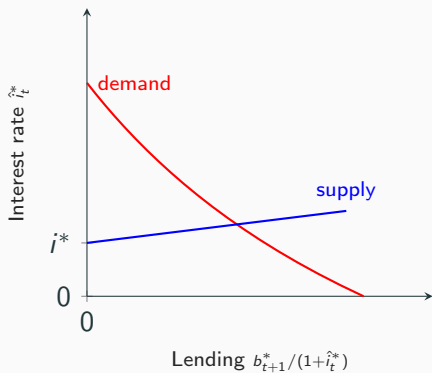
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- Possible microfoundations:
 - Balance sheet constraints
 - Reputation

Monetary Policy Dilemma

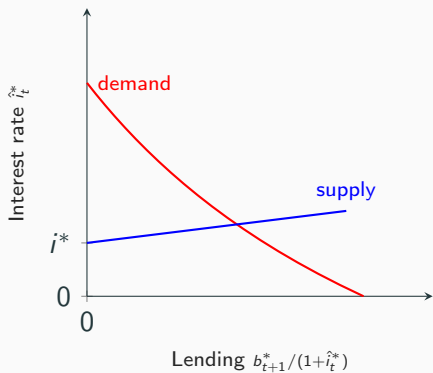
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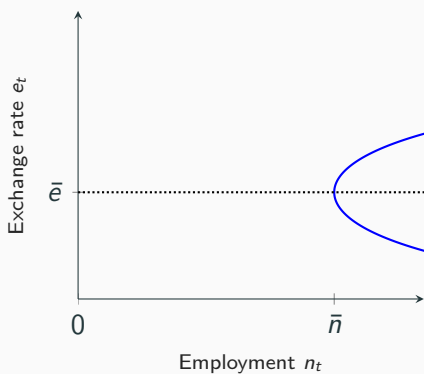
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Monetary Policy Dilemma

International loan market



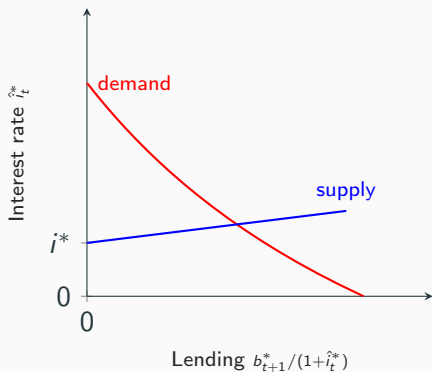
Domestic policy menu



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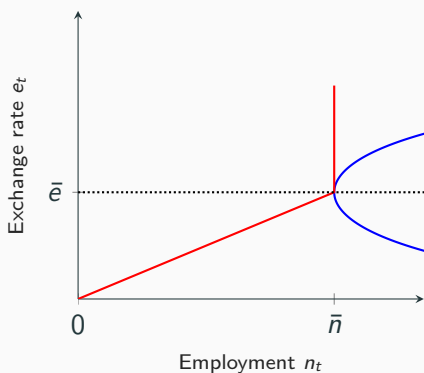
Monetary Policy Dilemma

International loan market



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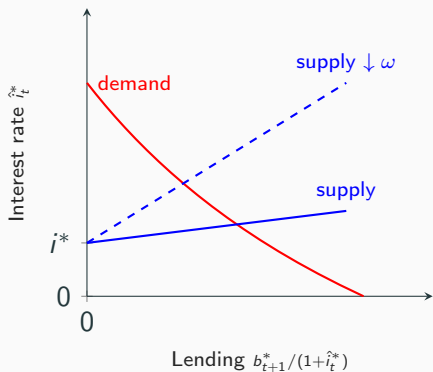
Domestic policy menu



$$n_t = \frac{1-\phi}{\phi} \left(\frac{w_t}{e_t} \right)^{-\frac{1}{\rho}} \left[y^T + a_t^* - b_t^* + \mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*) \right]$$

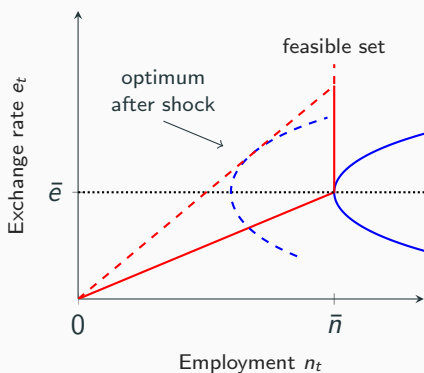
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Timeline



Timeline



Optimal ex-post policy

$$V_t(a_t^* - b_t^*, \omega_t) = \max_{\substack{c_t^T, c_{t+1}^T, c_t^N, \hat{i}_t^*, e_t \\ w_t, \tau_t}} (1 - \beta) U(c_t^T, c_t^N) - \Psi(e_t) + \beta U(c_{t+1}^T, \bar{n})$$

$$\text{s.t.} \quad (1 - \beta)(c_t^T - y^T - a_t^* + b_t^*) \leq \frac{1}{1 + \hat{i}_t^*} (y^T - c_{t+1}^T)$$

$$c_t^T - y^T - a_t^* + b_t^* \leq \omega_t (\hat{i}_t^* - i_t^*)$$

$$c_t^N \leq \frac{1 - \phi}{\phi} \left(\frac{w_t}{e_t} \right)^{-\frac{1}{\rho}} c_t^T$$

$$c_t^N \leq \bar{n}, \quad w_t \geq \underline{w}$$

$$U_T(c_t^T, c_t^N) = \beta(1 - \tau_t)(1 + i_t^*) U_T(c_{t+1}^T, \bar{n})$$

Optimal ex-post policy

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$$c_t^N \leq \bar{n}, \quad w_t \geq \underline{w}$$

Optimal ex-post policy

Assume wage rigidity binds

$$V_t(a_t^* - b_t^*, \omega_t) = \max_{c_t^T, c_{t+1}^T, c_t^N, \hat{i}_t^*, e_t} (1 - \beta) U(c_t^T, c_t^N) - \Psi(e_t) + \beta U(c_{t+1}^T, \bar{n})$$

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$$c_t^T - y^T - a_t^* + b_t^* \leq \omega_t (\hat{i}_t^* - i_t^*) \quad [\mu_t]$$

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Optimal ex-post policy

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First-order conditions:

$$(1 - \beta) U_T(c_t^T, c_t^N) + \nu_t \frac{c_t^N}{c_t^T} = (1 - \beta) \lambda_t + \mu_t$$

Optimal ex-post policy

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$$\lambda_t = \beta(1 + \hat{i}_t^*) U_T(c_{t+1}^T, \bar{n}), \quad \nu_t = (1 - \beta) U_N(c_t^T, c_t^N), \quad \mu_t = -\frac{\lambda_t}{\omega_t} \frac{c_{t+1}^T - y^T}{(1 + \hat{i}_t^*)^2}$$

Optimal ex-post policy

$$V_t(a_t^* - b_t^*, \omega_t) = \max_{c_t^T, c_{t+1}^T, c_t^N, \hat{i}_t^*, e_t} (1 - \beta) U(c_t^T, c_t^N) - \Psi(e_t) + \beta U(c_{t+1}^T, \bar{n})$$

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$$c_t^N \leq \frac{1 - \phi}{\phi} \left(\frac{w}{e_t} \right)^{-\frac{1}{\rho}} c_t^T \quad [\nu_t]$$

First-order conditions:

$$(1 - \beta) U_T(c_t^T, c_t^N) + \nu_t \frac{c_t^N}{c_t^T} = (1 - \beta) \lambda_t + \mu_t$$

$$\lambda_t = \beta(1 + \hat{i}_t^*) U_T(c_{t+1}^T, \bar{n}), \quad \nu_t = (1 - \beta) U_N(c_t^T, c_t^N), \quad \mu_t = -\frac{\lambda_t}{\omega_t} \frac{(1 - \beta) b_{t+1}^*}{(1 + \hat{i}_t^*)^2}$$

Do ex post capital controls help?

- Not quite...

$$U_T(c_t^T, c_t^N) + \underbrace{U_N(c_t^T, c_t^N) \frac{c_t^N}{c_t^T}}_{\text{AD Externality}} = \beta U_T(c_{t+1}^T, \bar{n}) \left(1 + \hat{i}_t^* + \underbrace{\frac{1}{\omega_t} \frac{b_{t+1}^*}{1 + \hat{i}_t^*}}_{\text{Borrowing prem. externality}} \right)$$

- Stimulating c^T raises employment but increases \hat{i}_t^*

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- Stimulating c^T raises employment but increases \hat{i}_t^*

Timeline



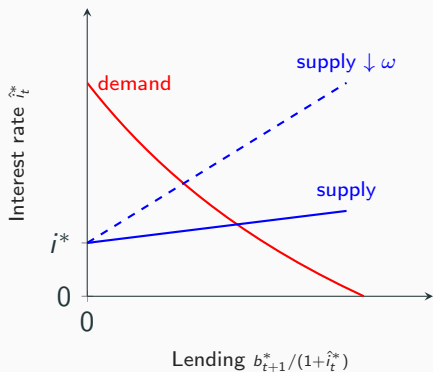
Timeline



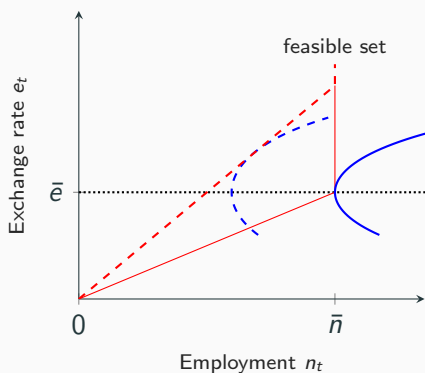
$$\Psi = 0$$

↑ NFA Ex Ante improves Policy Menu Ex Post

International loan market



Domestic policy menu

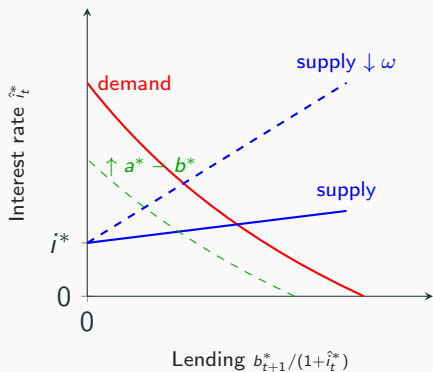


$$\mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*) = \omega_t (\hat{i}_t^* - i_t^*)$$

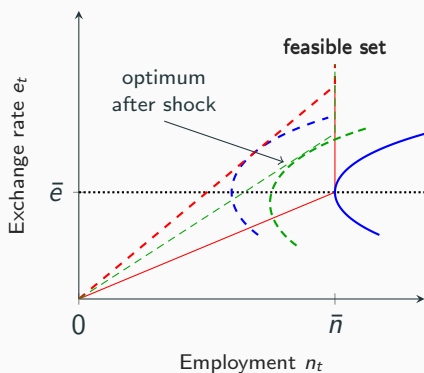
$$n_t = \frac{1-\phi}{\phi} \left(\frac{w_t}{e_t} \right)^{-\frac{1}{\rho}} \left[y^T + a_t^* - b_t^* + \mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*) \right]$$

↑ NFA Ex Ante improves Policy Menu Ex Post

International loan market



Domestic policy menu



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Optimal Ex Ante Policy

$$\max_{c_{t-1}^T, c_{t-1}^N, \hat{i}_{t-1}^*, a_t^*, b_t^*, e_{t-1}, w_{t-1}, \tau_{t-1}} (1-\beta)U(c_{t-1}^T, c_{t-1}^N) + \beta \mathbf{E}_{t-1} [V_t(a_t^* - b_t^*, \omega_t)],$$

$$\text{s.t. } c_{t-1}^T \leq y_{t-1}^T - \frac{1}{1+i_{t-1}^*} a_t^* + \frac{1}{1+\hat{i}_{t-1}^*} b_t^*$$

$$c_{t-1}^T - y_{t-1}^T \leq \omega_{t-1} (\hat{i}_t^* - i_t^*)$$

$$c_{t-1}^N = \frac{1-\phi}{\phi} \left(\frac{w_{t-1}}{e_{t-1}} \right)^{-\frac{1}{\rho}} c_{t-1}^T$$

$$c_{t-1}^N \leq \bar{n} \quad w_{t-1} \geq \underline{w}$$

$$U_T(c_{t-1}^T, c_{t-1}^N) = \beta(1 - \tau_{t-1})(1 + i_{t-1}^*) \mathbf{E}_{t-1} U_T(c_t^T, n_t)$$

Optimal Ex Ante Policy

$$\max_{c_{t-1}^T, c_{t-1}^N, \hat{i}_{t-1}^*, a_t^*, b_t^*, e_{t-1}, w_{t-1}} (1-\beta)U(c_{t-1}^T, c_{t-1}^N) + \beta \mathbf{E}_{t-1} [V_t(a_t^* - b_t^*, \omega_t)],$$

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First-order conditions:

$$(1-\beta)U_T(c_{t-1}^T, \bar{n}) + \mu_{t-1} = \lambda_{t-1}, \quad \frac{\lambda_{t-1}}{1+\hat{i}_{t-1}^*} = V_{a^*}(a_t^* - b_t^*, \omega_t)$$

Optimal Ex Ante Policy

$$\begin{aligned} \max_{c_{t-1}^T, c_{t-1}^N, \hat{i}_{t-1}^*, a_t^*, b_t^*, e_{t-1}} \quad & (1-\beta)U(c_{t-1}^T, \bar{n}) + \beta \mathbf{E}_{t-1} [V_t(a_t^* - b_t^*, \omega_t)], \\ \text{s.t.} \quad & c_{t-1}^T \leq y_{t-1}^T - \frac{1}{1+i_{t-1}^*} a_t^* + \frac{1}{1+\hat{i}_{t-1}^*} b_t^* \\ & c_{t-1}^T - y_{t-1}^T \leq \omega_{t-1} (\hat{i}_t^* - i_t^*) \end{aligned}$$

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Envelope conditions (from t): ▶ Problem period t

$$V_{a^*}(a_t^* - b_t^*, \omega_t) = (1-\beta) \left(U_T(c_t^T, c_t^N) + U_N(c_t^T, c_t^N) \frac{c_t^N}{c_t^T} \right)$$

Socially optimal choice of b_t^*

- Benefits of lower b_t^* : lower borrowing costs + higher demand menu)

$$U_T(c_{t-1}^T, \bar{n}) = \beta \left(1 + \hat{i}_{t-1}^* + \frac{1}{\omega_{t-1}} \frac{b_t^*}{1 + \hat{i}_{t-1}^*} \right) \\ E_{t-1} \left[U_T(c_t^T, c_t^N) + \iota_{n_t < \bar{n}} U_N(c_t^T, c_t^N) \frac{c_t^N}{c_t^T} \right]$$

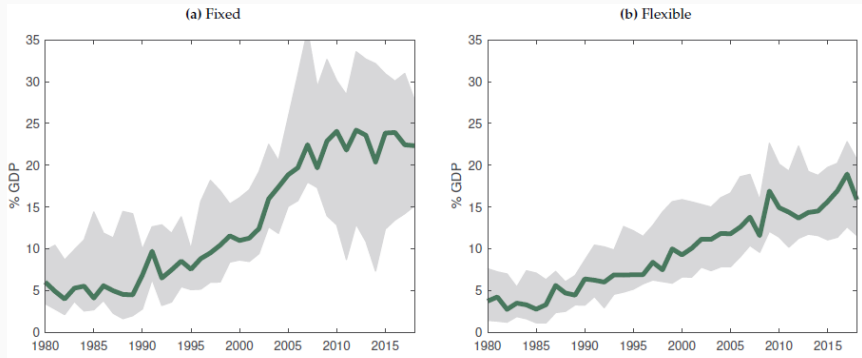
- Ex ante both externalities go in same direction

Connections

- Aggregate demand externalities with fixed exchange rates
 - Schmitt-Grohé and Uribe (2016); Farhi and Werning (2016); etc
- Pecuniary externalities and sudden stops
 - Caballero and Krishnamurthy (2003); Bianchi (2011); Korinek (2018); Bianchi and Mendoza (2018); Jeanne and Korinek (2019); etc
- Reduction in borrowing costs when intermediaries are constrained
 - Amador, Bianchi, Bocola and Perri (2018); Fanelli and Straub (2021); Basu, Boz, Gopinath, Roch, and Unsal (2020); etc
- Weaker case for ex-post policies under upward supply of funds
 - Bianchi, Ottonello and Presno (2019)

Foreign Currency Reserve Interventions

Reserve Accumulation and Exchange Rate Regimes



► Data Details

Ilzetzi, Reinhart, and Rogoff (2019); Bianchi and Sosa Padilla (2020)

Two Views of Reserve Accumulation

- **Precautionary view:** protect spending in a crisis
- **Exchange rate management view:** prevent large fluctuations in e_t

Two Views of Reserve Accumulation

- **Precautionary view:** protect spending in a crisis
- **Exchange rate management view:** prevent large fluctuations in e_t

Two sides of the same coin:

- More resources to spend, lower contracting in spending

$$U_T(c_{t-1}^T, \bar{n}) = \beta \left(1 + \hat{i}_{t-1}^* + \frac{1}{\omega_{t-1}} \frac{b_t^*}{1 + \hat{i}_{t-1}^*} \right) E_{t-1} \left[U_T(c_t^T, c_t^N) + \iota_{n_t < \bar{n}} U_N(c_t^T, c_t^N) \frac{c_t^N}{c_t^T} \right]$$

- Intervention prevents a large depreciation

$$U_T(c_{t-1}^T, \bar{n}) = \beta \left(1 + \hat{i}_{t-1}^* + \frac{1}{\omega_{t-1}} \frac{b_t^*}{1 + \hat{i}_{t-1}^*} \right) E_{t-1} \left[U_T(c_t^T, c_t^N) + \Psi'(e_t) \rho \frac{e_t}{c_t^T} \right]$$

FX Interventions

- Government purchases foreign reserves A_t^* at $t - 1$
- Financed with lump-sum taxes for simplicity (wlog)
- Questions:
 - Does this work to increase the country's NFA position?
 - What are the costs and benefits?

FX Interventions (ctd)

- Country budget constraint

$$\frac{1}{1 + i_{t-1}^*} (A_t^* + a_t^*) - \frac{1}{1 + \hat{i}_{t-1}^*} b_t^* + c_{t-1}^T = y_{t-1}^T.$$

- Assume initially households choose $a_t^* = 0$
- Keeping c_{t-1}^T requires higher b^* , but increase in \hat{i}^* discourages households from borrowing
- Increase in A^* leads to higher $A^* - b^*$ but less than 1:1

▶ figure

FX Interventions (ctd)

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▶ figure

FX Interventions (ctd)

- Country budget constraint

$$\frac{1}{1 + i_{t-1}^*} (A_t^* + \dots) - \frac{1}{1 + \hat{i}_{t-1}^*} b_t^* + c_{t-1}^T = y_{t-1}^T.$$

- Assume initially households choose $a_t^* = 0$
- Keeping c_{t-1}^T requires higher b^* , but increase in \hat{i}^* discourages households from borrowing
- Increase in A^* leads to higher $A^* - b^*$ but less than 1:1

▶ figure

Optimal FX Policy

$$\max_{e_{t-1}, c_{t-1}^T, b_t^*, A_t^*} (1 - \beta) U(c_{t-1}^T, c_{t-1}^N) + \beta \mathbf{E}_{t-1} [V_t(A_t^* - b_t^*, \omega_t)],$$

$$\text{s.t. } c_{t-1}^T \leq y_{t-1}^T - \frac{1}{1 + i_{t-1}^*} A_t^* + \frac{1}{1 + \hat{i}_{t-1}^*} b_t^*$$

$$c_{t-1}^T + \frac{A_t^*}{1 + i_{t-1}^*} - y_{t-1}^T \leq \omega_t (\hat{i}_t^* - i_t^*)$$

$$c_{t-1}^N = \frac{1 - \phi}{\phi} \left(\frac{w_{t-1}}{e_{t-1}} \right)^{-\frac{1}{\rho}} c_{t-1}^T$$

$$c_{t-1}^N \leq \bar{n} \quad w_{t-1} \geq \underline{w}$$

$$U_T(c_{t-1}^T, c_{t-1}^N) = \beta (1 + \hat{i}_{t-1}^*) U_T(c_t^T, n_t)$$

Optimal FX Policy

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$$U_T(c_{t-1}^T, c_{t-1}^N) = \beta(1 + \hat{i}_{t-1}^*)U_T(c_t^T, n_t)$$

Capital Controls or FX Interventions?

- Here: capital controls \succ FX interventions
- Both achieve increase in NFA, but FX carry losses
 - Equivalent if no losses (Arce, Bengui and Bianchi 2020)
- In practice, other costs from capital controls too
 - Circumvention: Bengui and Bianchi 2019

- Mechanics of FX interventions:
 - Gabaix and Maggiori (2015); Amador, Bianchi, Bocola and Perri (2020); Fanelli and Straub (2021)
- Prudential aspects:
 - Arce, Bengui and Bianchi (2019); Devereux, Davis and Yu (2020); Kim and Zhang (2020)
- Aggregate demand stabilization:
 - Bianchi and Sosa-Padilla (2020)

Controls on outflows

- So far, capital controls respect contractual returns

Controls on outflows

- So far, capital controls respect contractual returns
- A policy preventing investors from repatriating funds:

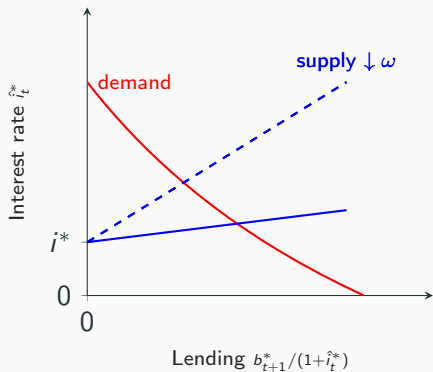
$$\max_{\{b_{t+1}^*\}} \sum_{t=0}^{\infty} \frac{1}{(1+i_t^*)^{t+1}} \left[b_{t+1}^* - \frac{1+i_t^*}{1+\hat{i}_t^*} b_{t+1}^* - \frac{1}{\omega_t} \Phi \left(\frac{b_{t+1}^*}{1+\hat{i}_t^*} \right) \right],$$

subject to

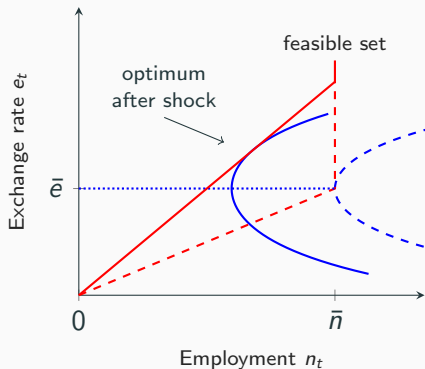
$$b_{t+1} \geq \Lambda_t b_t$$

Mechanics of Controls on Outflows

International loan market

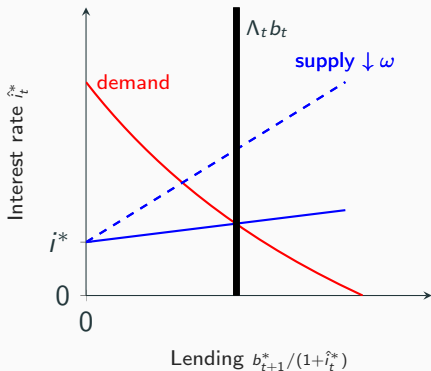


Domestic policy menu

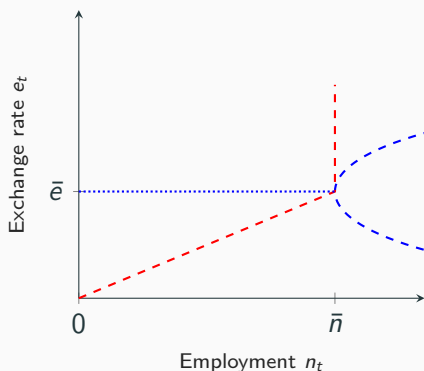


Mechanics of Controls on Outflows

International loan market



Domestic policy menu



- More similar to Malaysia in 1997 and Iceland 2011
- Costly ex ante once policy is anticipated

Balance-sheet effects and pecuniary externalities

Balance-Sheet Effects

- So far, scope for prudential policies due to rigidities in prices and monetary policy
- Similar macroeconomic externality with balance sheet constraints

$$\frac{b_{t+1}^*}{1 + i_t^*} \leq \kappa_t (y_t^T + \frac{p_t^N}{e_t} y^N)$$

- Lower demand when credit constraint binds reduces p^N and further tightens constraint

Optimal Ex Post Policy

Assume collateral constraint binds at time t (and also $\Phi = \Psi = 0$)

$$V_t(a_t^* - b_t^*, \kappa_t) = \max_{c_t^T, c_t^N \leq \bar{n}, e_t, w_t} (1 - \beta) U(c_t^T, c_t^N) + \beta U(c_{t+1}^T, \bar{n})$$

$$\text{s.t. } (1 - \beta) (c_t^T - y_t^T - a_t^* + b_t^*) = \frac{1}{1 + i_t^*} (y^T - c_{t+1}^T),$$

$$n_t = \frac{1 - \phi}{\phi} \left(\frac{w_t}{e_t} \right)^{-\frac{1}{\rho}} c_t^T,$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \kappa \left(y^T + \frac{w_t}{e_t} n_t \right) [\mu_t]$$

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- **Depreciating may be costly \Rightarrow endogenous Ψ**

- Farhi-Werning (2016); Ottonello (2021); Coulibaly (2021); Basu et al. (2021); Bianchi and Coulibaly (2022)

Optimal Ex Post Policy

Assume collateral constraint binds at time t (and also $\Phi = \Psi = 0$)

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$$c_t^T - y_t^T - a_t^* + b_t^* = \kappa \left(y^T + \frac{w_t}{e_t} n_t \right) [\mu_t]$$

- Higher c^T ex post raises borrowing capacity \Rightarrow **ex ante macropru:**

$$U_T(c_{t-1}^T, c_{t-1}^N) = (1 + i_{t-1}^*) \beta \mathbf{E}_{t-1} \left[U_T(c_t^T, c_t^N) + \kappa \frac{p_t^N c_t^N}{c_t^T} \rho \mu_t \right]$$

- Bianchi (2011)

Conclusions

- Growing literature on prudential capital flow management policies
- Pecuniary and aggregate demand externalities will continue to be playing a central role
- Many open questions
 - Better understanding of the source of fear of floating
 - Sources of capital flights (credit demand/supply, liquidity/solvency)
 - Interactions of externalities and combination of policy tools (IPF)
 - International coordination
 - Empirical-theoretical connections
 - Quantitative models and implementation

Extra Slides

Capital Controls and Reserves-to-GDP Ratios

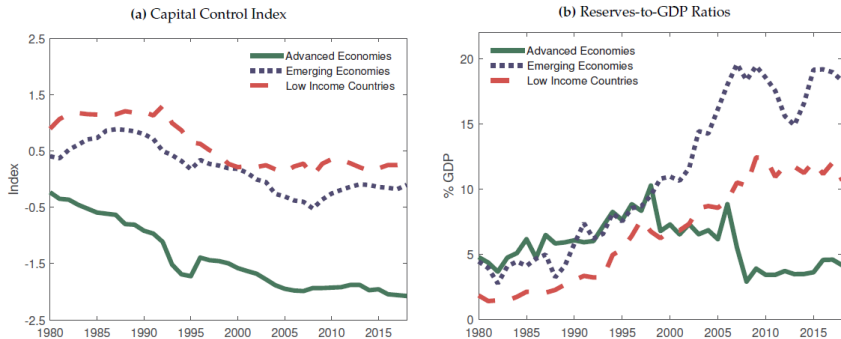


Figure 1: Capital Controls and Reserve Accumulation

Trends in International Reserves

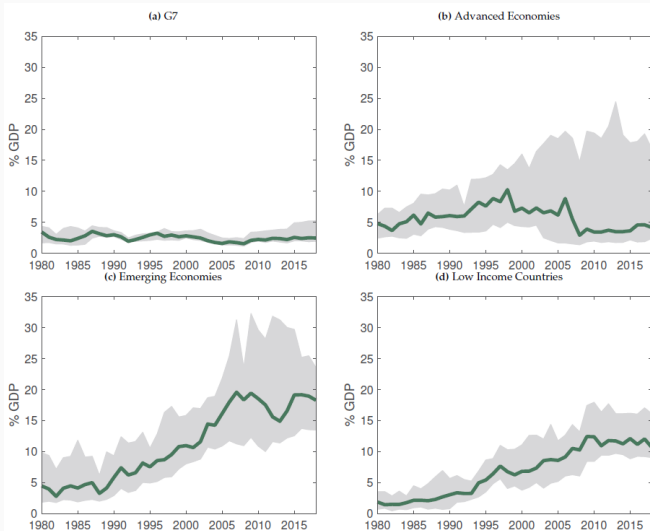


Figure 2: Trends in International Reserves

4 Equations in $(c_t^T, \hat{i}_t^*, n_t, e_t)$: General Case

- Equilibrium for period t [▶ Back](#)

$$U_T(c_t^T, n_t) = \beta(1+\hat{i}^*)U_T(y^T - (1+\hat{i}^*)(c^T - y^T - a_t^* + b_t^*), \bar{n})$$

$$c_t^T - y^T - a_t^* + b_t^* = \omega(\hat{i}_t^* - i^*)$$

$$\frac{e_{t+1}}{e_t} (1 + \hat{i}_t^*) = 1 + i_t$$

$$n_t = \frac{1-\phi}{\phi} \left(\frac{w_t}{e_t} \right)^{-\frac{1}{\rho}} c_t^T$$

Optimal ex-post policy

$$V(a_t^* - b_t^*, \omega_t) = \max_{c_t^T, c_{t+1}^T, c_t^N, e_t, \hat{i}_t^*} (1 - \beta) \left(U(c_t^T, c_t^N) - \Psi(e_t) \right) + \beta U(c_{t+1}^T, \bar{n})$$

$$\text{s.t.} \quad (1 - \beta) \left(c_t^T - y_t^T - a_t^* + b_t^* \right) = \frac{1}{1 + \hat{i}_t^*} \left(y^T - c_{t+1}^T \right),$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \omega_t \left(\hat{i}_t^* - i_t^* \right),$$

$$\frac{c_t^N}{c_t^T} = \frac{1 - \phi}{\phi} \left(\frac{w_t}{e_t} \right)^{-\frac{1}{\rho}},$$

Optimal ex-post policy

$$V(a_t^* - b_t^*, \omega_t) = \max_{c_t^T, c_{t+1}^T, c_t^N, e_t, \hat{i}_t^*} (1 - \beta) \left(U(c_t^T, c_t^N) - \Psi(e_t) \right) + \beta U(c_{t+1}^T, \bar{n})$$

$$\text{s.t.} \quad (1 - \beta) \left(c_t^T - y_t^T - a_t^* + b_t^* \right) = \frac{1}{1 + \hat{i}_t^*} \left(y_t^T - c_{t+1}^T \right),$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \omega_t \left(\hat{i}_t^* - i_t^* \right),$$

$$\frac{c_t^N}{c_t^T} = \frac{1 - \phi}{\phi} \left(\frac{w_t}{e_t} \right)^{-\frac{1}{\rho}},$$

Envelope and optimality:

$$V_{a^*} (a_t^* - b_t^*, \omega_t) = (1 - \beta) \lambda_t + \mu_t$$

Optimal ex-post policy

$$V(a_t^* - b_t^*, \omega_t) = \max_{c_t^T, c_{t+1}^T, c_t^N, e_t, \hat{i}_t^*} (1-\beta) \left(U(c_t^T, c_t^N) - \Psi(e_t) \right) + \beta U(c_{t+1}^T, \bar{n})$$

$$\text{s.t.} \quad (1-\beta) \left(c_t^T - y_t^T - a_t^* + b_t^* \right) = \frac{1}{1 + \hat{i}_t^*} \left(y^T - c_{t+1}^T \right),$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \omega_t \left(\hat{i}_t^* - i_t^* \right),$$

$$\frac{c_t^N}{c_t^T} = \frac{1-\phi}{\phi} \left(\frac{w_t}{e_t} \right)^{-\frac{1}{\rho}},$$

Envelope and optimality:

$$V_{a^*} (a_t^* - b_t^*, \omega_t) = (1-\beta) \lambda_t + \mu_t$$

$$(1-\beta) U_T (c_t^T, c_t^N) + \nu_t \frac{c_t^N}{c_t^T} = (1-\beta) \lambda_t + \mu_t, \quad \nu_t = (1-\beta) U_N (c_t^T, c_t^N)$$

Optimal ex-post policy

$$V(a_t^* - b_t^*, \omega_t) = \max_{c_t^T, c_{t+1}^T, c_t^N, e_t, \hat{i}_t^*} (1 - \beta) \left(U(c_t^T, c_t^N) - \psi(e_t) \right) + \beta U(c_{t+1}^T, \bar{n})$$

$$\text{s.t.} \quad (1 - \beta) \left(c_t^T - y_t^T - a_t^* + b_t^* \right) = \frac{1}{1 + \hat{i}_t^*} \left(y_t^T - c_{t+1}^T \right),$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \omega_t \left(\hat{i}_t^* - i_t^* \right),$$

$$\frac{c_t^N}{c_t^T} = \frac{1 - \phi}{\phi} \left(\frac{w_t}{e_t} \right)^{-\frac{1}{\rho}},$$

Envelope and optimality: [▶ Back](#)

$$V_{a^*} (a_t^* - b_t^*, \omega_t) = (1 - \beta) \left(U_T(c_t^T, c_t^N) + U_N(c_t^T, c_t^N) \right)$$

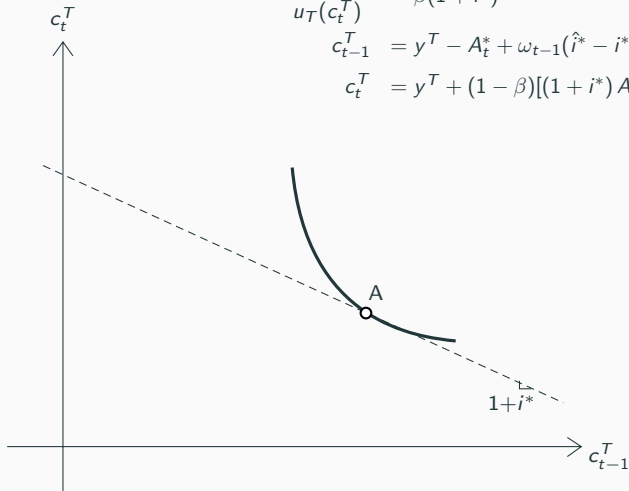
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$$\frac{u_T(c_{t-1}^T)}{u_T(c_t^T)} = \beta(1 + \hat{i}^*)$$

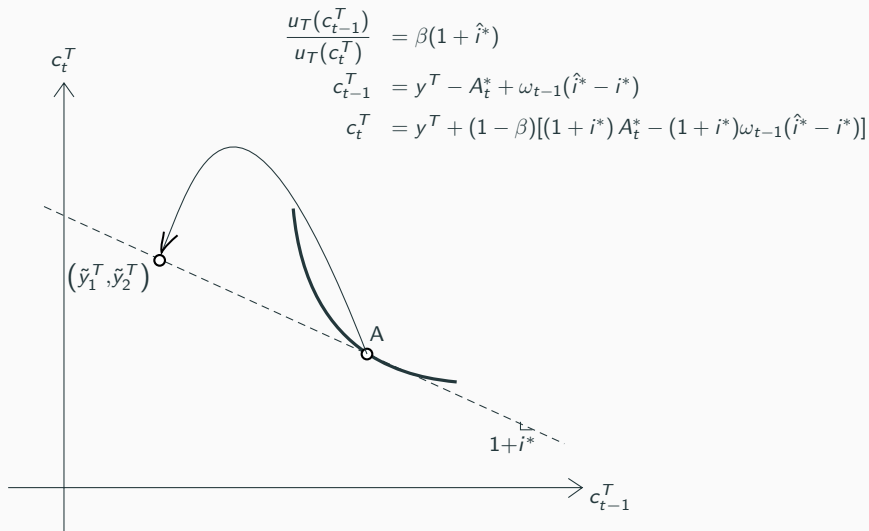
$$c_{t-1}^T = y^T - A_t^* + \omega_{t-1}(\hat{i}^* - i^*)$$

$$c_t^T = y^T + (1 - \beta)[(1 + i^*)A_t^* - (1 + i^*)\omega_{t-1}(\hat{i}^* - i^*)]$$



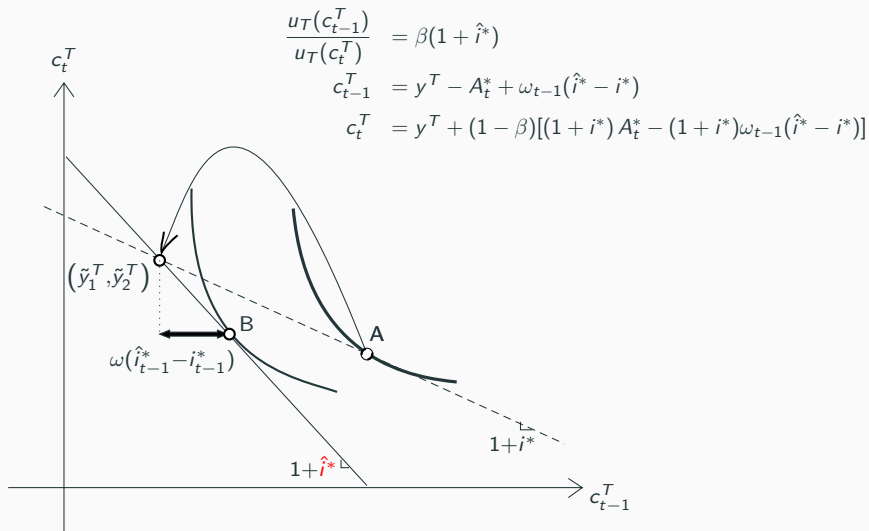
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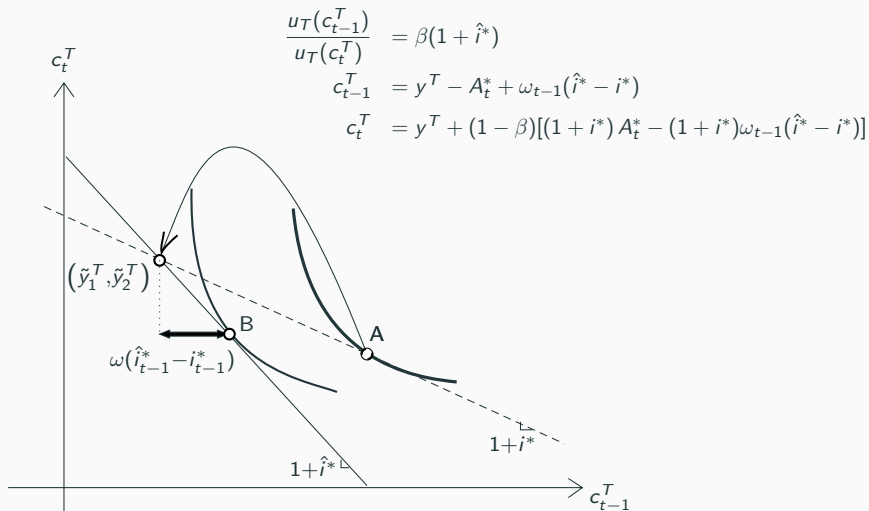
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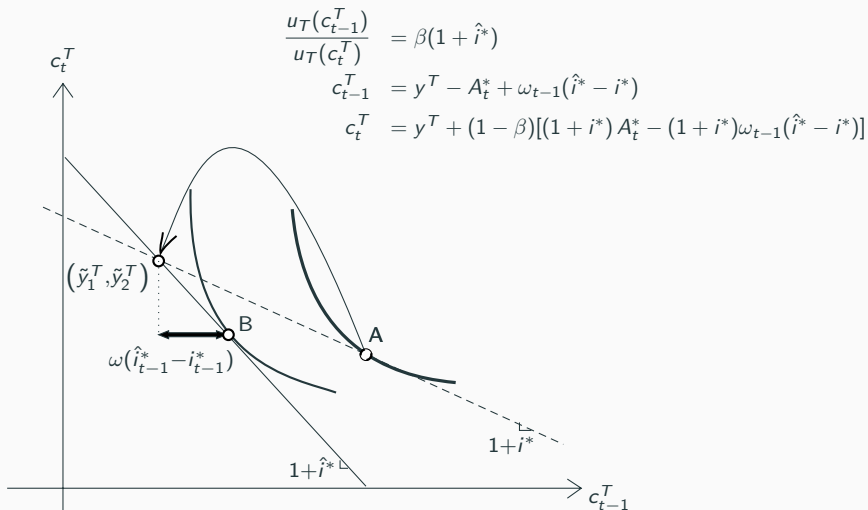


Amador; Bianchi; Bocola and Perri (2018)

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$\Delta c > 0$ desirable with agg. demand or pec. externalities



Monetary Policy Dilemma (ctd)

Proposition Assume $\sigma = \rho$ and suppose the nominal rate i_t is the only policy instrument and is chosen optimally at t . Suppose $a_t^* - b_t^*$ is such that consumers are borrowers. We have the following results:

- i) \hat{i}_t^* is decreasing in ω_t .
- ii) there is a cutoff $\hat{\omega}$ such that if $\omega_t < \hat{\omega}$ the equilibrium features unemployment $n_t < \bar{n}$ and a depreciated exchange rate $e_t > \bar{e}$.
- iii) For all $\omega < \hat{\omega}$, $dy^N/d\omega > 0$
- iii) For all $\omega < \hat{\omega}$, if $\rho > 1$, $de/d\omega < 0$